Special Relativity → Quantum Mechanics

The SRQM Interpretation of Quantum Mechanics

A Tensor Study of Physical 4-Vectors

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Using Special Relativity (SR) as a starting point, then noting a few empirical 4-Vector facts, i.e. classically measurable physical constants, one can instead *derive* the Principles that are normally considered to be the Axioms of Quantum Mechanics (QM).
Hence, [SR→QM] or [SRQM]

Since many of the QM Axioms are rather obscure, this approach is a far more logical and understandable paradigm than QM as a separate theory from SR, and sheds light on the physical origin, structure, and meaning of the QM Principles.

For instance, the properties of SR <Events>, encoded as components of 4-Vectors, ex. (time,position), (energy,momentum), etc., the things we measure, can be "quantized by the Metric", [time,energy], [position,momentum] while 4D SpaceTime & the GR Metric are not themselves "quantized", in agreement with all known experiments and observations to-date.

The SRQM or [SR→QM] Interpretation of Quantum Mechanics
A Tensor Study of Physical 4-Vectors

or: Why General Relativity (GR) is *NOT* wrong
or: Don't bet against Einstein ;)
or: QM, the easy way...

Recommended viewing:
via a .PDF Viewer/WebBrowser
with Fit-To-Page & Page-Up/Down

And yes,
I did the Math…
Ad Astra:Magnum Opus

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

(+,-,-,-) SR → QM Physics

4-Vector SRQM Interpretation of QM

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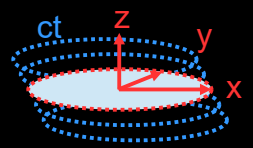
Tensors, especially { 4-Vectors = 4D (1,0)-Tensors }, are a fantastic language/tool for describing the physics of both relativistic and quantum phenomena. They easily show many interesting properties and relations of our Universe, and do so in a simple and concise mathematical way. Due to their tensorial nature, 4-Vectors are automatically **4D SpaceTime** coordinate-frame invariant, their components obey relativistic covariance, and these 4-Vectors can be used to generate *ALL* of the physical Lorentz Scalar 4D (0,0)-Tensors and higher-rank Tensors of Special Relativity (**SR**). Let me repeat: You can mathematically build *ALL* of the SR Lorentz Scalars and "larger" SR Tensors from empirically discovered SR 4-Vectors.

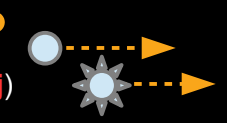
4D (Time·Space) SR 4-Vectors are likewise easily shown to be related to the standard physics 3D vectors { 3-vectors = 3D (1,0)-tensors } that are used in Newtonian Classical Mechanics (CM), Maxwellian Classical ElectroMagnetism (EM), and standard Quantum Mechanics (QM). In addition, each and every physical **SR 4-Vector** *also* fundamentally connects a special-relativistically-related temporal scalar to a **spatial 3-vector**:



Temporal time (t) & **Spatial** 3-position ($\mathbf{r}$)→(r^x, r^y, r^z) = (x, y, z) as **SR** 4-Position $\mathbf{R} = R^\mu = (ct,\mathbf{r})$

Temporal energy (E) & **Spatial** 3-momentum ($\mathbf{p}$)→(p^x, p^y, p^z) as **SR** 4-Momentum $\mathbf{P} = P^\mu = (E/c,\mathbf{p})$

Temporal charge-density (ρ) & **Spatial** 3-current-density ($\mathbf{j}$)→(j^x, j^y, j^z) as **SR** 4-CurrentDensity $\mathbf{J} = J^\mu = (\rho c,\mathbf{j})$



Why 4-Vectors and Tensors as opposed to some of the more abstract mathematical approaches to Quantum Mechanics?

Experiment is the ultimate arbiter of which theories actually correspond to reality:nature. If quantum-logics and string-theories give no testable/measurable predictions, and if invented/hypothetical/wished-for particles can't be detected, then they are basically useless for real, actual, empirical physics. The components of 4-Vectors *ARE* the physical properties of real particles that *CAN* actually be empirically detected/measured/tested, and Tensors are the well-known and well-established mathematical/physical objects which describe these concepts in an invariant, coordinate-independent way.

In this treatise, I will first ***extensively*** demonstrate how 4-Vectors are used in the context of Special Relativity (SR), and then show that their use in Relativistic Quantum Mechanics (RQM) and Quantum Mechanics (QM) is really not fundamentally different. Quantum Principles, without need of QM Axioms, then emerge in a natural and elegant way. SR is a theory of Measurement, even in QM.

I also introduce the SRQM (Physical/Scientific) Diagramming Method: a highly instructive, graphical charting/diagrammming-method, which visually shows how the SRQM 4-Vectors, Lorentz 4-Scalars, and higher rank 4-Tensors are all related to each other. This symbolic representation clarifies a lot of physics and is a great tool for teaching and understanding.

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
Many concepts herein inspired by works of SR/GR Expert Physicist Wolfgang Rindler

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Chapter Sections:

Mostly SR

Mostly QM

(+,-,-,-) SR → QM
Physics

A Tensor Study
of Physical 4-Vectors

SRQM

Some Physics:Mathematics Abbreviations & Notation

4-Vector SRQM Interpretation
of QM

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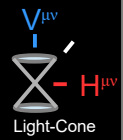
GR = General Relativity 4D = 4-Dimensional = {0,1,2,3}→{t,x,y,z}
SR = Special Relativity SR Metric-Convention →(+,-,-,-)
CM = Classical Mechanics SR Metric → Diag[+1,-1,-1,-1] {Cartesian Frame}
QM = Quantum Mechanics
RQM = Relativistic Quantum Mechanics
EM = ElectroMagnetism/ElectroMagnetics
NRQM = Non-Relativistic Quantum Mechanics = (standard QM)
QFT = Quantum Field Theory = (multiple particle QM)
QED = Quantum ElectroDynamics = QFT for (e^-)'s & photons
RWE/QWE = Relativistic/Quantum Wave Equation
KG = Klein-Gordon (Relativistic Quantum) Equation:Relation
PDE = Partial Differential Equation
MCRF = Momentarily Co-Moving Reference:Rest Frame
EoS = Equation of State (Scalar Invariant) = $w = p_o / \rho_{eo}$
$\mathbf{P}_T$ = 4-TotalMomentum = ($H/c=E_T/c,\mathbf{p}_T$) = Σ_n [All 4-Momenta $\mathbf{P}_n$]
H = The Hamiltonian = $\gamma(\mathbf{P}_T\cdot\mathbf{U})$ { "energy" used in advanced CM, (KE + PE) for |v| << c }
L = The Lagrangian = $-(\mathbf{P}_T\cdot\mathbf{U})/\gamma$ { "energy" used in advanced CM, (KE - PE) for |v| << c }
$\nabla = \nabla_r$ = 3-gradient (∇)→{rectangular basis} ($\partial_x,\partial_y,\partial_z$) = ($\partial/\partial x,\partial/\partial y,\partial/\partial z$)
$\partial^\mu = \partial/\partial R_\mu = \bar{\partial} = \partial = \partial_R$ = 4-Gradient (∂^μ) = ($\partial_t/c,-\nabla$), a (1,0)-Tensor
$\partial_\mu = \partial/\partial R^\mu = \underline{\partial}$ = Gradient One-Form (∂_μ) = ($\partial_t/c,\nabla$), a (0,1)-Tensor
S = S_{action} = The 4-Scalar Action (4-TotalMomentum $\mathbf{P}_T = -\partial[S]$)
Φ = Φ_{phase} = The 4-Scalar Phase (4-TotalWaveVector $\mathbf{K}_T = -\partial[\Phi]$)
Σ = Sum of Set = multiple (+) ; Π = Product of Set = multiple (×)
Δ = Difference ; d = Differential ; ∂ = Partial ={Calculus functions}
|v| << c: speed (v = |v|) much-less-than LightSpeed (c) approx.
$(1+x)^n \sim (1 + nx + O[x^2])$, |x| << 1: Classical-limit binomial approx.

$t_o = \tau$ = Proper Time (**Invariant** Rest Time) = t/γ : $t = \gamma t_o$ ←|*Time Dilation*|→
L_o = Proper Length (**Invariant** Rest Length) = γL : $L = L_o/\gamma$ →|*Length Contraction*|←
$\boldsymbol{\beta}$ = *Relativistic* Beta = $\mathbf{v}/c = \mathbf{u}/c = \{0..1\}\hat{\mathbf{n}}$; $\mathbf{v} = \mathbf{u} = c\boldsymbol{\beta}$ = 3-velocity = $\{0..c\}\hat{\mathbf{n}}$; v = u = |v|
γ = *Relativistic* Gamma = $\gamma_u = \gamma_\beta = 1/\sqrt{[1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}]} = 1/\sqrt{[1-|\boldsymbol{\beta}|^2]} = 1/\sqrt{[1-|\mathbf{u}/c|^2]} = dt/d\tau = \{1..\infty\}$
D = *Relativistic* Doppler = 1/[γ(1-|β|cos[θ])] ; can also be (D)imension, i.e. 4D,3D,1D
$\Lambda^{\mu'}{}_\nu$ = Lorentz (SpaceTime) Transform: prime (') specifies alternate reference frame RF, {boosts, rotations, reflections, identity}
$I_{(3)}$ = 3D Identity Matrix = Diag[1,1,1] : $I_{(4)}$ = 4D Identity Matrix = Diag[1,1,1,1]
$\delta^{ij} = \delta^i{}_j = \delta_{ij} = I_{(3)}$ = {1 if i=j, else 0} = Diag[1,1,1] 3D Kronecker delta
$\delta^{\mu\nu} = \delta^\mu{}_\nu = \delta_{\mu\nu} = I_{(4)}$ = {1 if μ=ν, else 0} = Diag[1,1,1,1] 4D Kronecker Delta (unique rank-2 isotropic tensor)
$\varepsilon^{ij}{}_k$ = {even:+1, odd:-1, else:0} 3D Levi-Civita anti-symmetric permutation (unique rank-3 isotropic)
$\varepsilon^{\mu\nu}{}_{\rho\sigma}$ = {even:+1, odd:-1, else:0} 4D Levi-Civita Anti-symmetric Permutation (one of a few...)
{other upper:lower index combinations possible for Levi-Civita symbol, but always anti-symmetric}

$\eta^{\mu\nu} \to \eta_{\mu\nu} \to \mathrm{Diag}[1,-I_{(3)}]_{rect} \leftarrow V^{\mu\nu} + H^{\mu\nu} = \eta^{\mu\nu}$ Minkowski:SR:"Flat" SpaceTime (+,-,-,-) Metric
$\eta^\mu{}_\nu = \delta^\mu{}_\nu = \mathrm{Diag}[1, I_{(3)}]_{any} = I_{(4)} = g^\mu{}_\nu$ {also true in GR} (1,1)-Tensor Identity Mixed-Metric
$V^{\mu\nu} = \|^{\mu\nu} = \underline{T}^\mu\underline{T}^\nu$ = Temporal "(V)ertical" Projection Tensor, also $V^\mu{}_\nu$ and $V_{\mu\nu}$
$H^{\mu\nu} = \perp^{\mu\nu} = \eta^{\mu\nu} - \underline{T}^\mu\underline{T}^\nu$ = **Spatial** "(H)orizontal" Projection Tensor, also $H^\mu{}_\nu$ and $H_{\mu\nu}$
$\underline{T}^\mu = \gamma(1,\boldsymbol{\beta})$ = 4-UnitTemporal = U^μ/c



Tensor-Index Notation & 4-Vector Notation:
$a^j = \mathbf{a} = (a^j) = (a^1,a^2,a^3) = (\mathbf{a})$: 3-vector [Latin Index {1,2,3} space-only]
$A^\mu = \mathbf{A} = (a^\mu) = (a^0,a^1,a^2,a^3) = (a^0,\mathbf{a})$: 4-Vector [Greek Index {0,1,2,3} Time·Space]
$A^\mu B_\mu = A_\nu B^\nu = \mathbf{A}\cdot\mathbf{B} = A^\mu\eta_{\mu\nu}B^\nu$: Einstein Sum : Dot Product : Inner Product
$A^\mu B^\nu = \mathbf{A}\otimes\mathbf{B}$: Tensor Product : Outer Product
$A^\mu B^\nu - A^\nu B^\mu = A^{[\mu}B^{\nu]} = \mathbf{A}\wedge\mathbf{B}$: Wedge Product : Exterior Product : Anti-Symmetric Product
$A^\mu B^\nu - A^\mu B^\nu = 0^{\mu\nu}$: (2,0)-Zero Tensor
$A^\mu B^\nu - B^\nu A^\mu = [A^\mu,B^\nu] = [\mathbf{A},\mathbf{B}]$: Commutation
$A^\mu B^\nu - B^\mu A^\nu$ = ???

Temporal object(+): blue, Spatial object(-): red
Mixed Time·Space object (<event>): purple
The mnemonic being blue and red mixed make purple
Null:Photonic:Light-like object(0): white
4D SpaceTime: I often write it as "Time·Space" just to match this ordering convention of 4-Vector (temporal, **spatial**) components

SRQM = The [SR→QM] Interpretation of Quantum Mechanics, by John B. Wilson

In full, with each a "validity-range" subset of the former: [GR→SR→RQM→QM→(EM & CM)]

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM

Some Physics:Mathematics Conventions & Notation

4-Vector SRQM Interpretation of QM

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4-Tensor $T^{\mu\nu}$: 4-Vector V^{μ} : 4-Scalar S — Conventions & Notation

Time·Space 4-Vector Name matches its **spatial** 3-vector component name
SR 4-Vector = (temporal 3-scalar, **spatial 3-vector**)
4-Vectors (4D) in **bold** UPPERCASE: ex. **A**
3-vectors (3D) in **bold** lowercase: ex. **a** : sometimes with vec=over-arrow $\bar{\mathbf{a}}$
Temporal scalars (1D) non-bold, usually lowercase, 0th component: ex. a^0, a_0
Individual non-vec scalar components non-bold: ex. $\mathbf{A} = (a^0,a^1,a^2,a^3) = (a^0,\mathbf{a})$
Rest-scalars (invariants) normal non-bold, denoted with naught ($_o$): ex. a_o
Tensor-index-notation normal non-bold: ex. $A^{\mu} = (a^{\mu}) = (a^0,a^k) = (a^0,a^1,a^2,a^3)$
4D Tensors use Greek indices: ex. { μ, ν, σ, ρ, …} : ex. 4-Position $R^{\mu} = (r^{\mu})$
3D tensors use Latin indices: ex. { i, j, k, …} : ex. 3-position $r^k = (r^k) = (r^1,r^2,r^3)$
Upper indices $A^{\mu} = (a^{\mu}) = (a^0,a^j)$: Lower indices $B_{\mu} = (b_{\mu}) = (b_0,b_j)$
LightSpeed Factor (c) in temporal component to match dimensional units
4-Vector: $\mathbf{A} = \bar{\mathbf{A}} = A^{\mu}$: ex. 4-Momentum $\mathbf{P} = P^{\mu} = (E/c,\mathbf{p}) = (mc,\mathbf{mu})$
4-CoVector = OneForm: $\underline{\mathbf{A}} = A_{\mu}$: ex. 4D GradientOneForm $\underline{\partial} = \partial_{\mu} = (\partial_t/c,\nabla)$
Null 4-Vector $\mathbf{N} \sim (|\mathbf{a}|,\mathbf{a}) = a(1,\hat{\mathbf{n}})$, with Lorentz Scalar Invariant $\mathbf{N}\cdot\mathbf{N} = N^{\mu}N_{\mu} = 0$

SR:Metric Convention: Particle-Physics,Temporal-0th-Positive (+,-,-,-)
that is used herein: West-Coast, Time-Like, Mostly-Minuses

Good object: green

Good, but limited-case, warning, etc.: yellow-orange

Good But Limited Idea

Old, outdated, wrong: red

Old Wrong Idea

SRQM Tensor Symbolic Representation

4-Scalar S (0 index): Ellipse

4-Vector V^{μ} (1 index): Rectangle

4-Tensor $T^{\mu\nu}$ (2 index): Octagon

Technically

All Tensors

Temporal object (+): blue, Spatial object (-): red
Mixed Time·Space object (generic <event>): purple
The mnemonic being blue and red mixed → make purple

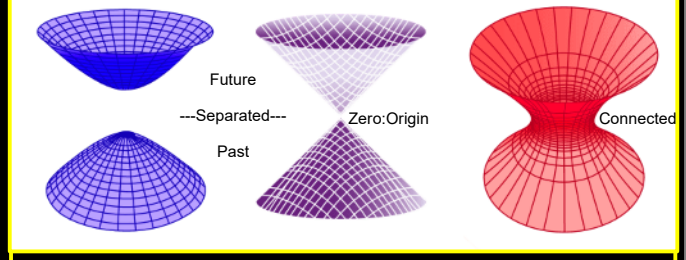
Null:Photonic:Light-like object (0): white

SpaceTime: I often write it as "Time·Space" just to match this ordering convention of 4-Vector (temporal, **spatial**) components

RQM & QM are derivable from principles of SR
Let that sink in...
Quantum Mechanics is derivable from Special Relativity
GR → SR → RQM → QM → {CM & EM}

(NormalMatter↔AntiMatter)
(Black Holes ↔ White Holes)

Both are the SpaceTime-reversed situations of the other… equivalent under CPT Symmetry



Temporal(+) Photonic=Null (0) Spatial(-)

Existing SR Rules
Quantum Principles

4-Position $\mathbf{R} = (ct,\mathbf{r}) = (c*\text{when},\mathbf{where}) \in$ <What Event>
4-Position $\mathbf{R} = (ct,\mathbf{r}) = (c*\text{then},\mathbf{there}) \in$ <That Event>
4-Origin $\mathbf{R_o} = (0,\mathbf{0}) = (c*\text{now},\mathbf{here}) \in$ <This Event>

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

3-Tensor 3D
(2,0)-Tensor T^{jk}
(1,1)-Tensor $T^j{}_k$ or $T_j{}^k$
(0,2)-Tensor T_{jk}

Classical (scalar 3-vector)
3D Galilean Invariant
not Lorentz Invariant

3-Scalar
3D (0,0)-Tensor S

$\text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

Special Relativity → Quantum Mechanics

The SRQM Interpretation: Links

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See also:

http://scirealm.org/SRQM.html (alt discussion)
http://scirealm.org/SRQM-RoadMap.html (main SRQM website)
http://scirealm.org/SRQM-Summary.pdf (SRQM Summary .pdf)
http://scirealm.org/SRQM-FundamentalConstants.pdf (SRQM Constants .pdf)
http://scirealm.org/4Vectors.html (4-Vector study)
http://scirealm.org/SRQM-Tensors.html (Tensor & 4-Vector Calculator)
http://scirealm.org/SciCalculator.html (Complex-capable RPN Calculator)

or Google "SRQM"

http://scirealm.org/SRQM.pdf (this .pdf document: most current version at SciRealm.org)

(+,-,-,-) SR → QM Physics

4

A Tensor Study of Physical 4-Vectors

SRQM Study: Physical / Mathematical Tensors

4D Tensor Types: 4-Scalar, 4-Vector, 4-Tensor

Component Types: Temporal, Spatial, Mixed

4-Vector SRQM Interpretation of QM

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Matrix Format | SRQM Diagram Format

Each 4D index = {0,1..3} ↔ Tensor Dimension = 4

1 Temporal + 3 Spatial = 4 SpaceTime Dimensions

SR:Minkowski Metric

$\partial[\mathbf{R}] = \partial^\mu[R^\nu] = \eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu} \rightarrow$

$\mathrm{Diag}[+1,-1,-1,-1] = \mathrm{Diag}[1,-\mathbf{I}_{(3)}] = \mathrm{Diag}[1,-\delta^{jk}]$

{in Cartesian form} "Particle Physics" Convention

$\{\eta_{\mu\nu}\} = 1/\{\eta^{\mu\nu}\} : \eta_\mu{}^\nu = \delta_\mu{}^\nu$ $\quad \mathrm{Tr}[\eta^{\mu\nu}]=4$

SR 4-Scalar S

a "number": magnitude only

S or S

SR 4-Scalar
4D (0,0)-Tensor S often as S_o
Lorentz Scalar

SRQM Diagram: Ellipse
4-Scalars, 0 index = rank 0
4*0 = 0 corners in diagram
4^0 = (1) = 1 component

(m,n)-Tensor has:
(m) # upper-indices
{ m+n = rank } (n) # lower-indices

4-Gradient $\partial = \partial_R$

$\partial^\mu = \partial/\partial R_\mu = (\partial_t/c, -\nabla)$

SR 4-Vector V^μ

an "arrow": magnitude and 1 direction

V^0	V^1	V^2	V^3

SR 4-Vector
4D (1,0)-Tensor $\mathbf{V} = \overline{\mathbf{V}}$
uses a single upper index:bar, contravariant

$V^\mu = (v^\mu) = (v^0, v^i) = (v^0, \mathbf{v})$
$=(v^0, v^1, v^2, v^3) \rightarrow (v^t, v^x, v^y, v^z)$

SRQM Diagram: Rectangle
4-Vectors, 1 index = rank 1
4-CoVectors,
4*1 = 4 corners in diagram
4^1 = (1+3) = 4 components

SR 4-CoVector = 4D One-Form
4D (0,1)-Tensor $\underline{\mathbf{C}}$ = "Dual" 4-Vector
uses a single lower index:bar, covariant

$C_\mu = \eta_{\mu\sigma}C^\sigma = (c_\mu) = (c_0, c_i) \rightarrow (c_t, c_x, c_y, c_z)$
$= (c^0, -\mathbf{c}) = (c^0, -c^i) \rightarrow (c^t, -c^x, -c^y, -c^z)$

4-Position R

$R^\mu = (ct, \mathbf{r}) \in$ <Event>

SpaceTime

$\partial\cdot\mathbf{R} = \partial_\mu R^\mu = 4$ Dimension

SR 4-Tensor $T^{\mu\nu} = T^{\text{row:column}}$

a "matrix" or "dyadic": magnitude and 2 directions

T^{00}	T^{01}	T^{02}	T^{03}
T^{10}	T^{11}	T^{12}	T^{13}
T^{20}	T^{21}	T^{22}	T^{23}
T^{30}	T^{31}	T^{32}	T^{33}

Temporal region: blue
Spatial region: red
Mixed Time·Space region: purple
The mnemonic being red and blue mixed → make purple

SR 4-Tensor
4D (2,0)-Tensor $\boldsymbol{T}$
$T^{\mu\nu} =$
$[\,T^{00},\ T^{0k}\,]$
$[\,T^{j0},\ T^{jk}\,]$
$\rightarrow$
$[T^{tt}, T^{tx}, T^{ty}, T^{tz}]$
$[T^{xt}, T^{xx}, T^{xy}, T^{xz}]$
$[T^{yt}, T^{yx}, T^{yy}, T^{yz}]$
$[T^{zt}, T^{zx}, T^{zy}, T^{zz}]$

SRQM Diagram: Octagon
4-Tensors, 2 index = rank 2
4*2 = 8 corners in diagram
4^2 = (1+6+9) = 16 components

for 2-index tensor components:
6 Anti-Symmetric (Skew)
+10 Symmetric
==========
16 General components

SR
Mixed 4-Tensor
4D (1,1)-Tensor
$T_\mu{}^\nu = \eta_{\mu\rho}T^{\rho\nu}$
=
$[\,T_0{}^0, T_0{}^k\,]$
$[\,T_j{}^0, T_j{}^k\,]$
=
$[\,+T^{00}, +T^{0k}\,]$
$[\,-T^{j0}, -T^{jk}\,]$

SR
Mixed 4-Tensor
4D (1,1)-Tensor
$T^\mu{}_\nu = \eta_{\rho\nu}T^{\mu\rho}$
=
$[\,T^0{}_0, T^0{}_k\,]$
$[\,T^j{}_0, T^j{}_k\,]$
=
$[\,+T^{00}, -T^{0k}\,]$
$[\,+T^{j0}, -T^{jk}\,]$

SR
Lowered 4-Tensor
4D (0,2)-Tensor
$T_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma}T^{\rho\sigma}$
=
$[\,T_{00}, T_{0k}\,]$
$[\,T_{j0}, T_{jk}\,]$
=
$[\,+T^{00}, -T^{0k}\,]$
$[\,-T^{j0}, +T^{jk}\,]$

Tensor Property:

Rank = # of indices
{0 = Scalar}
{1 = Vector}
{2 = Dyadic:Matrix}
etc...

Dimension = # of values a tensor index can take
{SR Tensors = 4D}
{ Classical = 3D}

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Technically, all these objects are "SR 4-Tensors", but usually reserve the name "4-Tensor" for 4D objects with 2 (or more) indices, and use the "#D (m,n)-Tensor" notation to specify all the objects more precisely.

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

3D Euclidean 3-vectors & 4D SR:Minkowski 4-Vectors

4-Vector SRQM Interpretation of QM

3,4

A Tensor Study of Physical 4-Vectors

Dimension	Regime	Metric	Transform:Invariance:Group
3D	Classical:QM	Euclidean	Galilean
4D	Relativistic:RQM	Minkowski	Lorentz⊂Poincaré

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Classical & QM

3D

approximation only
valid regime:
limited to |**v**| << c
& only weak gravity

Galilean Transformations G

$\nabla_j[r^{i'}]=\partial r^{i'}/\partial r^j=G^{i'}_j=\Lambda^{i'}_j$

$\text{Det}[\Lambda^{i'}_j]=\pm1$

[$G^{i'}_j = \Lambda^{i'}_j$] spatial components only, 3x3 matrix

$\Lambda^i_k(\Lambda^{-1})^k_j = \Lambda^{ik}(\Lambda^{-1})_{kj} = E^i_j = \delta^i_j = \text{Diag}[1,1,1]$

$\Lambda_{ij}\Lambda^{ij}=3=\Lambda_i^j\Lambda^i_j$

spatial-velocity-shifts, spatial-rotations, (P)arity, (T)ime-reversal

3D Classical:Euclidean Metric E

$\nabla[\mathbf{r}] = \nabla^j[r^k] = E^{jk} = -H^{jk} \rightarrow$ Kronecker delta δ^{jk}

$= \text{Diag}[+1,+1,+1] = \text{Diag}[\mathbf{I}_{(3)}] = [\delta^{jk}]$

{in Cartesian form}

$\{\delta_{kk}\} = 1/\{\delta^{kk}\}$

$\text{Tr}[\delta^{jk}]=\delta_1{}^1+\delta_2{}^2+\delta_3{}^3=1+1+1=3$

scalar 1-time t
$= (r^0/c) = (t)$
= <time>

3-vector 3-position **r**
$= r^j = (r^j) = (\mathbf{r}) \rightarrow (x,y,z)$
$= (r^1,r^2,r^3)$ = <location>

Galilean 3D Invariant

$\mathbf{r}\cdot\mathbf{r} = r^j\delta_{jk}r^k = (x)^2+(y)^2+(z)^2 = (r)^2$

length r = |**r**|

3D Space

$\nabla\cdot\mathbf{r} = \nabla^j\delta_{jk}r^k = \nabla_k r^k = 3$

$= (\partial/\partial x[x]+\partial/\partial y[y]+\partial/\partial z[z]) = (1+1+1)$

Dimension

Relativistic & RQM

4D

valid regime:
all cases except strong gravity GR
& still good "locally" at (then:there)

Lorentz Transformations Λ

$\partial_\nu[R^{\mu'}]=\partial R^{\mu'}/\partial R^\nu=\Lambda^{\mu'}{}_\nu$

[$\Lambda^{0'}{}_0,\Lambda^{0'}{}_j$] temporal:spatial:mixed

[$\Lambda^{i'}{}_0,\Lambda^{i'}{}_j$] components, 4x4 matrix

$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda_\mu{}^\nu\Lambda^\mu{}_\nu$

$\Lambda^\mu{}_\alpha(\Lambda^{-1})^\alpha{}_\nu = \Lambda^{\mu\alpha}(\Lambda^{-1})_{\alpha\nu} = \eta^\mu{}_\nu = \delta^\mu{}_\nu = \text{Diag}[1,1,1,1]$

$\text{Det}[\Lambda^{\mu'}{}_\nu]=\pm1$

Time·Space-Boosts, Spatial-Rotations, (CPT)

4D SR:Minkowski Metric η

$\partial[\mathbf{R}] = \partial^\mu[R^\nu] = \eta^{\mu\nu} = \eta_{\mu\nu} = V^{\mu\nu} + H^{\mu\nu} \rightarrow$

$\text{Diag}[+1,-1,-1,-1] = \text{Diag}[1,-\mathbf{I}_{(3)}] = \text{Diag}[1,-\delta^{jk}]$

{in Cartesian form} "Particle Physics" Convention

$\{\eta_{\mu\mu}\} = 1/\{\eta^{\mu\mu}\}$

others are zero

$\eta_\mu{}^\nu = \delta_\mu{}^\nu = \text{Diag}[1,1,1,1]$

$\text{Tr}[\eta^{\mu\nu}]=\eta_\nu{}^\nu=\eta_0{}^0+\eta_1{}^1+\eta_2{}^2+\eta_3{}^3=1+1+1+1=4$

Dimension: # of components required to specify a vector object, 3 for 3D, 4 for 4D. Tensor index range.

Regime: Classical (old = 1D time + 3D space)
Relativistic (modern = 4D Time·Space)

Metric: Specifies how the "intervals" between <events> are measured, ex. $\eta^{\mu\nu}$=Diagonal[1,-1,-1,-1]

Transform: Specifies how coordinate reference-frames are related to each other

Invariant: Tensor value which doesn't change with a change in the reference-frame: ex. c, h, m_o, e, G, k_B

Lorentz 4D Invariant

$\mathbf{R}\cdot\mathbf{R} = R^\mu\eta_{\mu\nu}R^\nu = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (c\tau)^2$

Interval $c\tau = ct_o = |\mathbf{R}|$

4-Vector 4-Position **R**
$= R^\mu = (r^\mu) = (r^0,r^j) = (ct,\mathbf{r})$
$= (r^0,r^1,r^2,r^3) \rightarrow (ct,x,y,z)$
∈ <Event> ∋ <time> & <location>

$V^{\mu\nu}$ |(+) (0) (-) $H^{\mu\nu}$

Light-Cone

4D SpaceTime

$\partial\cdot\mathbf{R} = \partial^\mu\eta_{\mu\nu}R^\nu = \partial_\nu R^\nu = 4$

$= (\partial/\partial ct[ct] - -\partial/\partial x[x] - -\partial/\partial y[y] - -\partial/\partial z[z])$

$= (\partial/\partial ct[ct]+\partial/\partial x[x]+\partial/\partial y[y]+\partial/\partial z[z])$

$= (1+1+1+1)$

Dimension

3D & 4D vector internal components labeled with superscript index, not an exponent
Only scalars outside of a vector will have exponents, such as in the Lorentz Scalar Product
$\mathbf{A} = A^\mu = (a^0,a^1,a^2,a^3) \leftarrow$ indices $\qquad \mathbf{A}\cdot\mathbf{A} = (a_o)^*(a_o) = (a_o)^2 \leftarrow$ exponent

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

3-Tensor 3D
(2,0)-Tensor T^{jk}
(1,1)-Tensor $T^j{}_k$ or $T_j{}^k$
(0,2)-Tensor T_{jk}

Classical (scalar 3-vector)
1D+3D Galilean Invariants
not Lorentz Invariant

3-Scalar
3D (0,0)-Tensor S

$\text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

3,4 • Σ

A Tensor Study of Physical 4-Vectors

Euclidean 3-Vectors & SR:Minkowski 4-Vectors

Dot Product, Lorentz Scalar Product

Einstein Summation Convention

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
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In Classical Mechanics (CM), an ex. of the magnitude of a 3-vector is the length $|\mathbf{\Delta r}|$ of a 3-displacement $\mathbf{\Delta r} = (\mathbf{r}_1 - \mathbf{r}_0)$.
Examine 3-position $\mathbf{r}$, which is a 3-displacement $\mathbf{\Delta r} \to \mathbf{r}$, with the base at the origin $\mathbf{r}_0 \to \mathbf{0} = (0,0,0)$ & $\mathbf{r}_1 \to \mathbf{r} = (x,y,z)$.
The 3D Dot Product: $\{ \mathbf{r'}\cdot\mathbf{r'} = \mathbf{r}\cdot\mathbf{r} = r^j\delta_{jk}r^k = r_k r^k = r^j r_j = (r^1r_1 + r^2r_2 + r^3r_3) = [(x)^2 + (y)^2 + (z)^2] = (r)^2 \}$ is the Pythagorean Theorem.
It uses the Euclidean Metric E_{jk} which in Cartesian form is equivalent to the 3D Kronecker delta $\delta_{jk} = \mathrm{Diag}[1,1,1] = \text{Identity } \mathbf{I}_{(3)}$.
The 3D magnitude^2 is $\mathbf{r}\cdot\mathbf{r}$. The |magnitude| is $\sqrt{[\mathbf{r}\cdot\mathbf{r}]} = \sqrt{[r^2]} = |\mathbf{r}| = r$. 3D magnitudes are always positive(+) spatial.
3D invariants are not invariant in 4D, ex. 3D ||Length|| but 4D |→Length Contraction←| & ←|Time Dilation|→

t & |r| are scalar invariants only in Euclidean 3D space. However, our universe is locally Minkowski 4D, not 3D.

scalar 1-time t
$= (r^0/c) = (t)$
= <time>

3-vector 3-position $\mathbf{r}$
$= r^j = (r^j) = (\mathbf{r}) \to (x,y,z)$
$= (r^1,r^2,r^3)$ = <location>

Galilean 3D Invariant
$\mathbf{r}\cdot\mathbf{r} = r^j\delta_{jk}r^k = (x)^2+(y)^2+(z)^2 = (r)^2$
length $r = |\mathbf{r}|$

3D Classical:Euclidean Metric E
$\nabla[\mathbf{r}] = \nabla^j[r^k] = E^{jk} = -H^{jk} \to$ Kronecker delta δ^{jk}
$= \mathrm{Diag}[+1,+1,+1] = \mathrm{Diag}[\mathbf{I}_{(3)}] = [\delta^{jk}]$
{in Cartesian form}
$\{\delta_{kk}\} = 1/\{\delta^{kk}\}$ $\mathrm{Tr}[\delta^{jk}]=\delta_1{}^1+\delta_2{}^2+\delta_3{}^3=1+1+1=3$

3D Space
$\nabla\cdot\mathbf{r} = \nabla^j\delta_{jk}r^k = \nabla_k r^k = 3$
$= (\partial/\partial x[x]+\partial/\partial y[y]+\partial/\partial z[z]) = (1+1+1)$
Dimension

The magnitude of an SR 4-Vector is very similar to the magnitude of a 3-vector, but there are some interesting differences.
One uses the Lorentz Scalar Product, a 4D Dot Product, which includes time & space components, and is based on the SR:Minkowski Metric Tensor. "Particle Physics" sign-convention {temporal,0th,+}→(+,-,-,-) of the Minkowski Metric gives $\eta_{\mu\nu} \to \mathrm{Diag}[+1,-1,-1,-1]$ {Cartesian form}, with the other entries zero. Note the 3D {spatial,1st2nd3rd,-} part is 4D negative(-).
Only the mixed (1,1)-Tensor form of Minkowski Metric $\eta_\mu{}^\nu$ is fully equivalent to the 4D Kronecker Delta $\delta_\mu{}^\nu = \mathrm{Diag}[1,1,1,1] = \mathbf{I}_{(4)}$.

$\mathbf{A'}\cdot\mathbf{A'} = \mathbf{A}\cdot\mathbf{A} = A^\mu\eta_{\mu\nu}A^\nu = (a^0a^0 - \mathbf{a}\cdot\mathbf{a}) = (a^0{}_o)^2 = (a^0a^0 - a^1a^1 - a^2a^2 - a^3a^3)$ = the rest-frame $(a^0{}_o)^2$
$= A^\mu A_\mu = \Sigma_{u=0..3}[a^u a_u] = (a^0a_0 + a^1a_1 + a^2a_2 + a^3a_3)$
$= A_\nu A^\nu = \Sigma_{\nu=0..3}[a_\nu a^\nu] = (a_0a^0 + a_1a^1 + a_2a^2 + a_3a^3)$

using Einstein Summation Convention which has upper-lower paired-indices summed over.

$\mathbf{R'}\cdot\mathbf{R'} = \mathbf{R}\cdot\mathbf{R} = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (ct)^2 - (x^2 + y^2 + z^2) = (ct_o)^2 = (c\tau)^2$
for 4-Position $\mathbf{R} = (ct,\mathbf{r})$ & LT'd 4-Position $\mathbf{R'} = (ct',\mathbf{r'})$
4D magnitude^2 can be: positive(+), zero:null(0), negative(-)
temporal , photonic , spatial

Lorentz 4D Invariant
$\mathbf{R}\cdot\mathbf{R} = R^\mu\eta_{\mu\nu}R^\nu = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (c\tau)^2$
Interval $c\tau = ct_o = |\mathbf{R}|$

4-Vector 4-Position $\mathbf{R}$
$= R^\mu = (r^\mu) = (r^0,r^j) = (ct,\mathbf{r})$
$= (r^0,r^1,r^2,r^3) \to (ct,x,y,z)$
∈ <Event> ∋ <time> & <location>

Time (t) & length |r| are **NOT** 4D SR invariants,they are 4-Vector components.
ProperTime (τ), Proper Length (L_o) & Interval Mag. |R| are 4D SR Lorentz Scalar Invariants.

4D SR:Minkowski Metric η
$\partial[\mathbf{R}] = \partial^\mu[R^\nu] = \eta^{\mu\nu} = \eta_{\mu\nu} = V^{\mu\nu} + H^{\mu\nu} \to$
$\mathrm{Diag}[+1,-1,-1,-1] = \mathrm{Diag}[1,-\mathbf{I}_{(3)}] = \mathrm{Diag}[1,-\delta^{jk}]$
{in Cartesian form} "Particle Physics" Convention
$\{\eta_{\mu\mu}\} = 1/\{\eta^{\mu\mu}\}$ $\eta_\mu{}^\nu = \delta_\mu{}^\nu = \mathrm{Diag}[1,1,1,1]$
others are zero
$\mathrm{Tr}[\eta^{\mu\nu}]=\eta_\nu{}^\nu=\eta_0{}^0+\eta_1{}^1+\eta_2{}^2+\eta_3{}^3=1+1+1+1=4$

4D SpaceTime
$\partial\cdot\mathbf{R} = \partial^\mu\eta_{\mu\nu}R^\nu = \partial_\nu R^\nu = 4$
$= (\partial/\partial ct[ct] - -\partial/\partial x[x] - -\partial/\partial y[y] - -\partial/\partial z[z])$
$= (\partial/\partial ct[ct]+\partial/\partial x[x]+\partial/\partial y[y]+\partial/\partial z[z])$
$= (1+1+1+1)$
Dimension

The 4-Vector version has the Pythagorean elements in the spatial components, the temporal component is of opposite sign.
This gives a "causality condition", with SpaceTime intervals (in the [+,-,-,-] SR:Minkowski Metric) that can be:

$\mathbf{\Delta R}\cdot\mathbf{\Delta R} = [(c\Delta t)^2 - \mathbf{\Delta r}\cdot\mathbf{\Delta r}] =$	$(c\Delta\tau)^2$	Time-like:Temporal	(+) {causal = 1D temporally-ordered, *spatially relative*}
	(0)	Light-like:Null:Photonic	(0) {causal & topological, maximum signal speed ($\lvert\mathbf{\Delta r}/\Delta t\rvert$=c)}
	$-(\Delta r_o)^2$	Space-like:Spatial	(–) {topological = 3D spatially-ordered, *temporally relative*}

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

3-Tensor 3D
(2,0)-Tensor T^{jk}
(1,1)-Tensor $T^j{}_k$ or $T_j{}^k$
(0,2)-Tensor T_{jk}

Classical (scalar 3-vector)
1D+3D Galilean Invariants
not Lorentz Invariant

3-Scalar
3D (0,0)-Tensor S

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

$\eta^{\mu\nu}$ Diagonal [1,-1,-1,-1] · $\eta_{\mu\nu}$

$\eta_\mu{}^\nu\ \eta^\mu{}_\nu$ Diagonal [1,1,1,1] δ all 4D types

SRQM Study: SR Minkowski SpaceTime

SR:Minkowski Metric [η] Operations

Invariant Lorentz Scalar Product

Tensor Index {Raising, Lowering, Gymnastics}

4-Vector SRQM Interpretation of QM

$\delta_j{}^k\ \delta_{jk}$ 3D Diagonal [1,1,1] $\delta_j{}_k\ \delta^{jk}$

$\delta_\mu{}^\nu\ \delta_{\mu\nu}$ 4D Diagonal [1,1,1,1] $\delta_\mu{}_\nu\ \delta^{\mu\nu}$

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Both GR and SR use a metric tensor ($g^{\mu\nu}$) to describe measurements in 4D SpaceTime (Time·Space).
SR uses the "flat" Minkowski Metric $g^{\mu\nu} \to \eta^{\mu\nu} \to \eta_{\mu\nu} \to \mathrm{Diag}[1,-\mathbf{I}_{(3)}] = \mathrm{Diag}[1,-\delta^{jk}] = \mathrm{Diag}[1,-1,-1,-1]_{\{\text{Cartesian form}\}}$, which is the {curvature~0 limit = low-mass limit} of the GR metric $g^{\mu\nu}$. SR is valid everywhere except extreme gravity, ex. near BH's.

4-Vectors are tensorial entities of Minkowski SpaceTime which maintain covariance for inertial observers, meaning that they may have different *relativistic* components for different observers, but describe the **same physical object**. (like viewing a sculpture from different angles – snapshot pictures "look" different, but it's actually the same object)
There are also { 4-CoVectors = One-Forms = 4D (0,1)-Tensors } which are dual to the { 4-Vectors = 4D (1,0)-Tensors }.

4-Vectors = 4D (1,0)-Tensors

$\mathbf{A} = \overline{\mathbf{A}} = A^\mu = \eta^{\mu\nu}A_\nu = (a^\mu) = (a^0,a^i) = (a^0,\mathbf{a}) = (a^0,a^1,a^2,a^3) \to (a^t,a^x,a^y,a^z)$
$\mathbf{B} = \overline{\mathbf{B}} = B^\mu = \eta^{\mu\nu}B_\nu = (b^\mu) = (b^0,b^i) = (b^0,\mathbf{b}) = (b^0,b^1,b^2,b^3) \to (b^t,b^x,b^y,b^z)$

4-CoVectors = 4D (0,1)-Tensors = 4D DualVectors = 4D OneForms

$\underline{\mathbf{A}} = A_\mu = \eta_{\mu\nu}A^\nu = (a_\mu) = (a_0,a_i) = (a_0,-\mathbf{a}) = (a_0, a_1, a_2, a_3) \to (a_t, a_x, a_y, a_z)$
$= (a_0,a_i) = (a^0,-\mathbf{a}) = (a^0,-a^1,-a^2,-a^3) \to (a^t,-a^x,-a^y,-a^z)$
$\underline{\mathbf{B}} = B_\mu = \eta_{\mu\nu}B^\nu = (b_\mu) = (b_0,b_i) = (b_0,-\mathbf{b}) = (b_0, b_1, b_2, b_3) \to (b_t, b_x, b_y, b_z)$
$= (b_0,b_i) = (b^0,-\mathbf{b}) = (b^0,-b^1,-b^2,-b^3) \to (b^t,-b^x,-b^y,-b^z)$

Index Raising & Lowering with SR:Minkowski Metric Tensor $\eta^{\mu\nu}$ or $\eta_{\mu\nu}$ (both → $\mathrm{Diag}[1,-\mathbf{I}_{(3)}]$)
This Metric is also used with other SR 4-Tensors to create raised, lowered, and mixed tensors

$\eta^{\mu\nu}$ Diagonal [1,-1,-1,-1] $\eta_{\mu\nu}$

ex. $T_{\mu\nu} = \eta_{\mu\alpha}T^\alpha{}_\nu = \eta_{\mu\alpha}\eta_{\nu\beta}T^{\alpha\beta}$

$\eta_{\mu\nu}\eta^{\mu\nu} = \eta^\mu{}_\mu = \delta^\mu{}_\mu = 4$
$= \mathrm{Trace}[\eta^\mu{}_\nu] = \mathrm{Tr}[\eta^\mu{}_\nu]$

$\eta^\mu{}_\nu$ Diagonal [1,1,1,1] $\delta^\mu{}_\nu$

$\mathbf{A'\cdot B'} = \mathbf{A\cdot B} = A^\mu\eta_{\mu\nu}B^\nu = A_\nu B^\nu = A^\mu B_\mu = \Sigma_{\nu=0..3}[a_\nu b^\nu] = \Sigma_{u=0..3}[a^u b_u] = (a^0b^0 - \mathbf{a\cdot b}) = (a^0b^0 - a^1b^1 - a^2b^2 - a^3b^3) = (a^0{}_o b^0{}_o)$
Lorentz Scalar Invariant Product using the Einstein Summation Convention which sums upper-lower-paired indices.

Proof of invariance (using Tensor gymnastics and the properties of the Minkowski Metric η & Lorentz Transforms Λ):
$\mathbf{A'\cdot B'} = A^{\mu'}\eta_{\mu'\nu'}B^{\nu'} =$
$(\Lambda^{\mu'}{}_\alpha A^\alpha)\ \eta_{\mu'\nu'}(\Lambda^{\nu'}{}_\beta B^\beta) = (\Lambda^{\mu'}{}_\alpha\eta_{\mu'\nu'}\Lambda^{\nu'}{}_\beta)\ A^\alpha B^\beta = (\Lambda_{\nu'\alpha}\Lambda^{\nu'}{}_\beta)\ A^\alpha B^\beta = (\eta_{\rho\alpha}\Lambda^\rho{}_{\nu'}\Lambda^{\nu'}{}_\beta)\ A^\alpha B^\beta = (\eta_{\alpha\rho}\delta^\rho{}_\beta)\ A^\alpha B^\beta = (\eta_{\alpha\beta})\ A^\alpha B^\beta =$
$A^\alpha(\eta_{\alpha\beta})B^\beta = \mathbf{A\cdot B}$

Lorentz Scalar Product of 4-Vectors ($\mathbf{A\cdot B}$) → Lorentz Invariant Scalars = 4D (0,0)-Tensors.
They have the same measured value for all inertial observers, i.e. same value in all 4D inertial reference-frames (IRF)

Index Raising

4-Vector $\mathbf{A}=\overline{\mathbf{A}}$
4D (1,0)-Tensor
$A^\mu=(a^\mu)=\eta^{\mu\nu}A_\nu$
$=(a^0,a^i)=(a^0,\mathbf{a})$

$\eta^{\mu\nu}$ Diagonal [1,-1,-1,-1] $\eta_{\mu\nu}$

4-CoVector $\underline{\mathbf{A}}$
4D (0,1)-Tensor
$A_\nu=(a_\nu)=\eta_{\mu\nu}A^\mu$
$=(a_0,a_i)=(+a^0,-\mathbf{a})$

Index Lowering

$A^\mu=\eta^{\mu\nu}A_\nu=\eta^{\mu\nu}\eta_{\alpha\nu}A^\alpha=\eta^\mu{}_\alpha A^\alpha=\delta^\mu{}_\alpha A^\alpha=A^\mu$

Index Gymnastics = Only Unique Upper-Lower Pairs Allowed

4-Vector
$\mathbf{A}=A^\nu=(a^0,\mathbf{a})$

$\eta_{\mu\nu}$ Diagonal [1,-1,-1,-1] ·

4-Vector
$\mathbf{B}=B^\nu=(b^0,\mathbf{b})$

Invariant Lorentz Scalar Product (0,0)-Tensor
$\mathbf{A\cdot B}=A_\nu B^\nu=A^\mu\eta_{\mu\nu}B^\nu=A^\mu B_\mu=\mathbf{A'\cdot B'}$
$= \underline{\mathbf{A}}\overline{\mathbf{B}} = \overline{\mathbf{A}}\cdot\overline{\mathbf{B}} = \overline{\mathbf{A}}\underline{\mathbf{B}}$
$=(a^0b^0 - \mathbf{a\cdot b})=(a^{0'}b^{0'} - \mathbf{a'\cdot b'})$
$=(a^0{}_o b^0{}_o)$

4-Vector
$\mathbf{A'}=A^{\mu'}=\Lambda^{\mu'}{}_\nu A^\nu=(a^{0'},\mathbf{a'})$

4-Vector
$\mathbf{B'}=B^{\mu'}=\Lambda^{\mu'}{}_\nu B^\nu=(b^{0'},\mathbf{b'})$

Lorentz Transform ($\Lambda^{\mu'}{}_\nu$)

$\mathrm{Det}[\Lambda^{\mu'}{}_\nu]=\pm1$

$\mathrm{Tr}[\Lambda^{\mu'}{}_\nu]=\{-\infty..+\infty\}$ =Lorentz Transform Type

$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda_\mu{}^\nu\Lambda^\mu{}_\nu$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

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(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Einstein & Lorentz "saw" the physics of SR,
Minkowski & Poincaré "saw" the mathematics of SR.
We are indebted to all of them for the simplicity, beauty, and power of how SR and 4-vectors work...

$\mathrm{Trace}[T^\mu{}_\nu] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V\cdot V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v\cdot v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4-Vector SRQM Interpretation of QM

Special Relativity → Quantum Mechanics
SRQM (Physics) Diagramming Method

A Tensor Study of Physical 4-Vectors

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The **SRQM (Physics) Diagramming Method** shows the properties & relationships of various physical objects/tensors in a graphical way. This "flowchart" method aids understanding.

Representation: 4-Scalars by ellipses, 4-Vectors by rectangles, 4-Tensors by octagons. Physical/mathematical equations and descriptions inside each shape/object. Sometimes there will be additional clarifying descriptions around a shape/object.

Relationships: Lorentz Scalar Products or tensor compositions of different 4-Vectors are on simple lines(—) between related 4-Vectors. Lorentz Scalar Products of a single 4-Vector, or Invariants of Tensors, are next to that object and often highlighted in a different color.

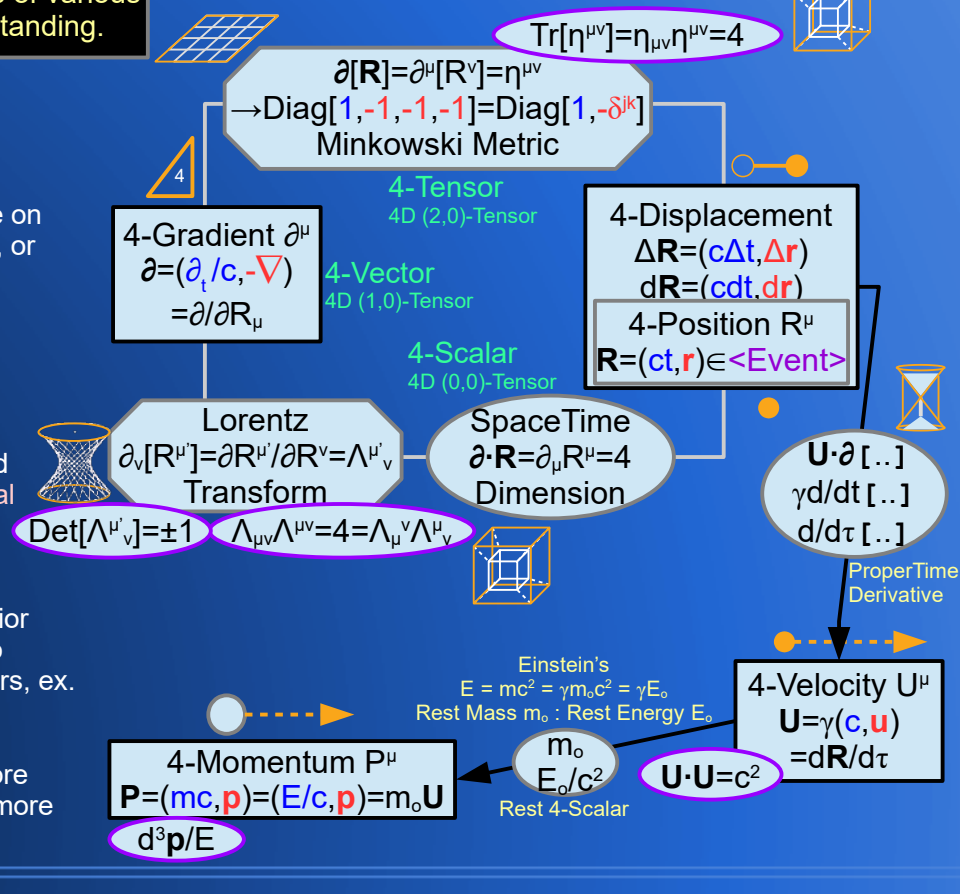
Flow: Objects that are some function of a Lorentz 4-Scalar with another 4-Vector or 4-Tensor are on lines with arrows(→) indicating the direction of flow. (ex. multiplication by SR 4-Scalar, or 4-Scalar function of SR 4-Vector indicated by [..])

Properties: Some objects will also have a symbol representing its properties nearby, and sometimes there will be color highlighting within the object to emphasize temporal:spatial properties. I use blue=Temporal ⊗ red=Spatial → purple=mixed Time·Space.

Alternate ways of writing 4-Vector expressions in physics:
($\mathbf{A}\cdot\mathbf{B}$) is a 4-Vector style, which uses vector-notation (ex. inner product "dot=·" or exterior product "wedge=^"), and is typically more compact, always using **bold** UPPERCASE to represent the 4-Vector, ex. ($\mathbf{A}\cdot\mathbf{B}$) = ($A^\mu \eta_{\mu\nu} B^\nu$), and **bold** lowercase to represent 3-vectors, ex. ($\mathbf{a}\cdot\mathbf{b}$) = ($a^j \delta_{jk} b^k$). Most 3-vector rules have analogues in 4-Vector mathematics.

($A^\mu\eta_{\mu\nu}B^\nu$) is a Ricci Calculus style, which uses tensor-index-notation and is useful for more complicated expressions, especially to clarify those expressions involving tensors with more than one index, such as the Faraday EM Tensor $F^{\mu\nu} = (\partial^\mu A^\nu - \partial^\nu A^\mu) = (\partial \wedge \mathbf{A})$

SRQM (Physics) Diagramming Method



SR 4-Tensor	SR 4-Vector	SR 4-Scalar
(2,0)-Tensor $T^{\mu\nu}$ (1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$ (0,2)-Tensor $T_{\mu\nu}$	(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$ **SR 4-CoVector:OneForm** (0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$	(0,0)-Tensor S or S_o Lorentz Scalar

Lorentz Factor = *Relativistic* Gamma $\gamma = 1/\sqrt{[1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: $\boldsymbol{\beta} = \mathbf{u}/c$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

Special Relativity → Quantum Mechanics
SRQM Tensor Invariants
Inherent 4D SpaceTime Properties

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

One of the extremely important properties of Tensor Mathematics is the fact that there are numerous ways to generate **Tensor Invariants**. These Invariants lead to Physical Properties that are fundamental in our Universe, and are totally independent of any coordinate-systems used to measure them. Thus, they represent symmetry properties that are inherent in the fabric of 4D SpaceTime (Time·Space). See the Cayley-Hamilton Theorem, esp. for the Anti-Symmetric Tensor Products.

Trace Tensor Invariant: $Tr[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T_{\nu}{}^{\nu} = T_{Tr} = \Sigma_n$ [EigenValues λ_n] for $T^{\mu}{}_{\nu}$

Determinant Tensor Invariant: $Det[T^{\mu\nu}] = \Pi_n$ [EigenValues λ_n] for $T^{\mu}{}_{\nu} \rightarrow (Pfaffian[T^{\mu\nu}])^2$ for 4D anti-symmetric

Inner Product Tensor Invariant: $IP[T^{\mu}] = LSP[T^{\mu},T^{\nu}] = T^{\mu}\eta_{\mu\nu}T^{\nu} = T^{\mu}T_{\mu} = \mathbf{T}\cdot\mathbf{T}$: $IP[T^{\mu\nu}] = T^{\mu\nu}T_{\mu\nu} = T_{IP}$

4-Divergence Tensor Invariant: $4\text{-}Div[T^{\mu}] = \partial_{\mu}T^{\mu} = \partial T^{\mu}/\partial X^{\mu} = \boldsymbol{\partial}\cdot\mathbf{T}$: $4\text{-}Div[T^{\mu\nu}] = \partial_{\mu}T^{\mu\nu} = \partial T^{\mu\nu}/\partial X^{\mu} = S^{\nu}$

Lorentz Scalar Product Tensor Invariant: $LSP[T^{\mu},S^{\nu}] = T^{\mu}\eta_{\mu\nu}S^{\nu} = T^{\mu}S_{\mu} = T_{\nu}S^{\nu} = \mathbf{T}\cdot\mathbf{S} = t^0s^0 - \mathbf{t}\cdot\mathbf{s} = t^0{}_o s^0{}_o$

Phase Space Tensor Invariant: $PS[T^{\mu}] = (d^3\mathbf{t} / t^0) = (dt^1\, dt^2\, dt^3 / t^0)$ for $(\mathbf{T}\cdot\mathbf{T}) = T^{\mu}T_{\mu}$ = constant

The Ratio of 4-Vector Magnitudes (Ratio of Rest Value 4-Scalars): $\mathbf{T}\cdot\mathbf{V} / \mathbf{S}\cdot\mathbf{V} = (t^0{}_o / s^0{}_o)$

Tensor EigenValues $\lambda_n = \{ \lambda_0, \lambda_1, \lambda_2, \lambda_3 \}$: indexed as 0..3, counted as 1..4

The 4D Anti-Symmetric Tensor Products: index bracket [ ] notation indicates anti-symmetric indices:

$T^{\alpha}{}_{\alpha}$ = Trace = Σ_n [EigenValues λ_n] for (1,1)-Tensors

$T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta]}$ = AntiSymm Bi-Product → Inner Product

$T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma]}$ = AntiSymm Tri-Product → ?Name?

$T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma}T^{\delta}{}_{\delta]}$ = AntiSymm Quad-Product → 4D Determinant = Π_n [EigenValues λ_n] for 4D (1,1)-Tensors

These invariants are not all always independent, some invariants are functions of other invariants.

SRQM (Physics) Diagramming Method

$Tr[\eta^{\mu\nu}]=\eta_{\mu\nu}\eta^{\mu\nu}=4$

Trace Tensor Invariant
SpaceTime Dimension from Tr[..] of Minkowski

$\partial[\mathbf{R}]=\partial^{\mu}[R^{\nu}]=\eta^{\mu\nu}$
$\rightarrow Diag[1,-1,-1,-1]=Diag[1,-\delta^{jk}]$
Minkowski Metric

4-Tensor
4D (2,0)-Tensor

4-Gradient ∂^{μ}
$\boldsymbol{\partial}=(\partial_t/c,-\nabla)$
$=\partial/\partial R_{\mu}$

4-Vector
4D (1,0)-Tensor

4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position R^{μ}
$\mathbf{R}=(ct,\mathbf{r})\in$<Event>

4-Scalar
4D (0,0)-Tensor

Lorentz
$\partial_{\nu}[R^{\mu'}]=\partial R^{\mu'}/\partial R^{\nu}=\Lambda^{\mu'}{}_{\nu}$
Transform

SpaceTime
$\boldsymbol{\partial}\cdot\mathbf{R}=\partial_{\mu}R^{\mu}=4$
Dimension

$\mathbf{U}\cdot\boldsymbol{\partial}$ [..]
$\gamma d/dt$ [..]
$d/d\tau$ [..]

ProperTime Derivative

$Det[\Lambda^{\mu'}{}_{\nu}]=\pm1$

Determinant Tensor Invariant
Affine Transform (Anti-)Unitary from Det[..] of Lorentz

$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda_{\mu}{}^{\nu}\Lambda^{\mu}{}_{\nu}$

Inner Product Tensor Invariant
SpaceTime Dimension from IP[..] of Lorentz

4-Divergence Tensor Invariant
SpaceTime Dimension from 4-Divergence of 4-Position

Einstein's
$E = mc^2 = \gamma m_o c^2 = \gamma E_o$
Rest Mass m_o : Rest Energy E_o

4-Velocity U^{μ}
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

4-Momentum P^{μ}
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})=m_o\mathbf{U}$

m_o
E_o/c^2
Rest 4-Scalar

$\mathbf{U}\cdot\mathbf{U}=c^2$

Lorentz Scalar Tensor Invariant
Speed of Light (c) from LSP[..] of 4-Velocity

$d^3\mathbf{p}/E$

Phase Space Tensor Invariant
4-Momentum Phase Space Weighting Factor

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Lorentz Factor =
Relativistic Gamma $\gamma = 1/\sqrt{[1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: $\boldsymbol{\beta} = \mathbf{u}/c$

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4-Vector SRQM Interpretation of QM

SRQM Study: Physical/Mathematical Tensors

Tensor Types: 4-Scalar, 4-Vector, 4-Tensor

Physics Examples – Venn Diagram

A Tensor Study of Physical 4-Vectors

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John B. Wilson
SciRealm@aol.com
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Physical 4-Tensors: Objects of Reality which have **Invariant 4D SpaceTime** (Time·Space) properties

0 index-count Tensors: rank 0

SR 4-Scalar
4D (0,0)-Tensors
Lorentz Scalar S

- EM Charge $(Q)=\int\rho d^3\mathbf{x}$
- Speed-of-Light $(c)=\sqrt{[\mathbf{U}\cdot\mathbf{U}]}$
- $d^3\mathbf{p}/E$
- $\#_{dimensionless}$
- RestMass (m_o)
- Planck's Const (h)
- Dirac $\delta^4[\mathbf{X}-\mathbf{X}_o]$
- ProperTime $\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$ Derivative
- $d^4\mathbf{X}=cdt\cdot dx\cdot dy\cdot dz$
- $V_o=\int\gamma d^3\mathbf{x}$
- SpaceTime $\partial\cdot\mathbf{R}=\partial_\mu R^\mu=4$ Dimension
- $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4$
- $Tr[\eta^{\mu\nu}]=4$
- $Det[\Lambda^{\mu'}{}_\nu]=\pm1$

1 index-count Tensors: rank 1

SR 4-Vector
4D (1,0)-Tensors
contravariant
$\mathbf{V}=\overline{\mathbf{V}}=V^\mu=(v^\mu)$
$=(v^0,\mathbf{v})=(v^0,v^i)\rightarrow(v^t,v^x,v^y,v^z)$

- 4-Position $\mathbf{R}=R^\mu=(ct,\mathbf{r})\in$<Event> $\rightarrow(ct,x,y,z)$
- 4-Velocity $\mathbf{U}=U^\mu=\gamma(c,\mathbf{u})=d\mathbf{R}/d\tau$
- 4-Momentum $\mathbf{P}=P^\mu=(mc,m\mathbf{u})=m_o\mathbf{U}=(E/c,\mathbf{p})=(E_o/c^2)\mathbf{U}$

SR 4-CoVector = "Dual" 4-Vector
4D (0,1)-Tensors = 4D One-Forms
covariant
$\underline{\mathbf{C}}=C_\mu=\eta_{\mu\sigma}C^\sigma=(c_\mu)=(c_0,c_i)\rightarrow(c_t,c_x,c_y,c_z)$
$=(c^0,-\mathbf{c})=(c^0,-c^i)\rightarrow(c^t,-c^x,-c^y,-c^z)$

4D Gradient One-Form
$\underline{\partial}=\partial_\mu=(\partial_t/c,\nabla)=\eta_{\mu\nu}\partial^\nu$
$=\partial/\partial R^\mu\rightarrow(\partial_t/c,\partial_x,\partial_y,\partial_z)$
$=(\partial/c\partial t,\partial/\partial x,\partial/\partial y,\partial/\partial z)$

2 index-count Tensors: rank 2

SR 4-Tensor
4D (2,0)-Tensors
$T^{\mu\nu}=$
$[T^{00},T^{0k}]$
$[T^{j0},T^{jk}]$

- Minkowski Metric $\eta^{\mu\nu}=\partial^\mu[R^\nu]=\partial[\mathbf{R}]=V^{\mu\nu}+H^{\mu\nu}$
- Faraday EM 4-Tensor $F^{\alpha\beta}=\partial^\alpha A^\beta-\partial^\beta A^\alpha=\partial\wedge\mathbf{A}$
- Perfect Fluid 4-Tensor $T^{\mu\nu}=(\rho_{eo})V^{\mu\nu}+(-p_o)H^{\mu\nu}$

SR Mixed 4-Tensor
4D (1,1)-Tensors
$T^\mu{}_\nu=\eta_{\rho\nu}T^{\mu\rho}$
$=$
$[T^0{}_0,T^0{}_k]$
$[T^j{}_0,T^j{}_k]$

Projection (Mixed) Tensors $P^\mu{}_\nu$

- Temporal Projection $P^\mu{}_\nu\rightarrow V^\mu{}_\nu$
- Spatial Projection $P^\mu{}_\nu\rightarrow H^\mu{}_\nu$

Lorentz Transform Tensors $\partial_\nu[R^{\mu'}]=\Lambda^{\mu'}{}_\nu$

- Boost $\Lambda^{\mu'}{}_\nu\rightarrow B^{\mu'}{}_\nu$
- Rotation $\Lambda^{\mu'}{}_\nu\rightarrow R^{\mu'}{}_\nu$
- ParityInverse $\Lambda^{\mu'}{}_\nu\rightarrow(PI)^{\mu'}{}_\nu$

SR Lowered 4-Tensor
4D (0,2)-Tensors
$T_{\mu\nu}=\eta_{\mu\rho}\eta_{\nu\sigma}T^{\rho\sigma}$
$=$
$[T_{00},T_{0k}]$
$[T_{j0},T_{jk}]$

- Lowered Minkowski Metric $\partial_\mu[R_\nu]=\eta_{\mu\nu}=(\cdot)=$ LSP

Projection Tensors $P_{\mu\nu}$

- Temporal Proj. $P_{\mu\nu}\rightarrow V_{\mu\nu}$
- Spatial Proj. $P_{\mu\nu}\rightarrow H_{\mu\nu}$

(>2) index-count Tensors:

SR & GR 4-Tensors $T^{\cdots}_{\cdots}$

- Riemann Curvature Tensor $R^\rho{}_{\sigma\mu\nu}=\partial_\mu\Gamma^\rho{}_{\nu\sigma}-\partial_\nu\Gamma^\rho{}_{\mu\sigma}+\Gamma^\rho{}_{\mu\lambda}\Gamma^\lambda{}_{\nu\sigma}-\Gamma^\rho{}_{\nu\lambda}\Gamma^\lambda{}_{\mu\sigma}\rightarrow0^\rho{}_{\sigma\mu\nu}$ for SR "Flat" Minkowski SpaceTime
- Ricci Tensor $R_{\mu\nu}$
- SchoutenTensor $P_{\mu\nu}$
- BachTensor $B_{\mu\nu}$
- Weyl (Conformal) Curvature Tensor $C^\rho{}_{\sigma\mu\nu}=$ Traceless part of Riemann $[R^\rho{}_{\sigma\mu\nu}]$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

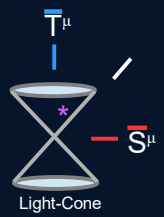
Ricci Decomposition of Riemann Tensor
$R^\rho{}_{\sigma\mu\nu}=S^\rho{}_{\sigma\mu\nu}$ (scalar part) $+E^\rho{}_{\sigma\mu\nu}$ (semi-traceless part) $+C^\rho{}_{\sigma\mu\nu}$ (traceless part)

$Trace[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
$=$ Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

4 SRQM Study: SRQM 4-Vectors = 4D (1,0)-Tensors Measurable objects of Reality



4-Vector SRQM Interpretation of QM

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John B. Wilson
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4-Vector = 4D (1,0)-Tensor	SI Dimensional Units	[Temporal : Spatial] components
4-Position $\mathbf{R} = R^\mu = (ct,\mathbf{r}) = \mathbf{X} = X^\mu$ {alt notation}	[m]	[Time (t) : Space/length/extent ($\mathbf{r}$)]
4-Differential $d\mathbf{R} = dR^\mu = (cdt,d\mathbf{r})$	[m]	[TimeDifferential (dt) : SpaceDifferential ($d\mathbf{r}$)] **Infinitesimal**
4-Displacement $\Delta\mathbf{R} = \Delta R^\mu = (c\Delta t,\Delta\mathbf{r})$	[m]	[TemporalDisplacement (Δt) : SpatialDisplacement ($\Delta\mathbf{r}$)] **Finite**
4-Velocity $\mathbf{U} = U^\mu = \gamma(c,\mathbf{u}) = (\gamma c,\gamma\mathbf{u})$	[m/s]	[Temporal "velocity" factor (γ) : Spatial "velocity" factor ($\gamma\mathbf{u}$), Spatial 3-velocity ($\mathbf{u}$)]
4-UnitTemporal $\bar{\mathbf{T}} = T^\mu = \gamma(1,\boldsymbol{\beta}) = (\gamma,\gamma\boldsymbol{\beta}) = \mathbf{U}/c$	[dimensionless = 1]	[Temporal "velocity" factor (γ) : Spatial normalized "velocity" factor ($\gamma\boldsymbol{\beta}$), Spatial 3-beta ($\boldsymbol{\beta}$)]
4-UnitSpatial $\bar{\mathbf{S}} = S^\mu = \gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{\mathbf{n}},\hat{\mathbf{n}}) = (\gamma_{\beta\hat{n}}\boldsymbol{\beta}\cdot\hat{\mathbf{n}},\gamma_{\beta\hat{n}}\hat{\mathbf{n}})$: $(\bar{\mathbf{T}} \perp \bar{\mathbf{S}})$	[dimensionless = 1]	[Temporal "velocity" factor ($\gamma_{\beta\hat{n}}\boldsymbol{\beta}\cdot\hat{\mathbf{n}}$) : Spatial normalized "velocity" factor ($\gamma_{\beta\hat{n}}\hat{\mathbf{n}}$), Spatial 3-beta ($\boldsymbol{\beta}\cdot\hat{\mathbf{n}}$)$\hat{\mathbf{n}}$]
4-Null $\bar{\mathbf{N}} = N^\mu = a(1,\hat{\mathbf{n}}) = (a,a\hat{\mathbf{n}}) = (\lvert\mathbf{a}\rvert,\mathbf{a})$: $(\bar{\mathbf{N}} \perp \bar{\mathbf{N}})$	[{a} SI Units]	[Temporal factor ($\lvert\mathbf{a}\rvert$) : Spatial factor ($\mathbf{a}$)] with $\bar{\mathbf{N}}\cdot\bar{\mathbf{N}} = 0$: ($\hat{\mathbf{n}}$) = unit-direction 3-vector
4-Momentum $\mathbf{P} = P^\mu = (E/c = mc,\mathbf{p} = m\mathbf{u})$	[N·s = kg·m/s]	[mass (m) : energy (E) : 3-momentum ($\mathbf{p} = m\mathbf{u}$)] with { $E = mc^2 = \gamma m_o c^2 = \gamma E_o$ } & { $\mathbf{p} = m\mathbf{u} = \gamma m_o\mathbf{u}$ }
4-TotalMomentum $\mathbf{P}_T = P_T^\mu = (E_T/c = H/c,\mathbf{p}_T) = \Sigma_n[\mathbf{P}_n]$	[N·s = kg·m/s]	[total-energy (E_T) = Hamiltonian (H) : 3-total-momentum ($\mathbf{p}_T$)]
4-MassFlux=4-MomentumDensity $\mathbf{G} = (\rho_m c,\mathbf{g}) = \mathbf{Q}/c^2 = n_o\mathbf{P}$	[N·s/m³ = kg/m²·s]	[mass density (ρ_m) : 3-mass-flux = 3-momentum-density ($\mathbf{g}$)]
4-HeatEnergyFlux $\mathbf{Q} = (\rho_E c = \mathbf{q}\cdot\boldsymbol{\beta},\mathbf{q}) = c^2\mathbf{G}$	[W/m² = J/m²·s = kg/s³]	[energy density (ρ_E) : 3-energy-flux=PoyntingVector ($\mathbf{q}$)]
4-Acceleration $\mathbf{A} = A^\mu = \gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$: { $\gamma' = d\gamma/dt$ }	[m/s²]	[relativistic Temporal acceleration (γ') : relativistic 3-acceleration ($\gamma'\mathbf{u}+\gamma\mathbf{a}$), 3-acceleration ($\mathbf{a} = \dot{\mathbf{u}}$)]
4-Force $\mathbf{F} = F^\mu = \gamma(\dot{E}/c,\mathbf{f} = \dot{\mathbf{p}}) = (\gamma\dot{E}/c,\gamma\mathbf{f} = \gamma\dot{\mathbf{p}})$	[N = kg·m/s²]	[relativistic power ($\gamma\dot{E}$), power ($\dot{E}$) : relativistic 3-force ($\gamma\mathbf{f}$), 3-force ($\mathbf{f} = \dot{\mathbf{p}}$)]
4-WaveVector $\mathbf{K} = K^\mu = (\omega/c = 1/c\bar{T},\mathbf{k} = \omega\hat{\mathbf{n}}/v_{phase})$	[rad/m]	[angular-frequency ($\omega = 2\pi\nu = 2\pi/T$) : 3-angular-wave-number ($\mathbf{k} = 2\pi\hat{\mathbf{n}}/\lambda = 2\pi\nu\hat{\mathbf{n}}/v_{phase} = \omega\hat{\mathbf{n}}/v_{phase}$)]
4-TotalWaveVector $\mathbf{K}_T = K_T^\mu = (\omega_T/c,\mathbf{k}_T) = \Sigma_n[\mathbf{K}_n]$	[rad/m]	[total-angular-frequency (ω_T) : 3-total-angular-wave-number ($\mathbf{k}_T$)]
4-(Particle:Dust)NumberFlux $\mathbf{N} = N^\mu = n(c,\mathbf{u}) = (nc,\mathbf{n} = n\mathbf{u})$	[#/m²·s = #·m/s·1/m³]	[Temporal number-density (n) : Spatial 3-number-flux ($\mathbf{n} = n\mathbf{u}$)] with { $n = \gamma n_o$ } & { $\mathbf{n} = n\mathbf{u} = \gamma n_o\mathbf{u}$ }
4-ChargeFlux=4-CurrentDensity $\mathbf{J} = J^\mu = q\mathbf{N} = (\rho c,\mathbf{j} = \rho\mathbf{u})$	[C/m²·s = C·m/s·1/m³]	[charge-density (ρ) : 3-charge-flux = 3-current-density ($\mathbf{j} = \rho\mathbf{u}$)] with { $\rho = \gamma\rho_o$ } & { $\mathbf{j} = \rho\mathbf{u} = \gamma\rho_o\mathbf{u}$ }
4-VectorPotential $\mathbf{A} = A^\mu = (\varphi/c,\mathbf{a}) \rightarrow \mathbf{A}_{EM}$	[T·m = kg·m/C·s]	[scalar-potential = voltage (φ) : 3-vector-potential ($\mathbf{a}$)], typically the EM versions (φ_{EM}) : ($\mathbf{a}_{EM}$)
4-PotentialMomentum $\mathbf{Q} = Q^\mu = q\mathbf{A} = (V/c = q\varphi/c,q\mathbf{a})$	[N·s = kg·m/s]	[potential-energy ($V = q\varphi$) : 3-potential-momentum ($\mathbf{q} = q\mathbf{a}$)], EM ver ($V_{EM} = q\varphi_{EM}$) : ($\mathbf{q}_{EM} = q\mathbf{a}_{EM}$)
4-Gradient $\partial_R = \partial_X = \partial = \partial^\mu = \partial/\partial R_\mu = \partial/\partial X_\mu = (\partial_t/c,-\nabla)$	[1/m]	[Temporal partial differential (∂_t) : Spatial 3-gradient=spatial partial differentials($\nabla = \partial_r \rightarrow (\partial_x,\partial_y,\partial_z)$)]
4-Spin $\mathbf{S} = S^\mu = (s^0 = \mathbf{s}\cdot\boldsymbol{\beta} = \mathbf{s}\cdot\mathbf{u}/c,\mathbf{s})$	[J·s = N·m·s = kg·m²/s]	[Temporal spin ($s^0 = \mathbf{s}\cdot\boldsymbol{\beta} = \mathbf{s}\cdot\mathbf{u}/c$) : Spatial 3-spin ($\mathbf{s}$)] { because $\mathbf{S} \perp \bar{\mathbf{T}} \leftrightarrow [\ \mathbf{S}\cdot\bar{\mathbf{T}} = 0 = \gamma(s^0 - \mathbf{s}\cdot\boldsymbol{\beta})\ ]$ }
4-ThermalVector $\boldsymbol{\theta} = \theta^\mu = (\theta^0,\boldsymbol{\theta}) = (c/k_B T,\mathbf{u}/k_B T)$	[1/N·s = s/kg·m]	[ThermodynamicBeta ($\theta^0/c = \beta = 1/k_B T$) : ThermodynamicBeta 3-MotionVector ($\mathbf{u}/k_B T$)]
4-PureEntropyFlux $\mathbf{S}_{ent} = (s_{ent\_pure}c,\mathbf{s}_{ent\_pure} = s_{ent\_pure}\mathbf{u})$	[J/°K·m²·s = kg/°K·s³]	[pure-entropy-density (s_{ent_pure}) : 3-pure-entropy-flux ($\mathbf{s}_{ent\_pure} = s_{ent\_pure}\mathbf{u}$)]
4-Jerk $\mathbf{J} = J^\mu = \gamma(c[\gamma\gamma']',[\gamma\gamma'\mathbf{u}+\gamma^2\mathbf{a}]')$	[m/s³]	[Temporal 4-Jerk stuff… : Spatial 4-Jerk stuff...]

{ Relativistic Gamma $\gamma = 1/\sqrt{[1-(u/c)^2]} = 1/\sqrt{[1-\beta^2]}$ }

Notational Clash Warnings: 4-Vectors

- 4-Acceleration **A** vs. 4-VectorPotential **A**
- 4-HeatEnergyFlux **Q** vs. 4-PotentialMomentum **Q**
- 4-UnitSpatial $\bar{\mathbf{S}}$ vs. 4-Spin **S**
- 4-SurfaceNormal $\hat{\mathbf{N}}$ vs 4-NumberFlux **N**
- 4-Jerk **J** vs 4-CurrentDensity **J**

Lorentz Factor:
Relativistic Gamma $\gamma = 1/\sqrt{[\ 1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}\ ]}$: $\boldsymbol{\beta} = \mathbf{u}/c$
$\gamma' = d\gamma/dt = \gamma^3\boldsymbol{\beta}\cdot\mathbf{a}/c = \gamma^3\mathbf{u}\cdot\mathbf{a}/c^2$

Notational Clash Warnings: Other

Relativistic Beta $\boldsymbol{\beta} = \mathbf{v}/c = \{\ 0..1\ \}\hat{\mathbf{n}}$ [dimensionless]
Thermodynamic Beta $\beta = 1/k_B T$ [1/J] with k_B=Boltzmann's Const & T = AbsoluteTemperature

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

4-Tensors can be constructed from the Tensor Products of 4-Vectors. Technically, 4-Tensors refer to all SR objects (4-Scalars, 4-Vectors, etc), but typically reserve the name 4-Tensor for SR Tensors of 2 or more indices. Use #D (m,n)-Tensor notation to specify types more precisely.

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

4

SRQM Study:

SRQM 4-Tensors = 4D (2,0)-Tensors {or higher index #}

Made from 4-Vector relations

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

4-Tensor = 4D (2,0)-Tensor

SI Dimensional Units [T = kg/C·s]

[Temporal-Temporal : Temporal-Spatial : Spatial-Spatial] components

Faraday EM Tensor $F^{\mu\nu}$ = [0 , $-e^{0j}/c$]
Antisymmetric [$+e^{i0}/c$, $-\varepsilon^{ij}{}_k b^k$]

[(0) : 3-electric-field ($\mathbf{e} = e^i = e^{i0}$) : 3-magnetic-field ($\mathbf{b} = b^k$)] $F^{\mu\nu} = \partial\wedge\mathbf{A} = \partial^\mu A^\nu - \partial^\nu A^\mu$

4-Angular Momentum $M^{\mu\nu}$ = [0 , $-cn^{0j}$]
Antisymmetric Tensor [$+cn^{i0}$, $-\varepsilon^{ij}{}_k I^k$]

[J·s = N·m·s = kg·m²/s] [(0) : 3-mass-moment ($\mathbf{n} = n^i = n^{i0}$) : 3-angular-momentum ($\mathbf{I} = I^k$)] $M^{\mu\nu} = \mathbf{X}\wedge\mathbf{P} = X^\mu P^\nu - X^\nu P^\mu$

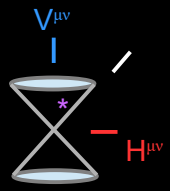
Minkowski Metric $\eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu} \rightarrow \mathrm{Diag}[1,-\delta^{jk}]_{MCRF}$

[dimensionless = 1] [1 : **0** : $-\mathbf{I}_{(3)} = -\delta^{jk}$]

Temporal Projection Tensor $V^{\mu\nu} \rightarrow \mathrm{Diag}[1, 0^{jk}]_{MCRF}$
Spatial Projection Tensor $H^{\mu\nu} \rightarrow \mathrm{Diag}[0,-\delta^{jk}]_{MCRF}$

[dimensionless = 1] [1 : **0** : $\mathbf{0}_{(3)} = 0^{jk}$]
[dimensionless = 1] [0 : **0** : $-\mathbf{I}_{(3)} = -\delta^{jk}$]

SR 4-Tensor
4D (2,0)-Tensor $\boldsymbol{T}$
$T^{\mu\nu}$ =
[T^{00}, T^{0k}]
[T^{j0} , T^{jk}]



$\eta^{\mu\nu} = \partial^\mu[R^\nu] = V^{\mu\nu} + H^{\mu\nu}$

$V^{\mu\nu} = \bar{T}^\mu\bar{T}^\nu$ (L-C Vertical)
$H^{\mu\nu} = \eta^{\mu\nu} - \bar{T}^\mu\bar{T}^\nu$ (L-C Horizontal)

Relativistic-Fluid Stress-Energy(Density) [ρ_e , $+cq^{0j}$]
Symmetric Tensor $T_{relativisticfluid}{}^{\mu\nu} \rightarrow$ [$+cq^{j0}$, $p\delta^{ij} + \Pi^{ij}$]$_{MCRF}$

[Pa=J/m³=N/m²=kg/m·s²] [energy-density ρ_e : 3-heat-flux ($\mathbf{q} = q^i = q^{i0}$) :3-pressure & 3-viscous-shear $p\delta^{ij} + \Pi^{ij}$]
$T_{relfluid}{}^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} + (\bar{T}^\mu H^\nu{}_\sigma Q^\sigma + Q^\sigma H^\mu{}_\sigma \bar{T}^\nu)/c + H^\mu{}_\rho H^\nu{}_\sigma \Pi^{\sigma\rho}$

Perfect-Fluid Stress-Energy(Density)
Symmetric Tensor $T_{perfectfluid}{}^{\mu\nu} \rightarrow \mathrm{Diag}[\rho_e,p,p,p]_{MCRF}$

[Pa=J/m³=N/m²=kg/m·s²] [ρ_e : **0** : $p\mathbf{I}_{(3)} = p\delta^{jk}$] $T_{perfluid}{}^{\mu\nu} = (\rho_{eo}+p_o)\bar{T}^\mu\bar{T}^\nu - (p_o)\partial^\mu[R^\nu] = (\rho_{eo}+p_o)U^\mu U^\nu/c^2 - (p_o)\eta^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu}$

Matter-Dust Stress-Energy(Density)
Symmetric Tensor $T_{matterdust}{}^{\mu\nu} \rightarrow \mathrm{Diag}[\rho_e,0,0,0]_{MCRF}$

[Pa=J/m³=N/m²=kg/m·s²] [ρ_e : **0** : $\mathbf{0}_{(3)} = 0^{jk}$] $T_{matterdust}{}^{\mu\nu} = (\rho_{eo})\bar{T}^\mu\bar{T}^\nu = (\rho_{eo})V^{\mu\nu} = (\rho_{eo})U^\mu U^\nu/c^2 = (m_o)U^\mu(n_o)U^\nu/c^2 = P^\mu N^\nu/c^2$

Any {index>0} SR Tensor can be decomposed into separate orthogonal parts: T^μ = [$\mathrm{Temporal}^\mu$ + $\mathrm{Spatial}^\mu$] , $T^{\mu\nu}$ = [$\mathrm{Temporal}^{\mu\nu}$ + $\mathrm{Mixed}^{\mu\nu}$ + $\mathrm{Spatial}^{\mu\nu}$]
by using combinations of (V)ertical & (H)orizontal Projection Tensors, with ($\eta^\mu{}_\nu = V^\mu{}_\nu + H^\mu{}_\nu$)
and contraction orthogonality ($V^\mu{}_\alpha H^\alpha{}_\nu$) = ($V^\mu{}_\alpha(\eta^\alpha{}_\nu - V^\alpha{}_\nu)$) = ($V^\mu{}_\alpha\eta^\alpha{}_\nu - V^\mu{}_\alpha V^\alpha{}_\nu$) = ($V^\mu{}_\nu - V^\mu{}_\nu$) = $0^\mu{}_\nu$

$\mathrm{Temporal}^\mu = V^\mu{}_\alpha(T^\alpha)$: $\mathrm{IP}[\mathrm{Temporal}^\mu] = \eta_{\mu\nu}V^\mu{}_\alpha(T^\alpha)V^\nu{}_\beta(T^\beta) = \{T^0\}^2$

$\mathrm{Spatial}^\mu = H^\mu{}_\alpha(T^\alpha)$: $\mathrm{IP}[\mathrm{Spatial}^\mu] = \eta_{\mu\nu}H^\mu{}_\alpha(T^\alpha)H^\nu{}_\beta(T^\beta) = -\{(T^1)^2+(T^2)^2+(T^3)^2\}$

$\mathrm{Temporal}^{\mu\nu} = V^\mu{}_\alpha V^\nu{}_\beta(T^{\alpha\beta})$: $\mathrm{Tr}[\mathrm{Temporal}^{\mu\nu}] = \eta_{\mu\nu}V^\mu{}_\alpha V^\nu{}_\beta(T^{\alpha\beta}) = V_{\nu\alpha}V^\nu{}_\beta(T^{\alpha\beta}) = V_{\alpha\beta}(T^{\alpha\beta}) = \{T^{00}\}$
$\mathrm{Mixed}^{\mu\nu} = V^\mu{}_\alpha H^\nu{}_\beta(T^{\alpha\beta}) + H^\mu{}_\alpha V^\nu{}_\beta(T^{\alpha\beta})$: $\mathrm{Tr}[\mathrm{Mixed}^{\mu\nu}] = \eta_{\mu\nu}(V^\mu{}_\alpha H^\nu{}_\beta(T^{\alpha\beta})+H^\mu{}_\alpha V^\nu{}_\beta(T^{\alpha\beta})) = (V_{\nu\alpha}H^\nu{}_\beta+H_{\nu\alpha}V^\nu{}_\beta)(T^{\alpha\beta}) = \{0\}$
$\mathrm{Spatial}^{\mu\nu} = H^\mu{}_\alpha H^\nu{}_\beta(T^{\alpha\beta})$: $\mathrm{Tr}[\mathrm{Spatial}^{\mu\nu}] = \eta_{\mu\nu}H^\mu{}_\alpha H^\nu{}_\beta(T^{\alpha\beta}) = H_{\nu\alpha}H^\nu{}_\beta(T^{\alpha\beta}) = H_{\alpha\beta}(T^{\alpha\beta}) = -\{T^{11}+T^{22}+T^{33}\}$

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SRQM Study:

4-Scalars = 4D (0,0)-Tensors = Lorentz Scalars = 4D SR Invariants ↔ Physical Constants

Made from 4-Vector relations

4-Vector SRQM Interpretation of QM

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4-Scalar = 4D (0,0)-Tensor = SR Invariant	SI Dimensional Units
RestTime:ProperTime ($t_o = \tau$)	[s]
RestTime:ProperTime Differential ($dt_o = d\tau$)	[s]
ProperTimeDerivative ($d/dt_o = d/d\tau$)	[1/s]
FormalDifferential (d[..])	[dimensionless = 1]
MaxSignalSpeed = LightSpeed (c)	[m/s]
RestMass ($m_o = E_o/c^2 = \hbar\omega_o/c^2$)	[kg]
RestEnergy ($E_o = m_oc^2 = \hbar\omega_o$)	[$J = kg\cdot m^2/s^2$]
RestThermodynamicBeta ($\beta_o = 1/k_BT_o$)	[$1/J = s^2/kg\cdot m^2$]
RestAngFrequency ($\omega_o = E_o/\hbar$)	[rad/s]
RestChargeDensity ($\rho_o = qn_o$)	[C/m^3]
RestScalarPotential (φ_o)	[$V = J/C = kg\cdot m^2/C\cdot s^2$]
RestNumberDensity (n_o)	[$\#/m^3$]
SR Phase ($\Phi_{phase} = S_{action}/\hbar$)	$[rad]_{angle}$
SR Action ($S_{action} = \hbar\Phi_{phase}$)	$[J\cdot s]_{action}$
Planck Constant ($h = \hbar*2\pi$)	[$J\cdot s = N\cdot m\cdot s = kg\cdot m^2/s$] /cyc
Dirac:Planck-Reduced Constant ($\hbar = h/2\pi$)	[$J\cdot s = N\cdot m\cdot s = kg\cdot m^2/s$] /rad
SpaceTime Dimension (4) → [4D]	[dimensionless = 1]
Electric Constant (ε_o)	[$F/m = C^2\cdot s^2/kg\cdot m^3$]
Magnetic Constant (μ_o)	[$H/m = kg\cdot m/C^2$]
EM Charge (q)	[$C = A\cdot s$]
EM Charge (Q) *alt method*	[$C = A\cdot s$]
Particle # ($N = n_oV_o$)	[#]
Rest Volume (V_o)	[m^3]
Rest(MCRF) EnergyDensity ($\rho_{eo} = n_oE_o$)	[$Pa = J/m^3 = N/m^2 = kg/m\cdot s^2$]
Rest(MCRF) Pressure (p_o)	[$Pa = J/m^3 = N/m^2 = kg/m\cdot s^2$]
Faraday EM InnerProduct Invariant $2(\mathbf{b}\cdot\mathbf{b}-\mathbf{e}\cdot\mathbf{e}/c^2)$	[$T^2 = kg^2/C^2\cdot s^2$]
Faraday EM Determinant Invariant $(\mathbf{e}\cdot\mathbf{b}/c)^2$	[$T^4 = kg^4/C^4\cdot s^4$]

4-Scalar = 4D (0,0)-Tensor {generally composed of 4-Vector combinations using LSP}

$(\tau) = [\mathbf{R}\cdot\mathbf{U}]/[\mathbf{U}\cdot\mathbf{U}] = [\mathbf{R}\cdot\mathbf{R}]/[\mathbf{U}\cdot\mathbf{R}] = [\mathbf{R}\cdot\mathbf{K}]/[\mathbf{U}\cdot\mathbf{K}]$ **Time as measured in the at-rest frame**

$(d\tau) = [d\mathbf{R}\cdot\mathbf{U}]/[\mathbf{U}\cdot\mathbf{U}]$ **Differential Time as measured in the at-rest frame**

$(d/d\tau) = [\mathbf{U}\cdot\partial] = \gamma(d/dt)$ **Note that the 4-Gradient operator is to the right of 4-Velocity**

$(d[..]) = [d\mathbf{R}\cdot\partial][..]$ **Shows that the formal differential is an Invariant operation**

$(c) = \sqrt{[\mathbf{U}\cdot\mathbf{U}]} = [\bar{\mathbf{T}}\cdot\mathbf{U}]$ with 4-UnitTemporal $\bar{\mathbf{T}} = \gamma(1,\boldsymbol{\beta}) = \mathbf{U}/c$ & $[\bar{\mathbf{T}}\cdot\bar{\mathbf{T}}] = +1$ = "Unit"

$(m_o) = [\mathbf{P}\cdot\mathbf{U}]/[\mathbf{U}\cdot\mathbf{U}] = [\mathbf{P}\cdot\mathbf{R}]/[\mathbf{U}\cdot\mathbf{R}]$ $(m_o \to m_e)$ as Electron RestMass

$(E_o) = [\mathbf{P}\cdot\mathbf{U}]$

$(\beta_o = 1 / k_BT_o) = [\boldsymbol{\Theta}\cdot\mathbf{U}]/[\mathbf{U}\cdot\mathbf{U}]$ with k_B = Boltzmann's Const & T_o = RestTemperature

$(\omega_o) = [\mathbf{K}\cdot\mathbf{U}]$

$(\rho_o) = [\mathbf{J}\cdot\mathbf{U}]/[\mathbf{U}\cdot\mathbf{U}] = (q)[\mathbf{N}\cdot\mathbf{U}]/[\mathbf{U}\cdot\mathbf{U}] = (q)(n_o)$

$(\varphi_o) = [\mathbf{A}\cdot\mathbf{U}]$ $(\varphi_o \to \varphi_{EMo})$ as the EM version RestScalarPotential

$(n_o) = [\mathbf{N}\cdot\mathbf{U}]/[\mathbf{U}\cdot\mathbf{U}]$ = Particle# N / RestVolume V_o

$(\Phi_{phase,free}) = -[\mathbf{K}\cdot\mathbf{R}] = (\mathbf{k}\cdot\mathbf{r} - \omega t)$: $(\Phi_{phase}) = -[\mathbf{K}_T\cdot\mathbf{R}] = (\mathbf{k}_T\cdot\mathbf{r} - \omega_T t)$ **Units [Angle] = [WaveVec.]·[Length] = [Freq.]·[Time]**

$(S_{action,free}) = -[\mathbf{P}\cdot\mathbf{R}] = (\mathbf{p}\cdot\mathbf{r} - Et)$: $(S_{action}) = -[\mathbf{P}_T\cdot\mathbf{R}] = (\mathbf{p}_T\cdot\mathbf{r} - E_Tt)$ **Units [Action] = [Momentum]·[Length] = [Energy]·[Time]**

$(h) = [\mathbf{P}\cdot\mathbf{U}]/[\mathbf{K}_{cyc}\cdot\mathbf{U}] = [\mathbf{P}\cdot\mathbf{R}]/[\mathbf{K}_{cyc}\cdot\mathbf{R}]$: $\mathbf{K}_{cyc} = \mathbf{K}/(2\pi)$

$(\hbar) = [\mathbf{P}\cdot\mathbf{U}]/[\mathbf{K}\cdot\mathbf{U}] = [\mathbf{P}\cdot\mathbf{R}]/[\mathbf{K}\cdot\mathbf{R}]$: $\mathbf{K} = (2\pi)\mathbf{K}_{cyc}$

$(4) = [\partial\cdot\mathbf{R}] = Tr[\eta^{\alpha\beta}] = \Lambda_{\mu\nu}\Lambda^{\mu\nu}$ SR Dim = 4, InnerProduct[any Lorentz Transf{cont.,discrete}] = 4

$\partial\cdot F^{\alpha\beta} = (\mu_o)\mathbf{J} = (1/\varepsilon_oc^2)\mathbf{J}$ Maxwell EM Eqn. w/ source (no spin) $\mu_o\varepsilon_o = 1/c^2$

$\partial\cdot F^{\alpha\beta} = (\mu_o)\mathbf{J} = (1/\varepsilon_oc^2)\mathbf{J}$ Maxwell EM Eqn. w/ source (no spin) $\mu_o\varepsilon_o = 1/c^2$

$\mathbf{U}\cdot F^{\alpha\beta} = (1/q)\mathbf{F}$ Lorentz Force Eqn. $(q \to -e)$ as Electron Charge

$(Q) = \int\rho(dxdydz) = \int\rho d^3\mathbf{x} = \int\rho_o\gamma d^3\mathbf{x} = \int(\rho_o)(dA)(\gamma dr)$ Integration of volume charge density

$(N) = \int n(dxdydz) = \int nd^3\mathbf{x} = \int n_o\gamma d^3\mathbf{x} = \int(n_o)(dA)(\gamma dr)$ Integration of volume number density

$(V_o) = \int\gamma(dxdydz) = \int\gamma d^3\mathbf{x} = \int(dA)(\gamma dr)$ Integration of volume elements (Riemannian Volume Form)

$(\rho_{eo}) = V_{\alpha\beta}T^{\alpha\beta}$ = Temporal "(V)ertical" Projection of (Rel:Perfect)Fluid Stress-Energy Tensor

$(p_o) = (-1/3)H_{\alpha\beta}T^{\alpha\beta}$ = **Spatial** "(H)orizontal" Projection of (Rel:Perfect)Fluid Stress-Energy Tensor

$2(\mathbf{b}\cdot\mathbf{b}-\mathbf{e}\cdot\mathbf{e}/c^2) = IP[F^{\alpha\beta}] = F^{\alpha\beta}F_{\alpha\beta}$

$(\mathbf{e}\cdot\mathbf{b}/c)^2 = Det[F^{\alpha\beta}] \to (Pfaffian[F^{\alpha\beta}])^2$, since $F^{\alpha\beta}$ is $(2n \times 2n)$ square anti-symmetric

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Lorentz Scalars = 4D (0,0)-Tensors can be constructed from the Lorentz Scalar Products (LSP) of 4-Vectors: ex. "multiplication" $(\mathbf{A}\cdot\mathbf{B})$ & "division" $(\mathbf{A}\cdot\mathbf{V})/(\mathbf{B}\cdot\mathbf{V})$. Both of these give invariant scalar products. A "mediating" 4-Vector $\mathbf{V}$ is required for division, since vectors by themselves don't divide, they must be made into scalars 1st.

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SRQM Study: Physical 4-Vectors

Some SR 4-Vectors and Symbols

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4-Gradient [1/m]
4-"Position"Gradient
$\partial=\partial_R=\partial_X=\partial^\mu=(\partial_t/c,-\nabla)$
$=\partial/\partial R_\mu \rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Gradient 4-Vector [operator]
$\partial=\bar{\partial}=\partial^\mu=(\partial_t/c,-\nabla)$
$\underline{\partial}=\partial_\mu=(\partial_t/c,\nabla)$
Gradient One-Form [operator]

4-Displacement [m]
$\Delta\mathbf{R}=\Delta R^\mu=(c\Delta t,\Delta\mathbf{r})=\mathbf{R}_2-\mathbf{R}_1$ {finite}
$d\mathbf{R}=dR^\mu=(cdt,d\mathbf{r})$ {infinitesimal}

4-Position [m]
$\mathbf{R}=R^\mu=(ct,\mathbf{r})\in$<Event>
$\rightarrow(ct,x,y,z)_{\{rect\}}$
alt. notation $\mathbf{X}=X^\mu$

Lorentz Invariant, but not Poincaré Invariant

[1] **4-UnitTemporal**
$\bar{\mathbf{T}}=T^\mu=\gamma(1,\boldsymbol{\beta})$
$=\gamma(1,\mathbf{u}/c)=\mathbf{U}/c$

4-Velocity [m/s]
$\mathbf{U}=U^\mu=\gamma(c,\mathbf{u})$
$=c\bar{\mathbf{T}}=c^2\partial[\tau]=d\mathbf{R}/d\tau$

4-Acceleration [m/s²]
$\mathbf{A}=A^\mu=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\dot{\mathbf{u}})$
$=d\mathbf{U}/d\tau=d^2\mathbf{R}/d\tau^2$: $\{\gamma'=d\gamma/dt\}$

[1] **4-UnitSpatial**
$\bar{\mathbf{S}}=S^\mu=\gamma_{\beta n}(\boldsymbol{\beta}\cdot\hat{\mathbf{n}},\hat{\mathbf{n}})$
(depends on direction $\hat{\mathbf{n}}$)

4-Spin [J·s = kg·m²/s]
$\mathbf{S}_{spin}=S_{spin}{}^\mu$
$=(s^0=\boldsymbol{\beta}\cdot\mathbf{s},\mathbf{s})=s_o\bar{\mathbf{S}}$

4-Momentum [kg·m/s = N·s]
$\mathbf{P}=P^\mu=(mc,m\mathbf{u})=m_o\gamma(c,\mathbf{u})=m_o\mathbf{U}=-\partial[S_{action}]$
$=(E/c,\mathbf{p})=(E_o/c^2)\gamma(c,\mathbf{u})=(E_o/c^2)\mathbf{U}=(E_o/c)\bar{\mathbf{T}}$

4-WaveVector [rad/m]
$\mathbf{K}=K^\mu=(\omega/c=2\pi\nu/c,\mathbf{k})=(\omega_o/c^2)\mathbf{U}=-\partial[\Phi_{phase}]$
$=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})=(1/c\bar{\lambda},\hat{\mathbf{n}}/\lambda)=(\omega_o/c)\bar{\mathbf{T}}$

4-(EM)VectorPotential [kg·m/C·s = T·m]
$\mathbf{A}=A^\mu=(\varphi/c,\mathbf{a})=(\varphi_o/c^2)\mathbf{U}=(\varphi_o/c)\bar{\mathbf{T}}$
$\mathbf{A}_{EM}=A_{EM}{}^\mu=(\varphi_{EM}/c,\mathbf{a}_{EM})$

q

4-(EM)VectorPotentialMomentum [kg·m/s = N·s]
$\mathbf{Q}=Q^\mu=(q\varphi/c,q\mathbf{a})=(V/c,\mathbf{q})$
$=q\mathbf{A}=(q\varphi_o/c^2)\mathbf{U}=(V_o/c^2)\mathbf{U}=(V_o/c)\bar{\mathbf{T}}$

4-ChargeFlux : 4-CurrentDensity [C/m²·s = A/m²]
$\mathbf{J}=J^\mu=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})=\rho_o\gamma(c,\mathbf{u})=\rho_o\mathbf{U}$
$=qn_o\mathbf{U}=q\mathbf{N}=\rho_o c\bar{\mathbf{T}}$

q

4-(Particle:Dust:Number)Flux [#/m²·s = #/m³·m/s]
$\mathbf{N}=N^\mu=(nc,\mathbf{n})=n(c,\mathbf{u})=n_o\gamma(c,\mathbf{u})$
$=n_o\mathbf{U}=n_oc\bar{\mathbf{T}}$

4-ThermalVector [m/J·s = 1/N·s]
4-InverseTemperatureMomentum
$\boldsymbol{\Theta}=\Theta^\mu=(\theta^0,\boldsymbol{\theta})=(c/k_BT,\mathbf{u}/k_BT)=(\theta_o/c)\mathbf{U}=(\theta_o)\bar{\mathbf{T}}$
$=(1/k_BT)(c,\mathbf{u})=(1/k_B\gamma T)\mathbf{U}=(1/k_BT_o)\mathbf{U}$

4-(Minkowski)Force [kg·m/s² = N]
$\mathbf{F}=F^\mu=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$
$=d\mathbf{P}/d\tau=\gamma d\mathbf{P}/dt$
sometimes written as $\mathbf{K}$ or $\mathbf{M}$

4-MassFlux [N·s/m³ = kg/m²·s]
4-MomentumDensity
$\mathbf{G}=G^\mu=(\rho_mc,\mathbf{g})=\rho_m(c,\mathbf{u})$
$=m_o\mathbf{N}=n_om_o\mathbf{U}=\rho_{mo}\mathbf{U}=n_o\mathbf{P}=\mathbf{Q}/c^2$

c²

4-HeatEnergyFlux
$\mathbf{Q}=Q^\mu=(\rho_Ec,\mathbf{q})=\rho_E(c,\mathbf{u})$
$=E_o\mathbf{N}=n_oE_o\mathbf{U}=\rho_{Eo}\mathbf{U}=c^2\mathbf{G}$
$=(\boldsymbol{\beta}\cdot\mathbf{q},\mathbf{q})$ because $\bar{T}_\mu Q^\mu=0$ [W/m² = J/m²·s = kg/s³]

4-PureEntropyFlux [W/°K·m² = J/°K·m²·s = kg/°K·s³]
$\mathbf{S}_{ent\_pure}=S_{ent\_pure}{}^\mu$
$=(s_{ent\_pure}{}^0,\mathbf{s}_{ent\_pure})$
$=S_{ent}\mathbf{N}=k_B\,Ln[\Omega]\mathbf{N}=n_oS_{ent}\mathbf{U}$

4-HeatEntropyFlux
$\mathbf{S}_{ent\_heat}=S_{ent\_heat}{}^\mu$
$=(s_{ent\_heat}{}^0,\mathbf{s}_{ent\_heat})$
$=S_{ent}\mathbf{N}+\mathbf{Q}/T_o=S_{ent}\mathbf{N}+E_o\mathbf{N}/T_o$
$=n_o(S_{ent}+E_o/T_o)\mathbf{U}$

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Lorentz Scalar

4-Vector V $=V^\mu=(v^\mu)=(v^0,v^i)=(v^0,\mathbf{v})$
SR 4-Vector V $=V^\mu=(\text{scalar}*c^{\pm1},\text{3-vector})$

$\dot{\mathbf{v}}=d\mathbf{v}/dt$
$\ddot{\mathbf{v}}=d^2\mathbf{v}/dt^2$

$\text{Trace}[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

$\overline{T}$ $\overline{S}$

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SRQM Study:

Primary/Primitive/Elemental 4-Vectors: 4-UnitTemporal $\overline{T}$ & 4-UnitSpatial $\overline{S}$

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4-UnitTemporal, [dimensionless=1]
Magnitude^2 = (+1)
"Magnitude" = (±1)
|Magnitude| = (1)

$\overline{T}\cdot\overline{T} = \gamma(1,\boldsymbol{\beta})\cdot\gamma(1,\boldsymbol{\beta})$
$= \gamma^2(1*1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}) = \gamma^2(1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta})$
$= +1$

4-UnitTemporal
$\overline{T} = T^\mu = \gamma(1,\boldsymbol{\beta})$
$= \gamma(1,\mathbf{u}/c) = \mathbf{U}/c$

LightSpeed Invariant (c) [m/s]

c

$\mathbf{U}\cdot\mathbf{U}=c^2$

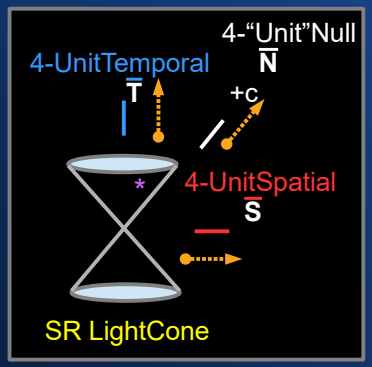
4-Velocity
$\mathbf{U} = U^\mu = \gamma(c,\mathbf{u}) = c\gamma(1,\boldsymbol{\beta})$
$= (\mathbf{U}\cdot\partial)\mathbf{R} = d\mathbf{R}/d\tau = c\overline{T}$

a "Temporal" 4-Vector, [m/s]
Magnitude^2 = $+(c)^2$
"Magnitude" = (±c)
|Magnitude| = (c)

3 independent components

4-UnitTemporal orthogonal-to (⊥) 4-UnitSpatial [dimensionless=1]
Magnitude^2 = (0)
"Magnitude" = (0)
|Magnitude| = (0)

$\overline{T}\cdot\overline{S}$
$= \gamma(1,\boldsymbol{\beta})\cdot\gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{n},\hat{n})$
$= \gamma^*\gamma_{\beta\hat{n}}(1^*\boldsymbol{\beta}\cdot\hat{n} - \boldsymbol{\beta}\cdot\hat{n})$
$= \gamma^*\gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{n} - \boldsymbol{\beta}\cdot\hat{n})$
$= 0 \leftrightarrow (\overline{T} \perp \overline{S})$



Relativistic Lorentz Gamma γ $\boldsymbol{\beta} = \mathbf{u}/c$
$\gamma = 1/\sqrt{[1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}]} = 1/\sqrt{[1 - |\boldsymbol{\beta}|^2]}$
$\gamma_{\beta\hat{n}} = 1/\sqrt{[1 - \boldsymbol{\beta}_{\hat{n}}\cdot\boldsymbol{\beta}_{\hat{n}}]} = 1/\sqrt{[1 - |\boldsymbol{\beta}_{\hat{n}}|^2]}$

with $\boldsymbol{\beta}_{\hat{n}} = (\boldsymbol{\beta}\cdot\hat{n})\hat{n}$ = component of vector $\boldsymbol{\beta}$ along the $\hat{n}$-direction, ($|\hat{n}|=1$)

In the RestFrame ($\boldsymbol{\beta} = \mathbf{0}$) of a massive particle ($m_o > 0$):
4-Velocity **U** appears totally Temporal $\mathbf{U}_o = c(1,\mathbf{0})$
4-Spin **S** appears totally Spatial $\mathbf{S}_o = s_o(0,\hat{n})$

In the Light:NullFrame ($\boldsymbol{\beta} = \hat{n}$) of a massless particle ($m_o = 0$) still have $\mathbf{U}\cdot\mathbf{U}=c^2$
4-Null $\overline{\mathbf{N}}$: no RestFrame : 4-Velocity **U** undetermined
4-Spin **S** loses 1 dim, helical about propagation dir.

{ 4-UnitTemporal $\overline{\mathbf{T}}$: 4-UnitSpatial $\overline{\mathbf{S}}$: 4-"Unit"Null $\overline{\mathbf{N}}$ } are dimensionless, which allows them to make dimensional 4-Vectors via multiplication by dimensional 4-Scalars, as shown here.

a "Spatial" 4-Vector, [J·s]
Magnitude^2 = $-(s_o)^2$
"Magnitude" = $(\pm is_o)$
|Magnitude| = (s_o)

3 independent components

4-UnitSpatial
$\overline{S} = S^\mu = \gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{n},\hat{n})$
(depends on direction $\hat{n}$)

Spin (s_o) Invariant [J·s]

s_o

4-Spin
$\mathbf{S}_{spin} = S_{spin}^\mu = s_o\overline{S}$
$(s^0,\mathbf{s}) = (\boldsymbol{\beta}\cdot\mathbf{s},\mathbf{s})$
$= s_o\gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{n},\hat{n}) = s(\boldsymbol{\beta}\cdot\hat{n},\hat{n})$

$\mathbf{S}_{spin}\cdot\mathbf{S}_{spin}$
$= (s^0,\mathbf{s})\cdot(s^0,\mathbf{s})$
$= (s^{0*}s^0 - \mathbf{s}\cdot\mathbf{s})$
$= -(s_o)^2$

4-UnitSpatial, [dimensionless=1]
Magnitude^2 = (-1)
"Magnitude" = (±i)
|Magnitude| = (1) = √[(Mag*)(Mag)]

$\overline{S}\cdot\overline{S} = \gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{n},\hat{n})\cdot\gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{n},\hat{n})$
$= \gamma_{\beta\hat{n}}^2(\boldsymbol{\beta}\cdot\hat{n}^*\boldsymbol{\beta}\cdot\hat{n} - \hat{n}\cdot\hat{n}) = -\gamma_{\beta\hat{n}}^2(\hat{n}\cdot\hat{n} - (\boldsymbol{\beta}\cdot\hat{n})^2)$
$= -\gamma_{\beta\hat{n}}^2(1 - (\boldsymbol{\beta}\cdot\hat{n})^2)$
$= -1$

4-Scalar (a) Dirac-δ Invariant [{a_{zero}}]

a_{zero}

4-"Unit"Null [dimensionless=1]
Magnitude^2 = (0/0)
"Magnitude" = (0/0)
|Magnitude| = (0/0)

4-Null [SI Units = {a_{zero}} = {a}]
Magnitude^2 = (0)
"Magnitude" = (0)
|Magnitude| = (0)

4-Null $\mathbf{N} = a_{zero}\overline{\mathbf{N}} = (|\mathbf{a}|,\mathbf{a})$
$= a(1,\hat{n}) = \gamma_\infty a_{zero}(1,\hat{n})$

3 independent components

$\mathbf{N}\cdot\mathbf{N}$
$= a^*a[(1)^*(1)-\hat{n}\cdot\hat{n}]$
$= 0 \leftrightarrow (\mathbf{N} \perp \mathbf{N})$

4-"Unit"Null $\overline{\mathbf{N}}$
$= \gamma_\infty(1,\hat{n})$

2 independent components

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

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SR 4-CoVector:OneForm
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(0,0)-Tensor S or S_o
Lorentz Scalar

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: Primary/Primitive/Elemental 4-Vectors: 4-"Unit"Null $\bar{N}$

4-Vector SRQM Interpretation of QM

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3 independent components [dimensionless = 1]

$\bar{T}\cdot\bar{T} = +1$

4-UnitTemporal
$\bar{T} = T^{\mu} = \gamma(1,\beta) = \gamma(1,u/c) = U/c$

0 independent components – because universal constant [m/s]

c

LightSpeed Invariant (c)

3 independent components [m/s]

$U\cdot U = c^2$

4-Velocity U^{μ}
$U = \gamma(c,u) = c\gamma(1,\beta) = (U\cdot\partial)R = dR/d\tau = c\bar{T}$

$P\cdot U = \gamma(c,u)\cdot(E/c,p) = \gamma(E - p\cdot u) = E_o$

$P_{null}\cdot U = \gamma(c,u)\cdot(|p|,p) = \gamma(|p|c - p\cdot u) = |p_{null}|c = E_{null}$

1 independent component [kg]

m_o
E_o/c^2

Einstein's
$E = mc^2 = \gamma m_o c^2 = \gamma E_o$
Rest Mass m_o : Rest Energy E_o

$\bar{N}\cdot\bar{N} = 0/0 = \text{undefined}$

4-"Unit"Null $\bar{N}$
$= \gamma_{\infty}(1,\hat{n})$

Photonic (Null) limit
$\beta \to 1*\hat{n}$
$\gamma \to \gamma_{\infty} = \infty$

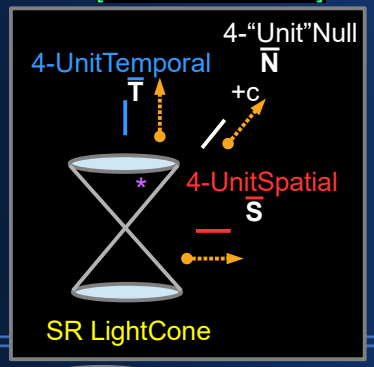
2 independent components – limiting process leaves only 2D "spherical" direction $\hat{n}$ [dimensionless = 1]

$\bar{T}\cdot\bar{S} = 0 \leftrightarrow (\bar{T} \perp \bar{S})$

4-UnitSpatial
$\bar{S} = S^{\mu} = \gamma_{\beta\hat{n}}(\beta\cdot\hat{n},\hat{n})$
(depends on direction $\hat{n}$)

$\bar{S}\cdot\bar{S} = -1$

3 independent components [dimensionless = 1]



Null Momentum Dirac-δ Invariant (p_{zero})

p_{zero}

1 independent component [kg·m/s = N·s]

4-Momentum P^{μ}
$P = (mc,p) = (E/c,p) = m_o U$

4 independent components [kg·m/s = N·s]

$P\cdot P = (m_o c)^2 = (E_o/c)^2 = (E/c)^2 - p\cdot p$

Photonic (Null) limit
$m_o : E_o \to 0$
$E = |p|c$

4-NullMomentum P_{null}^{μ}
$= \text{4-PhotonicMomentum}$
$= P_{null} = p_{zero}\bar{N} = (E/c=|p|,p)$
$= p(1,\hat{n}) = \gamma_{\infty}p_{zero}(1,\hat{n})$

$P_{null}\cdot P_{null} = 0$

3 independent components [kg·m/s = N·s]

SR is one physical case which has (∞*0 = finite)
$p = \gamma_{\infty}p_{zero}$
see Dirac Delta Function

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(0,0)-Tensor S or S_o
Lorentz Scalar

$\text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

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SRQM Study:

SR Lorentz Factor = Relativistic Gamma

$$\gamma = 1/\sqrt{[1-\beta^2]} = dt/d\tau = \partial\tau/\partial t$$

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The Lorentz Gamma Factor ($\gamma = 1/\sqrt{[1 - \beta\cdot\beta]}$) defines how relativistic time (t) is related to proper:rest time (τ). { $t = \gamma\tau = \gamma t_o$} = { ←*Time Dilation*→ }

Many relativistic relations use γ: $\mathbf{P} = (E/c,\mathbf{p}) = m_o\mathbf{U} = m_o\gamma(c,\mathbf{u}) = m(c,\mathbf{u})$
Time·Space factors to Einstein's { $E = \gamma E_o = \gamma m_o c^2 = mc^2$ } & { $\mathbf{p} = m\mathbf{u} = \gamma m_o\mathbf{u}$ }

Let generic 4-Vector $\mathbf{A} = (a^0,\mathbf{a})$
4-Velocity $\mathbf{U} = \gamma(c,\mathbf{u})$: $\mathbf{U}\cdot\mathbf{U} = c^2$

$\mathbf{A}\cdot\mathbf{U} = (a^0,\mathbf{a})\cdot\gamma(c,\mathbf{u}) = \gamma(a^0c - \mathbf{a}\cdot\mathbf{u}) = a^0_oc = (a^0_o/c)c^2 = (a^0_o/c)\mathbf{U}\cdot\mathbf{U}$

$\mathbf{A} = (a^0_o/c)\mathbf{U} = (a^0,\mathbf{a}) = (a^0_o/c)\gamma(c,\mathbf{u}) = (\gamma a^0_o/c)(c,\mathbf{u})$

Temporal part: $a^0 = (\gamma a^0_o)$
Spatial part: $\mathbf{a} = (\gamma a^0_o/c)\mathbf{u} = (a^0/c)\mathbf{u}$

$(\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\mathbf{A}\cdot\mathbf{U}) = (\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})(a^0_oc) = (\gamma\mathbf{u}\cdot\mathbf{u})(a^0_o/c) = (\gamma a^0_o/c)(\mathbf{u}\cdot\mathbf{u}) = \mathbf{a}\cdot\mathbf{u}$

$(\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\mathbf{A}\cdot\mathbf{U}) = (\mathbf{a}\cdot\mathbf{u})$
Important result used in Hamiltonian:Lagrangian connection

ex. $(\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\mathbf{P}\cdot\mathbf{U}) = (\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})(E_o) = (\gamma\mathbf{u}\cdot\mathbf{u})(E_o/c^2) = (E\mathbf{u}/c^2)\cdot\mathbf{u} = (\mathbf{p}\cdot\mathbf{u})$
since $(E\mathbf{u}/c^2) = \mathbf{p}$

$(d\tau/dt) = (\partial\tau/\partial t) + \mathbf{u}\cdot\nabla[\tau]$
$(1/\gamma) = (\gamma) + (-\gamma\beta^2)$

$\gamma' = d\gamma/dt$
$= (-1/2)[1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}]^{-(3/2)}(-2\boldsymbol{\beta})\cdot d\boldsymbol{\beta}/dt$
$= \gamma^3\boldsymbol{\beta}\cdot(d\mathbf{u}/dt)/c$
$= \gamma^3\boldsymbol{\beta}\cdot\mathbf{a}/c = \gamma^3\boldsymbol{\beta}\cdot\boldsymbol{\beta}'$
$= \gamma^3\mathbf{u}\cdot\mathbf{a}/c^2$

Rest Hamiltonian
$\mathbf{P_T}\cdot\mathbf{U} = (H/c,\mathbf{p_T})\cdot\gamma(c,\mathbf{u})$
$= \gamma(H - \mathbf{p_T}\cdot\mathbf{u}) = H_o$
$= -L_o$

4-Velocity = $\gamma(c,\mathbf{u})$: most known 4-Vector with γ exposed

4-Velocity
$\mathbf{U} = U^\mu = \gamma(c,\mathbf{u})$
$= dR^\mu/d\tau = (\mathbf{U}\cdot\partial)\mathbf{R}$
$= c^2\partial[\tau] = (\mathbf{U}\cdot\mathbf{U})\partial[\tau]$

4-TotalMomentum
$\mathbf{P_T} = (E_T/c = H/c,\mathbf{p_T})$
$= -\partial_R[S_{action}]$
$= (-\partial_t/c[S_{action}],\nabla[S_{action}])$
$= -\partial_U[L_o] = \partial_U[H_o]$

T

Invariant ProperTime Derivative
$\mathbf{U}\cdot\partial = \gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla) = \gamma(\partial_t + \mathbf{u}\cdot\nabla)$
$= \gamma d/dt = d/d\tau$

4-Gradient
$\partial = \partial_R = \partial_X = \partial^\mu = (\partial_t/c,-\nabla)$
$= \partial/\partial R_\mu \rightarrow (\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$= (\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

4

Lorentz Factor (γ)
Relativistic Gamma
$\mathbf{U}\cdot\mathbf{U} = c^2$
$\gamma(c,\mathbf{u})\cdot\gamma(c,\mathbf{u}) = c^2$
$\gamma^2(c^2-\mathbf{u}\cdot\mathbf{u}) = c^2$
$\gamma^2(1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}) = 1$
$\gamma^2 = 1/(1-\boldsymbol{\beta}\cdot\boldsymbol{\beta})$
$\gamma = 1/\sqrt{[1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$

$\boldsymbol{\beta} = \mathbf{u}/c$

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$\gamma^2 = 1/(1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta})$
$\gamma^2(1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}) = 1$
$\gamma^2 - \gamma^2\boldsymbol{\beta}\cdot\boldsymbol{\beta} = 1$
$\gamma^2 - 1 = \gamma^2\boldsymbol{\beta}\cdot\boldsymbol{\beta}$
$\gamma - 1/\gamma = \gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta}$

$\boldsymbol{\beta} = \mathbf{u}/c$

$\gamma = 1/\sqrt{[1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$
$(\gamma - 1/\gamma) = (\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})$
$(\gamma - 1/\gamma)(\mathbf{P_T}\cdot\mathbf{U}) = (\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\mathbf{P_T}\cdot\mathbf{U})$
$\gamma(\mathbf{P_T}\cdot\mathbf{U}) + -(\mathbf{P_T}\cdot\mathbf{U})/\gamma = (\mathbf{p_T}\cdot\mathbf{u})$
$\{ H \} + \{ L \} = (\mathbf{p_T}\cdot\mathbf{u})$
The {Hamiltonian:Lagrangian} Connection

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
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Lorentz Factor:
Relativistic Gamma $\gamma = 1/\sqrt{[1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: $\boldsymbol{\beta} = \mathbf{u}/c$
$\gamma' = d\gamma/dt = \gamma^3\boldsymbol{\beta}\cdot\mathbf{a}/c = \gamma^3\mathbf{u}\cdot\mathbf{a}/c^2$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0_o)^2$
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The Lorentz Gamma Factor ($\gamma = 1/\sqrt{[1 - \beta\cdot\beta]}$) defines how relativistic time (t) is related to proper:rest time (τ). { $t = \gamma\tau = \gamma t_o$} = { ←*Time Dilation*→ }

Many relativistic relations use γ: $\mathbf{P} = (E/c,\mathbf{p}) = m_o\mathbf{U} = m_o\gamma(c,\mathbf{u}) = m(c,\mathbf{u})$
Time·Space factors to Einstein's { $E = \gamma E_o = \gamma m_o c^2 = mc^2$ } & { $\mathbf{p} = m\mathbf{u} = \gamma m_o\mathbf{u}$ }

The SR Lorentz factor also plays a role in the total differential vs. the partial, and in defining the Hamiltonian (H) and Lagrangian (L) Energies.

$(\mathbf{U}\cdot\partial) = \gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla) = \gamma(\partial_t + \mathbf{u}\cdot\nabla) = \gamma d/dt = d/d\tau$, giving $(dt/d\tau = \gamma) : (d\tau/dt = 1/\gamma)$
$(\mathbf{U}\cdot\partial)\tau = \gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)\tau = \gamma(\partial_t + \mathbf{u}\cdot\nabla)\tau = \gamma d\tau/dt = d\tau/d\tau = 1$
$c^2(\mathbf{U}\cdot\partial)\tau = c^2\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)\tau = c^2\gamma(\partial_t + \mathbf{u}\cdot\nabla)\tau = c^2\gamma d\tau/dt = c^2 d\tau/d\tau = c^2 = (\mathbf{U}\cdot\mathbf{U})$
$c^2(\mathbf{U}\cdot\partial)\tau = (\mathbf{U}\cdot\mathbf{U})$
$c^2(\partial)\tau = (\mathbf{U})$
$c^2\partial[\tau] = c^2(\partial_t/c,-\nabla)[\tau] = \gamma(c,\mathbf{u}) = \mathbf{U}$
$\partial\tau/\partial t = \gamma$: $-c^2\nabla[\tau] = \gamma\mathbf{u}$
$c^2(-\mathbf{u}\cdot\nabla)[\tau] = \gamma(\mathbf{u}\cdot\mathbf{u})$
$(\mathbf{u}\cdot\nabla)[\tau] = -\gamma(\boldsymbol{\beta}\cdot\boldsymbol{\beta})$

$d\tau = (\partial\tau/\partial t)dt + (\partial\tau/\partial x)dx + (\partial\tau/\partial y)dy + (\partial\tau/\partial z)dz = (d\mathbf{R}\cdot\partial)[\tau]$
$d\tau/dt = (\partial\tau/\partial t)dt/dt + (\partial\tau/\partial x)dx/dt + (\partial\tau/\partial y)dy/dt + (\partial\tau/\partial z)dz/dt$
$d\tau/dt = (dt/dt)(\partial\tau/\partial t) + \{ (dx/dt)(\partial\tau/\partial x) + (dy/dt)(\partial\tau/\partial y) + (dz/dt)(\partial\tau/\partial z) \}$
$d\tau/dt = (\partial\tau/\partial t) + \mathbf{u}\cdot\nabla[\tau] = 1/\gamma$

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$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM: Some Basic 4-Vectors

4-Position, 4-Velocity, 4-Differential, 4-Gradient

SR SpaceTime Calculus & Invariants

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
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$(\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{C}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{C})$
$(\mathbf{A}\cdot\mathbf{C})(\mathbf{B}) = (\mathbf{A})(\mathbf{B}\cdot\mathbf{C})$ if $\mathbf{B}=k\mathbf{A}$
$(\mathbf{B}) = (\mathbf{A})(\mathbf{B}\cdot\mathbf{C})/(\mathbf{A}\cdot\mathbf{C})$
SR Vector Projection works for 4-Vectors in same Lorentz Class

Invariant ProperTime
$\mathbf{R}\cdot\mathbf{U}/\mathbf{U}\cdot\mathbf{U} = (ct,\mathbf{r})\cdot\gamma(c,\mathbf{u})/c^2 = \gamma(c^2t - \mathbf{u}\cdot\mathbf{r})/c^2 = (c^2t_o)/c^2$
$\mathbf{R}\cdot\mathbf{K}/\mathbf{U}\cdot\mathbf{K} = (t\omega - \mathbf{r}\cdot\mathbf{k})/[\gamma(\omega - \mathbf{u}\cdot\mathbf{k})] = (t\omega_o)/(\gamma\omega_o) = t/\gamma = t_o$
$\mathbf{R}\cdot\mathbf{R}/\mathbf{U}\cdot\mathbf{R} = (ct_o)^2/(c^2t_o)$
$= |\mathbf{R}|/|\mathbf{U}| = (ct_o)/(c) = t_o = \tau$

Can also use $\mathbf{R}\cdot\mathbf{V}/\mathbf{U}\cdot\mathbf{V} = \tau$, with $\mathbf{V}$ as any SR 4-Vector. The main idea is the $(\mathbf{R}/\mathbf{U})$; SI Units [m/(m/s)] = [s], but they need a mediating 4-Vector $\mathbf{V}$ to make them into scalars so that division is possible {vectors don't divide}.
$\mathbf{R}\cdot\mathbf{V}/\mathbf{U}\cdot\mathbf{V} = (ct,\mathbf{r})\cdot(v^0,\mathbf{v})/\gamma(c,\mathbf{u})\cdot(v^0,\mathbf{v})$ = Invariant Lorentz Scalar.
Change to frame-reference with $\mathbf{v} = \mathbf{0}$. Since the result is frame-invariant:
$\mathbf{R}\cdot\mathbf{V}/\mathbf{U}\cdot\mathbf{V} = ctv^0/\gamma cv^0 = t/\gamma = \tau = t_o$ = ProperTime = RestTime

4-Position [m]
$\mathbf{R}=R^\mu=(ct,\mathbf{r})\in$ <Event>
$\rightarrow(ct,x,y,z)$

$\mathbf{U}\cdot\partial[..]$
$d/d\tau[..] = d[..]/d\tau$
$\gamma d/dt[..]$

4-Velocity [m/s]
$\mathbf{U}=U^\mu=\gamma(c,\mathbf{u})$
$=dR^\mu/d\tau=(\mathbf{U}\cdot\partial)\mathbf{R}=c^2\partial[\tau]$

Invariant LightSpeed
$\mathbf{U}\cdot\mathbf{U} = \gamma^2(c^2-\mathbf{u}\cdot\mathbf{u})$
$= c^2\gamma^2(1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}) = c^2$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R} = (ct)^2-\mathbf{r}\cdot\mathbf{r} = (c\tau)^2$

$\partial/\partial[\eta_{\mu\nu}..]$

$d\tau$
dt/γ

$d\mathbf{R}\cdot\partial[..]$
$d[..]$

Invariant ProperTime Derivative
$\mathbf{U}\cdot\partial = \gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla) = \gamma(\partial_t + \mathbf{u}\cdot\nabla)$
$= \gamma d/dt = d/d\tau$

Invariant Differential
$d\mathbf{R}\cdot d\mathbf{R} = (cdt)^2-d\mathbf{r}\cdot d\mathbf{r} = (cd\tau)^2$
$\Delta\mathbf{R}\cdot\Delta\mathbf{R} = (c\Delta t)^2-\Delta\mathbf{r}\cdot\Delta\mathbf{r} = (c\Delta\tau)^2$
Interval

4-Differential [m]
$d\mathbf{R}=dR^\mu=(cdt,d\mathbf{r})$ {infintesimal}
$\Delta\mathbf{R}=\Delta R^\mu=(c\Delta t,\Delta\mathbf{r})=\mathbf{R}_2 - \mathbf{R}_1$ {finite}

4-Gradient [1/m]
$\partial=\partial_R=\partial_X=\partial^\nu=(\partial_t/c,-\nabla)$
$=\partial/\partial R_\nu \rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Invariant d'Alembertian
$\partial\cdot\partial = (\partial_t/c)^2-\nabla\cdot\nabla = (\partial/c\partial\tau)^2 = (\partial_\tau/c)^2$
Wave Equation

The 4-Velocity is interesting in that it sort of bootstraps itself into existence:

$\mathbf{U}$
$= \gamma(c,\mathbf{u})$
$= d\mathbf{R}/d\tau$
$= (d/d\tau)[\mathbf{R}]$
$= (\mathbf{U}\cdot\partial)[\mathbf{R}]$
$= U^\sigma\eta_{\alpha\beta}\partial^\beta R^\sigma$
$= U_\beta\partial^\beta R^\sigma$
$= U_\beta\eta^{\beta\sigma}$
$= U^\sigma$
$\mathbf{U}$

The bootstrap is because $d/d\tau = (\mathbf{U}\cdot\partial)$

$\mathbf{U} = (d/d\tau)[\mathbf{R}]$
$\mathbf{U} = (\mathbf{U}\cdot\partial)[\mathbf{R}]$

$(\mathbf{U}\cdot\mathbf{R})(\mathbf{U}\cdot\partial) = (\mathbf{U}\cdot\mathbf{R})(\mathbf{U}\cdot\partial)$
$(\mathbf{U}\cdot\mathbf{R})(\partial) = (\mathbf{R})(\mathbf{U}\cdot\partial)$ if $\partial = k\mathbf{R}$
$\partial = [\mathbf{R}/(\mathbf{U}\cdot\mathbf{R})]^*(\mathbf{U}\cdot\partial)$
$\partial = [\mathbf{R}/(\mathbf{U}\cdot\mathbf{R})]^*(d/d\tau) =$
$[\mathbf{R}/(c^2\tau)]^*(d/d\tau)$
$\partial[\tau] = [\mathbf{R}/(\mathbf{U}\cdot\mathbf{R})] = [\mathbf{R}/(c^2\tau)]$
with $\mathbf{R} = \mathbf{U}\tau$

Invariant Calculus 4D
$d\mathbf{R}\cdot\partial = (cdt,d\mathbf{r})\cdot(\partial_t/c,-\nabla) = d[..]$
$dR^\mu\eta_{\mu\nu}(\partial^\nu) = dR^\mu(\partial_\mu) = dR^\mu(\partial/\partial R^\mu) = (dt\,\partial_t + d\mathbf{r}\cdot\nabla)$
$= dt(\partial/\partial t) + dx(\partial/\partial x) + dy(\partial/\partial y) + dz(\partial/\partial z)$
Total Derivative Chain Rule

Math/Phys Wave Solution

4-WaveVector
$\mathbf{K}=K^\mu=(\omega/c=2\pi\nu/c,\mathbf{k})= -\partial[\Phi]$
$=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})=(1/c\mathcal{T},\hat{\mathbf{n}}/\lambda\!\!\!^{-})$

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(0,2)-Tensor $T_{\mu\nu}$

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[s] ProperTime
$d\mathbf{R}\cdot\partial_U=[..]d\tau$
Multiplier

[s] $d\tau$

[1/s] $1/d\tau$

[1] General Invariant
$\mathbf{U}\cdot\partial_U[..]=[..]$

[1] General Invariant
$\mathbf{R}\cdot\partial_R[..]=[..]$

[1] $\int[..]$

[1] $d[..]$

[1] Chain Rule
$d\mathbf{R}\cdot\partial_R=d[..]$ for $f[\mathbf{R}]$
$d\mathbf{U}\cdot\partial_U=d[..]$ for $f[\mathbf{U}]$
Differential

Relativistic 4-Velocity Relations
$\mathbf{R} = \int d\tau\, \mathbf{U}$
$\mathbf{U} = (d/d\tau)\mathbf{R}$

[m] 4-Position
$\mathbf{R}=(ct,\mathbf{r})=(\mathbf{R}\cdot\partial_U)\mathbf{U}$
$=R^\alpha=(\int d\tau)\mathbf{U}=(\int d\tau)d\mathbf{R}/d\tau=\int d\mathbf{R}$

[s] $\mathbf{R}\cdot\partial_U[..]$
$\int[..]d\tau$

[1/s] $\mathbf{U}\cdot\partial_R[..]$
$d/d\tau[..]$

[m/s] 4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})=(\mathbf{U}\cdot\partial_R)\mathbf{R}$
$=U^\alpha=(d/d\tau)\mathbf{R}$

[1] $\int[..]$

[1] $d[..]$

[s] $d\tau$

[1/s] $1/d\tau$

[s] Proper Time
$\mathbf{R}\cdot\partial_U=(\int d\mathbf{R})\cdot\partial_U=(\int \mathbf{U}d\tau)\cdot\partial_U$
$=\int[..]d\tau=\int[..](1/\gamma)dt$
$= \mathbf{U}\cdot\partial_A = \mathbf{P}\cdot\partial_F$
Integral

$\partial_R\cdot\mathbf{R}=4$
SpaceTime Dimension

[1]

$\partial_U\cdot\mathbf{U}=4$
SpaceTime Dimension

$\partial_U[\mathbf{U}]=\eta^{\alpha\beta}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$
Minkowski Metric

[1]

$\partial_R[\mathbf{R}]=\eta^{\alpha\beta}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$
Minkowski Metric

[1/s] Proper Time
$\mathbf{U}\cdot\partial_R=d\mathbf{R}/d\tau\cdot\partial_R$
$=d/d\tau=\gamma d/dt$
$= \mathbf{A}\cdot\partial_U = \mathbf{F}\cdot\partial_P$
Derivative

4

$\mathbf{U}\cdot\partial_R = \mathbf{U}\cdot(d/d\tau)\partial_U$
$\mathbf{A}\cdot\partial_U = (d/d\tau)\mathbf{U}\cdot\partial_U$
$\mathbf{F}\cdot\partial_P = (d/d\tau)\mathbf{P}\cdot\partial_P$
$= (d/d\tau)$

[1/s]

[s/m] 4-VelocityGradient
$\partial_U{}^\beta=\partial_U=\partial/\partial U_\beta=(\partial_{U_t}/c,-\nabla_U)$
$=(\partial/\partial u_0,-\partial/\partial u_1,-\partial/\partial u_2,-\partial/\partial u_3)$
$\rightarrow(\partial/\partial\gamma c,-\partial/\partial\gamma u_x,-\partial/\partial\gamma u_y,-\partial/\partial\gamma u_z)$

[1/s] $\mathbf{U}\cdot\partial_R[..]$
$d/d\tau[..]$

[s] $\mathbf{R}\cdot\partial_U[..]$
$\int[..]d\tau$

[1/m] 4-PositionGradient:4-Gradient
$\partial_R{}^\beta=\partial_R=\partial/\partial R_\beta=\partial=(\partial_t/c,-\nabla)$
$=(\partial/\partial r_0,-\partial/\partial r_1,-\partial/\partial r_2,-\partial/\partial r_3)$
$\rightarrow(\partial/\partial ct,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Relativistic 4D Euler-Lagrange Relation
$\partial_R = (d/d\tau)\partial_U$
$\partial_R[L] = (d/d\tau)\partial_U[L]$

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[s] ProperTime
$d\mathbf{R}\cdot\partial_U=[..]d\tau$
Multiplier

[1] $\int[..]$ [1] $d[..]$

[m] 4-Position
$\mathbf{R}=(ct,\mathbf{r})=(\mathbf{R}\cdot\partial_U)\mathbf{U}$
$=R^\alpha=(\int d\tau)\mathbf{U}=(\int d\tau)d\mathbf{R}/d\tau=\int d\mathbf{R}$

Relativistic 4-Velocity Relation
$\mathbf{U}=(d/d\tau)\mathbf{R}$

$\mathbf{U}\cdot\partial_R[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$
[1/s]

[m/s] 4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})=(\mathbf{U}\cdot\partial_R)\mathbf{R}$
$=U^\alpha=(d/d\tau)\mathbf{R}$

[1] Chain Rule
$d\mathbf{R}\cdot\partial_R=d[..]$
Differential

$d\tau$ [s]

$\partial_R\cdot\mathbf{R}=4$
SpaceTime Dimension
[1]

$\partial_U[\mathbf{U}]=\eta^{\alpha\beta}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$
Minkowski Metric
[1]

[s] Proper Time
$\mathbf{R}\cdot\partial_U=(\int d\mathbf{R})\cdot\partial_U=(\int\mathbf{U}d\tau)\cdot\partial_U=\int[..]d\tau=\int[..](1/\gamma)dt$
Integral

[1/s] Proper Time
$\mathbf{U}\cdot\partial_R=d\mathbf{R}/d\tau\cdot\partial_R=d/d\tau=\gamma d/dt$
Derivative

$\partial_U\cdot\mathbf{U}=4$
SpaceTime Dimension
[1]

$\partial_R[\mathbf{R}]=\eta^{\alpha\beta}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$
Minkowski Metric
[1]

[s/m] 4-VelocityGradient
$\partial_U{}^\beta=\partial_U=\partial/\partial U_\beta=(\partial_t/c,-\nabla_U)$
$\rightarrow(\partial/\partial\gamma c,-\partial/\partial\gamma u_x,-\partial/\partial\gamma u_y,-\partial/\partial\gamma u_z)$

$\mathbf{U}\cdot\partial_R[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$
[1/s]

[1/m] 4-PositionGradient:4-Gradient
$\partial_R{}^\beta=\partial_R=\partial/\partial R_\beta=\partial=(\partial_t/c,-\nabla)$
$\rightarrow(\partial/\partial ct,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Relativistic 4D Euler-Lagrange Relation
$\partial_R=(d/d\tau)\partial_U$
$\partial_R[L]=(d/d\tau)\partial_U[L]$

$(\mathbf{R}\cdot\partial_U)[..]=\int[..]d\tau=\int[..](1/\gamma)dt$

$(\mathbf{R}\cdot\partial_U)[1]=\int[1]d\tau=\tau$
$(\mathbf{R}\cdot\partial_U)[\gamma]=\int[\gamma](1/\gamma)dt=t$

$(\mathbf{R}\cdot\partial_U)[\mathbf{U}]=\mathbf{R}$
$(\mathbf{R}\cdot\partial_U)[\mathbf{A}]=\mathbf{U}$
$(\mathbf{R}\cdot\partial_U)[\mathbf{F}]=\mathbf{P}$
$(\mathbf{R}\cdot\partial_U)[\mathbf{R}\wedge\mathbf{F}]=\mathbf{R}\wedge\mathbf{P}$

$(\mathbf{R}\cdot\partial_U)[L_o]=S_{action}$
$(\mathbf{R}\cdot\partial_U)[\omega_o]=\Phi_{phase}$

$(\mathbf{R}\cdot\partial_U)[\partial_R]=\partial_U$

$(\mathbf{R}\cdot\partial_U)[(\mathbf{U}\cdot\partial_R)[..]]=[..]$

$(\mathbf{U}\cdot\partial_R)=d/d\tau=\gamma d/dt$

$(\mathbf{U}\cdot\partial_R)[\tau]=d\tau/d\tau=1$
$(\mathbf{U}\cdot\partial_R)[t]=\gamma dt/dt=\gamma$

$(\mathbf{U}\cdot\partial_R)[\mathbf{R}]=\mathbf{U}$
$(\mathbf{U}\cdot\partial_R)[\mathbf{U}]=\mathbf{A}$
$(\mathbf{U}\cdot\partial_R)[\mathbf{P}]=\mathbf{F}$
$(\mathbf{U}\cdot\partial_R)[\mathbf{R}\wedge\mathbf{P}]=\mathbf{R}\wedge\mathbf{F}$

$(\mathbf{U}\cdot\partial_R)[S_{action}]=L_o$
$(\mathbf{U}\cdot\partial_R)[\Phi_{phase}]=\omega_o$

$(\mathbf{U}\cdot\partial_R)[\partial_U]=\partial_R$

$(\mathbf{U}\cdot\partial_R)[(\mathbf{R}\cdot\partial_U)[..]]=[..]$

Lorentz Factor:
Relativistic Gamma $\gamma=1/\sqrt{[1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: $\boldsymbol{\beta}=\mathbf{u}/c$
$\gamma'=d\gamma/dt=\gamma^3\boldsymbol{\beta}\cdot\mathbf{a}/c=\gamma^3\mathbf{u}\cdot\mathbf{a}/c^2$

Euler-Lagrange Eqn
$(d/d\tau)[\partial_U]=\partial_R$
$\mathbf{U}\cdot\partial_R[\partial_U]=\partial_R$
$U^\rho\eta_{\rho\sigma}\partial_R{}^\sigma[\partial_U{}^\mu]=\partial_R{}^\mu$
$\eta_{\rho\sigma}U^\rho\partial_U{}^\mu\partial_R{}^\sigma=\partial_R{}^\mu$
$U_\sigma\partial_U{}^\mu\partial_R{}^\sigma=\partial_R{}^\mu$

$\eta_\sigma{}^\mu\partial_R{}^\sigma=\partial_R{}^\mu$ if $L_o\sim\mathbf{U}\cdot f[\mathbf{R}]$
Rest Lagrangian
$L_o=-(\mathbf{P}_T\cdot\mathbf{U})$

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= Lorentz Scalar Invariant

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A Tensor Study of Physical 4-Vectors

SRQM: Some SR Calculus, Basics

ProperTime Derivative $(\mathbf{U}\cdot\partial_R)[..] = d[..]/d\tau = \gamma d[..]/dt$

ProperTime Integral $(\mathbf{R}\cdot\partial_U)[..] = \int[..]d\tau = \int[..](1/\gamma)dt$

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R,**S**,**T** are generic 4-Vectors

$\mathbf{R} = R^\mu = (r^0,r^1,r^2,r^3) = (r^0,\mathbf{r})$
$\mathbf{S} = S^\mu = (s^0,s^1,s^2,s^3) = (s^0,\mathbf{s})$
$\mathbf{T} = T^\mu = (t^0,t^1,t^2,t^3) = (t^0,\mathbf{t})$

$\partial_R,\partial_S,\partial_T$ are their 4-Gradients

$\partial_R = \partial_{R^\mu} = (\partial/\partial r^0,-\partial/\partial r^1,-\partial/\partial r^2,-\partial/\partial r^3) = (\partial/\partial r^0,-\nabla_r)$
$\partial_S = \partial_{S^\mu} = (\partial/\partial s^0,-\partial/\partial s^1,-\partial/\partial s^2,-\partial/\partial s^3) = (\partial/\partial s^0,-\nabla_s)$
$\partial_T = \partial_{T^\mu} = (\partial/\partial t^0,-\partial/\partial t^1,-\partial/\partial t^2,-\partial/\partial t^3) = (\partial/\partial t^0,-\nabla_t)$
The $_{R,S,T}$ here are gradient ID's, not indexes

$(\mathbf{R}\cdot\partial_S)$
$= (r^0,r^1,r^2,r^3)\cdot(\partial/\partial s^0,-\partial/\partial s^1,-\partial/\partial s^2,-\partial/\partial s^3)$
$= (r^0\partial/\partial s^0+r^1\partial/\partial s^1+r^2\partial/\partial s^2+r^3\partial/\partial s^3)$

$(\mathbf{R}\cdot\partial_S)[\mathbf{S}]$
$= (r^0\partial/\partial s^0+r^1\partial/\partial s^1+r^2\partial/\partial s^2+r^3\partial/\partial s^3)[(s^0,s^1,s^2,s^3)]$

$= ((r^0\partial/\partial s^0+r^1\partial/\partial s^1+r^2\partial/\partial s^2+r^3\partial/\partial s^3)s^0,$
$(r^0\partial/\partial s^0+r^1\partial/\partial s^1+r^2\partial/\partial s^2+r^3\partial/\partial s^3)s^1,$
$(r^0\partial/\partial s^0+r^1\partial/\partial s^1+r^2\partial/\partial s^2+r^3\partial/\partial s^3)s^2,$
$(r^0\partial/\partial s^0+r^1\partial/\partial s^1+r^2\partial/\partial s^2+r^3\partial/\partial s^3)s^3)$

$= (r^0,r^1,r^2,r^3) = \mathbf{R}$

$(\mathbf{S}\cdot\mathbf{T}) = (s^0,s^1,s^2,s^3)\cdot(t^0,t^1,t^2,t^3) = (s^0t^0-s^1t^1-s^2t^2-s^3t^3)$

$(\mathbf{R}\cdot\partial_S)[(\mathbf{S}\cdot\mathbf{T})]$
$= (r^0\partial/\partial s^0+r^1\partial/\partial s^1+r^2\partial/\partial s^2+r^3\partial/\partial s^3)[(s^0t^0-s^1t^1-s^2t^2-s^3t^3)]$
$= (r^0t^0-r^1t^1-r^2t^2-r^3t^3)$
$= (\mathbf{R}\cdot\mathbf{T})$

$\partial_R = \partial_{R^\mu} = \partial/\partial R_\mu = (\partial/\partial r^0,-\nabla_r)$ $\quad R_\mu = \eta_{\mu\nu}R^\mu$

$(d\mathbf{R}\cdot\partial_R)[\,f\,]$
$= (dr^0,dr^1,dr^2,dr^3)\cdot(\partial/\partial r^0,-\partial/\partial r^1,-\partial/\partial r^2,-\partial/\partial r^3)$
$= (dr^0\partial/\partial r^0+dr^1\partial/\partial r^1+dr^2\partial/\partial r^2+dr^3\partial/\partial r^3)$
$= d[\,f\,]$ = Derivative Chain Rule
for $f = f\,[r^0,r^1,r^2,r^3]$

$(\partial_R\cdot\mathbf{R})$
$= (\partial/\partial r^0,-\partial/\partial r^1,-\partial/\partial r^2,-\partial/\partial r^3)\cdot(r^0,r^1,r^2,r^3)$
$= (\partial r^0/\partial r^0+\partial r^1/\partial r^1+\partial r^2/\partial r^2+\partial r^3/\partial r^3)$
$= (1+1+1+1)$
$= 4$

==========

The generic 4-Gradient Rules:

$(\mathbf{R}\cdot\partial_S)[\mathbf{S}] = \mathbf{R}$ if $(\mathbf{R}\cdot\partial_S) = d/d\tau$
$(\mathbf{S}\cdot\partial_R)[\mathbf{R}] = \mathbf{S}$ then $(\mathbf{S}\cdot\partial_R) = \int d\tau$

$(\mathbf{S}\cdot\partial_R)[(\mathbf{R}\cdot\partial_S)[\mathbf{S}]] = \mathbf{S}$
$(\mathbf{S}\cdot\partial_R)[(\mathbf{R}\cdot\partial_S)[..]] = [..]$
Invariant Lorentz Scalars act generally

$(\mathbf{R}\cdot\partial_S)[(\mathbf{S}\cdot\mathbf{T})] = (\mathbf{R}\cdot\partial_S)[(\mathbf{T}\cdot\mathbf{S})] = (\mathbf{R}\cdot\mathbf{T}) = (\mathbf{T}\cdot\mathbf{R})$
$(\mathbf{T}\cdot\partial_S)[(\mathbf{S}\cdot\mathbf{R})] = (\mathbf{T}\cdot\partial_S)[(\mathbf{S}\cdot\mathbf{R})] = (\mathbf{R}\cdot\mathbf{T}) = (\mathbf{T}\cdot\mathbf{R})$

$(d\mathbf{R}\cdot\partial_R)[..] = d[..]$ Derivative Chain Rule

$(\mathbf{R}\cdot\partial_R) = 4$

Now, apply to SR 4-Vector physics:

$\mathbf{R} = R^\mu = (ct,\mathbf{r})$ 4-Position
$\mathbf{U} = U^\mu = (\gamma c,\gamma\mathbf{u})$ 4-Velocity

$\partial_R = \partial_{R^\mu} = \partial/\partial R_\mu = (\partial/\partial ct,-\nabla_r)$ 4-PosGradient
$\partial_U = \partial_{U^\mu} = \partial/\partial U_\mu = (\partial/\partial\gamma c,-\nabla_{\gamma u})$ 4-VelGradient

$(\mathbf{U}\cdot\partial_R)[\mathbf{R}] = \mathbf{U}$
$(d/d\tau)[\mathbf{R}] = \mathbf{U}$
==========
$\therefore\ (\mathbf{U}\cdot\partial_R) = d/d\tau$
ProperTime Derivative is Lorentz Scalar

$(\mathbf{R}\cdot\partial_U)[\mathbf{U}] = \mathbf{R}$
$(\int d\tau)[\mathbf{U}] = \mathbf{R}$
=========
$\therefore\ (\mathbf{R}\cdot\partial_U) = \int d\tau$
ProperTime Integral is Lorentz Scalar

$(\mathbf{R}\cdot\partial_U)\mathbf{U} = \int d\tau\ \mathbf{U} = \int d\tau\ d\mathbf{R}/d\tau = \int d\mathbf{R} = \mathbf{R}$

$(\mathbf{R}\cdot\partial_U) = \int d\tau$ $\quad (d\mathbf{R}\cdot\partial_R) = d[..]$

$(\mathbf{R}\cdot\partial_U)[(\mathbf{U}\cdot\partial_R)[..]] = [..]$ $\quad = (\mathbf{R}\cdot\partial_R)[..]$
$(\int d\tau)[(d/d\tau)[..]] = [..]$ $\quad = (\int d\mathbf{R}\cdot\partial_R)[..]$
$\int d\tau[d/d\tau[..]] = [..]$ $\quad = \int d[..] = [..]$
$\int d[..] = [..]$ from ChainRule $f(\mathbf{R})$

SR 4-Tensor
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(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

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(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathbf{A}\cdot\mathbf{B} = A^\mu\eta_{\mu\nu}B^\nu = [(a^0)(b^0) - \mathbf{a}\cdot\mathbf{b}] = (a^0{}_o)(b^0{}_o)$
= Lorentz Scalar Invariant
using Minkowski Metric $\eta_{\mu\nu} = (+1,-1,-1,-1)$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM: Some SR Calculus, Basics

ProperTime Derivative $(\mathbf{U}\cdot\partial_R)[..] = d[..]/d\tau = \gamma d[..]/dt$

ProperTime Integral $(\mathbf{R}\cdot\partial_U)[..] = \int[..]d\tau = \int[..](1/\gamma)dt$

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
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Using 4-Position & its 4-PositionGradient:

$\mathbf{R} = R^\mu = (ct,x,y,z)$
$\partial_R = \partial_{R^\mu} = \partial/\partial R_\mu = (\partial/\partial ct,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$ $\quad R_\mu = \eta_{\mu\nu}R^\mu$

$f = f[t,x,y,z]$ = a math function of SpaceTime coords. **R**

$d[f] = dt(\partial f/\partial t)+dx(\partial f/\partial x)+dy(\partial f/\partial y)+dz(\partial f/\partial z) = (d\mathbf{R}\cdot\partial_R)[f]$

$df/dt =$
$= dt/dt\ (\partial f/\partial t) + dx/dt\ (\partial f/\partial x) + dy/dt\ (\partial f/\partial y) + dz/dt\ (\partial f/\partial z)$
$= (\partial f/\partial t) + u^x(\partial f/\partial x) + u^y(\partial f/\partial y) + u^z(\partial f/\partial z)$
$= (\partial/\partial t)[f] + \mathbf{u}\cdot\nabla[f]$

$d/dt = (\partial/\partial t) + \mathbf{u}\cdot\nabla = (\partial_t + \mathbf{u}\cdot\nabla)$

$\gamma(d/dt) = \gamma(\partial_t + \mathbf{u}\cdot\nabla) = d/d\tau = \mathbf{U}\cdot\partial_R = d\mathbf{R}/d\tau\cdot\partial = d\mathbf{R}\cdot\partial/d\tau$

$(\mathbf{U}\cdot\partial_R) = \gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla) = \gamma(\partial_t+\mathbf{u}\cdot\nabla) = d/d\tau$

also: $d[f] = (d\mathbf{R}\cdot\partial_R)[f]$

Using Generic 4-Vector & its associated 4-Gradient:

$\mathbf{A} = A^\mu = (a^0,a^1,a^2,a^3)$
$\partial_A = \partial_{A^\mu} = \partial/\partial A_\mu = (\partial/\partial a^0,-\partial/\partial a^1,-\partial/\partial a^2,-\partial/\partial a^3)$ $\quad A_\mu = \eta_{\mu\nu}A^\mu$

$f = f[a^0,a^1,a^2,a^3]$ = a math function of components of 4-Vector **A**

$d[f] = da^0(\partial f/\partial a^0)+da^1(\partial f/\partial a^1)+da^2(\partial f/\partial a^2)+da^3(\partial f/\partial a^3) = (d\mathbf{A}\cdot\partial_A)[f]$

$d[..] = (d\mathbf{A}\cdot\partial_A)[..]$

$d/da^0_o = (d\mathbf{A}\cdot\partial_A)/da^0_o = (d\mathbf{A}/da^0_o\cdot\partial_A) = (\mathbf{A'}\cdot\partial_A)$

==========
$(\mathbf{A'}\cdot\partial_A) = (d\mathbf{A}/da^0_o\cdot\partial_A) = (d\mathbf{A}\cdot\partial_A)/da^0_o = d/da^0_o$
==========

Apply general rule to SR 4-Position
$\mathbf{R} = R^\mu = (r^0,r^1,r^2,r^3) = (ct,\mathbf{r})$
$r^0_o = ct_o = c\tau$

$(d\mathbf{R}/dc\tau\cdot\partial_R) = (d\mathbf{R}\cdot\partial_R)/dc\tau = d/dc\tau$
$(d\mathbf{R}/d\tau\cdot\partial_R) = (d\mathbf{R}\cdot\partial_R)/d\tau = d/d\tau$
$(\mathbf{U}\cdot\partial_R) = (d\mathbf{R}\cdot\partial_R)/d\tau = d/d\tau$
$(\mathbf{U}\cdot\partial_R) = d/d\tau$

$d[f] = (d\mathbf{A}\cdot\partial_A)[f]$

$d[..] = (d\mathbf{A}\cdot\partial_A)[..]$

Apply the general rule to SR 4-Vectors

$d[..] = (d\mathbf{R}\cdot\partial_R)[..]$
$d[..] = (d\mathbf{U}\cdot\partial_U)[..]$
$d[..] = (d\mathbf{A}\cdot\partial_A)[..]$

The differential operator can be applied to various functions with different sets of variables.

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$(\mathbf{U}\cdot\partial_R)[\mathbf{R}] = \mathbf{U}$
$(d/d\tau)[\mathbf{R}] = \mathbf{U}$
==========
$\therefore (\mathbf{U}\cdot\partial_R) = d/d\tau$

$(\mathbf{A}\cdot\partial_U)[\mathbf{U}] = \mathbf{A}$
$(d/d\tau)[\mathbf{U}] = \mathbf{A}$
==========
$\therefore (\mathbf{A}\cdot\partial_U) = d/d\tau$

$(\mathbf{A}\cdot\partial_U) = d/d\tau$
$[(d/d\tau)(d/d\tau)\mathbf{R}\cdot\partial_U] = d/d\tau$
$[(d/d\tau)\mathbf{R}\cdot\partial_U] = 1$ $\qquad (\mathbf{U}\cdot\partial_U)[..] = 1[..]$
$d\mathbf{R}\cdot\partial_U = d\tau$
$\int d\mathbf{R}\cdot\partial_U = \int d\tau$
$(\mathbf{R}\cdot\partial_U) = \int d\tau$
$\therefore$
$(\mathbf{U}\cdot\partial_R) = d/d\tau$ ProperTime Derivative (PTD)
$(\mathbf{R}\cdot\partial_U) = \int d\tau$ Propertime Integral (PTI)

$(\mathbf{U}\cdot\partial_R) = d/d\tau = (\mathbf{A}\cdot\partial_U)$
$(\mathbf{U}\cdot\partial_R) = (\mathbf{A}\cdot\partial_U)$
$(\mathbf{U}\cdot\partial_R) = (d\mathbf{U}/d\tau\cdot\partial_U)$
$(\mathbf{U}\cdot\partial_R) =?= (\mathbf{U}\cdot d\partial_U/d\tau)$ This step is Euler-Lagrange
$(\mathbf{U}\cdot\partial_R) = (\mathbf{U}\cdot\partial_R)$

Relativistic 4D Euler-Lagrange Relation
$\partial_R = (d/d\tau)\partial_U$

$(\mathbf{R}\cdot\partial_U)[..]$
$=\int d\mathbf{R}\cdot\partial_U[..]$
$=\int d\tau\, d\mathbf{R}/d\tau\cdot\partial_U[..]$
$=\int d\tau\, \mathbf{U}\cdot\partial_U[..]$

$=\int d\tau[..]$ if $(\mathbf{U}\cdot\partial_U)[..] = [..]$
==========
$(\mathbf{R}\cdot\partial_U)(\mathbf{U}\cdot\partial_R)[..] = [..]$ $\qquad = (\mathbf{R}\cdot\partial_U)(d/d\tau)[..] = [..]$
$(\mathbf{R}\cdot\partial_R)[..] = [..]$ $\qquad =(\int d\tau)(d/d\tau)[..] = [..]$
$\int(d\mathbf{R}\cdot\partial_R)[..] = [..]$ $\qquad =\int d[..] = [..]$
$\int d[..] = [..]$ $\qquad [..] = [..]$
==========
$(\mathbf{R}\cdot\partial_U)[\mathbf{A}] = (\mathbf{R}\cdot\partial_U)[d\mathbf{U}/d\tau] = \int d\tau\, d\mathbf{U}/d\tau = \int d\mathbf{U} = \mathbf{U}$

$(\mathbf{U}\cdot\partial_R)[..] = (\mathbf{U}\cdot\partial_R)[..]$
$(\mathbf{U}\cdot\partial_R)(\mathbf{U}\cdot\partial_U)[..] = (\mathbf{U}\cdot\partial_R)[..]$
$d/d\tau(\mathbf{U}\cdot\partial_U)[..] = (\mathbf{U}\cdot\partial_R)[..]$
...
$d/d\tau(\partial_U)[..] = (\partial_R)[..]$

4-Velocity **U** is a good common factor:
==========
$(\mathbf{U}\cdot\partial_R) = d/d\tau$ ProperTime Derivative (PTD)
$(\mathbf{U}\cdot\partial_U) = 1$
$(\mathbf{U}\cdot\partial_A) = \int d\tau$ ProperTime Integral (PTI)

$\mathbf{R} = d^0\mathbf{R}/d\tau^0$	4-Position
$\mathbf{U} = d^1\mathbf{R}/d\tau^1 = d\mathbf{R}/d\tau$	4-Velocity
$\mathbf{A} = d^2\mathbf{R}/d\tau^2 = d\mathbf{U}/d\tau$	4-Acceleration
$\mathbf{J} = d^3\mathbf{R}/d\tau^3 = d\mathbf{A}/d\tau$	4-Jerk
$\mathbf{P} = d^0\mathbf{P}/d\tau^0 = m_o\mathbf{U}$	4-Momentum
$\mathbf{F} = d^1\mathbf{P}/d\tau^1 = d\mathbf{P}/d\tau$	4-Force
$\partial_R = \partial_{R^\mu} = \partial/\partial R_\mu$	4-PositionGradient
$\partial_U = \partial_{U^\mu} = \partial/\partial U_\mu$	4-VelocityGradient
$\partial_A = \partial_{A^\mu} = \partial/\partial A_\mu$	4-AccelGradient
$\partial_J = \partial_{J^\mu} = \partial/\partial J_\mu$	4-JerkGradient
$\partial_P = \partial_{P^\mu} = \partial/\partial P_\mu$	4-MomentumGradient
$\partial_F = \partial_{F^\mu} = \partial/\partial F_\mu$	4-ForceGradient

ProperTime Derivatives (PTD)
$(\mathbf{U}\cdot\partial_R) = d/d\tau$
$(\mathbf{A}\cdot\partial_U) = d/d\tau$
$(\mathbf{J}\cdot\partial_A) = d/d\tau$
$(\mathbf{F}\cdot\partial_P) = d/d\tau$

ProperTime Unit (PTU)
$(\mathbf{U}\cdot\partial_U) = 1$

ProperTime Integrals (PTI)
$(\mathbf{R}\cdot\partial_U) = \int d\tau$
$(\mathbf{U}\cdot\partial_A) = \int d\tau$
$(\mathbf{A}\cdot\partial_J) = \int d\tau$
$(\mathbf{P}\cdot\partial_F) = \int d\tau$

Cauchy's Integral Formula:
from: Augustin-Louis Cauchy

it is a central statement in complex analysis. It expresses the fact that a holomorphic function defined on a disk is completely determined by its values on the boundary of the disk, and it provides integral formulas for all derivatives of a holomorphic function.

Cauchy's formula shows that, in complex analysis, "differentiation is equivalent to integration": complex differentiation, like integration, behaves well under uniform limits...

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(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM: Some SR Calculus

ProperTime Derivative $(\mathbf{U}\cdot\partial_R)[..] = d[..]/d\tau = \gamma d[..]/dt$

ProperTime Integral $(\mathbf{R}\cdot\partial_U)[..] = \int[..]d\tau = \int[..](1/\gamma)dt$

4-Vector SRQM Interpretation of QM

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(PTD) ProperTime Derivative $(\mathbf{U}\cdot\partial_R)[..] = d[..]/d\tau = \gamma d[..]/dt$

PTD of ProperTime τ : $(\mathbf{U}\cdot\partial_R)[\tau] = (d/d\tau)[\tau] = d\tau/d\tau = 1$
PTD of RelativisticTime t : $(\mathbf{U}\cdot\partial_R)[t] = (d/d\tau)[t] = \gamma dt/dt = \gamma$

PTD of 4-Position $\mathbf{R}$: $(\mathbf{U}\cdot\partial_R)[\mathbf{R}] = (d/d\tau)[\mathbf{R}] = (d\mathbf{R}/d\tau) = \mathbf{U}$
PTD of 4-Velocity $\mathbf{U}$: $(\mathbf{A}\cdot\partial_U)[\mathbf{U}] = (\mathbf{U}\cdot\partial_R)[\mathbf{U}] = (d/d\tau)[\mathbf{U}] = (d\mathbf{U}/d\tau) = \mathbf{A}$
PTD of 4-Momentum $\mathbf{P}$: $(\mathbf{F}\cdot\partial_P)[\mathbf{P}] = (\mathbf{U}\cdot\partial_R)[\mathbf{P}] = (d/d\tau)[\mathbf{P}] = (d\mathbf{P}/d\tau) = \mathbf{F}$

PTD of 4-AngMomentum $\mathbf{R}\wedge\mathbf{P}$: $(\mathbf{F}\cdot\partial_P)[\mathbf{R}\wedge\mathbf{P}] = (\mathbf{U}\cdot\partial_R)[\mathbf{R}\wedge\mathbf{P}] = (d/d\tau)[\mathbf{R}\wedge\mathbf{P}] =$
$d(\mathbf{R}\wedge\mathbf{P}/d\tau) = (d\mathbf{R}/d\tau\wedge\mathbf{P}+\mathbf{R}\wedge d\mathbf{P}/d\tau) = (\mathbf{U}\wedge m_o\mathbf{U}+\mathbf{R}\wedge\mathbf{F}) = (0^{\mu\nu}+\mathbf{R}\wedge\mathbf{F}) = \mathbf{R}\wedge\mathbf{F}$

PTD of ActionInvariant S_{action} : $(\mathbf{U}\cdot\partial_R)[S_{action}] = (\mathbf{U}\cdot\partial_R)[-\mathbf{P}_T\cdot\mathbf{R}] = -\mathbf{P}_T\cdot\mathbf{U} = L_o$
$L_o = (d/d\tau)S_{action} = (\gamma d/dt)S_{action}$: $L = L_o/\gamma = (d/dt)S_{action}$

PTD of WavePhase Φ_{phase} : $(\mathbf{U}\cdot\partial_R)[\Phi_{phase}] = (\mathbf{U}\cdot\partial_R)[-\mathbf{K}\cdot\mathbf{R}] = -\mathbf{K}\cdot\mathbf{U} = \omega_o$
$\omega_o = (d/d\tau)\Phi_{phase} = (\gamma d/dt)\Phi_{phase}$

PTD of RestMass m_o : $(\mathbf{U}\cdot\partial_R)[m_o] = (d/d\tau)[m_o] = \mathbf{U}\cdot\mathbf{F}/c^2$
{ from $\mathbf{F} = d\mathbf{P}/d\tau = d[m_o\mathbf{U}]/d\tau = d[m_o]/d\tau\ \mathbf{U} + m_o\mathbf{A}$: $\mathbf{U}\cdot\mathbf{U} = c^2$: $\mathbf{U}\cdot\mathbf{A} = 0$ }

Relativistic 4D Euler-Lagrange Relation
PTD of 4-"Velocity"Gradient ∂_U : $(\mathbf{U}\cdot\partial_R)[\partial_U] = (d/d\tau)[\partial_U] = \partial_R$

In theory, an integration constant can be added to any of these inputs.

(PTI) ProperTime Integral $(\mathbf{R}\cdot\partial_U)[..] = \int[..]d\tau = \int[..](1/\gamma)dt$

PTI of Unit 1 : $(\mathbf{R}\cdot\partial_U)[1] = \int[1]d\tau = \tau$
PTI of LorentzFactor γ : $(\mathbf{R}\cdot\partial_U)[\gamma] = \int[\gamma](1/\gamma)dt = t$

PTI of 4-Velocity $\mathbf{U}$: $(\mathbf{R}\cdot\partial_U)[\mathbf{U}] = (\int d\tau)[\mathbf{U}] = (\int d\tau)[d\mathbf{R}/d\tau] = \int[d\mathbf{R}] = \mathbf{R}$
PTI of 4-Acceleration $\mathbf{A}$: $(\mathbf{U}\cdot\partial_A)[\mathbf{A}] = (\mathbf{R}\cdot\partial_U)[\mathbf{A}] = (\int d\tau)[\mathbf{A}] = (\int d\tau)[d\mathbf{U}/d\tau] = \int[d\mathbf{U}] = \mathbf{U}$
PTI of 4-Force $\mathbf{F}$: $(\mathbf{P}\cdot\partial_F)[\mathbf{F}] = (\mathbf{R}\cdot\partial_U)[\mathbf{F}] = (\int d\tau)[\mathbf{F}] = (\int d\tau)[d\mathbf{P}/d\tau] = \int[d\mathbf{P}] = \mathbf{P}$

PTI of 4-Torque $\mathbf{R}\wedge\mathbf{F}$: $(\mathbf{P}\cdot\partial_F)[\mathbf{R}\wedge\mathbf{F}] = (\mathbf{R}\cdot\partial_U)[\mathbf{R}\wedge\mathbf{F}]$
$= (\int d\tau)[\mathbf{R}\wedge\mathbf{F}] = (\int d\tau)[\mathbf{R}\wedge d\mathbf{P}/d\tau] = \int[\mathbf{R}\wedge d\mathbf{P}] = \mathbf{R}\wedge\mathbf{P}$

PTI of RestLagrangian L_o : $(\mathbf{R}\cdot\partial_U)[L_o] = (\mathbf{R}\cdot\partial_U)[-\mathbf{P}_T\cdot\mathbf{U}] = [-\mathbf{P}_T\cdot\mathbf{R}] = S_{action}$
$S_{action} = (\int d\tau)L_o = (\int dt/\gamma)\gamma L = (\int dt)L = \int L dt$

PTI of RestAngFreq ω_o : $(\mathbf{R}\cdot\partial_U)[\omega_o] = (\mathbf{R}\cdot\partial_U)[-\mathbf{K}\cdot\mathbf{U}] = [-\mathbf{K}\cdot\mathbf{R}] = \Phi_{phase}$
$\Phi_{phase} = (\int d\tau)\omega_o$

PTI of "ForceScalar" : $(\mathbf{R}\cdot\partial_U)[\mathbf{U}\cdot\mathbf{F}/c^2] = \mathbf{R}\cdot\mathbf{F}/c^2 = \int\mathbf{U}d\tau\cdot d\mathbf{P}/d\tau/c^2 = \mathbf{U}\cdot\mathbf{P}/c^2 = E_o/c^2 = m_o$
$(\int d\tau)[\mathbf{U}\cdot\mathbf{F}/c^2] = (\int d\tau)[(d/d\tau)[m_o]] = \int d[m_o] = m_o$

"Inverted" Relativistic 4D Euler-Lagrange Relation
PTI of 4-"Position"Gradient ∂_R : $(\mathbf{R}\cdot\partial_U)[\partial_R] = (\int d\tau)[\partial_R] = \partial_U$

In theory, an integration constant can be added to any of these result outputs.

Writing the ProperTime Derivative = $(\mathbf{U}\cdot\partial_R) = d[..]/d\tau$ and the ProperTime Integral = $(\mathbf{R}\cdot\partial_U) = \int[..]d\tau$ in this format shows that these operations are Lorentz Invariant Scalars.

Lorentz Factor:
Relativistic Gamma $\gamma = 1/\sqrt{[1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: $\boldsymbol{\beta} = \mathbf{u}/c$
$\gamma' = d\gamma/dt = \gamma^3\boldsymbol{\beta}\cdot\mathbf{a}/c = \gamma^3\mathbf{u}\cdot\mathbf{a}/c^2$

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SRQM Study:
Physical 4-Vectors
Some 4-Position R^μ Relations

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$\partial/\partial\,[\eta_{\mu\nu}..]$

[1/m]

4-Gradient: 4-(Position)Gradient

$\partial = \partial_R = \partial_x = \partial^\mu = (\partial_t/c, -\nabla)$

$= \partial/\partial R_\mu \rightarrow (\partial_t/c, -\partial_x, -\partial_y, -\partial_z)$

$= (\partial/c\partial t, -\partial/\partial x, -\partial/\partial y, -\partial/\partial z)$

Minkowski
$\partial[\mathbf{R}] = \partial^\mu[R^\nu] = \eta^{\mu\nu}$
Metric

Lorentz
$\partial_\nu[R^{\mu'}] = \Lambda^{\mu'}{}_\nu$
Transform

SpaceTime
$\partial\cdot\mathbf{R} = \partial_\mu R^\mu = 4$
Dimension

SRQM Non-Zero Commutation
$[\partial,\mathbf{R}] = [\partial^\mu, R^\nu]$
$= \partial^\mu R^\nu - R^\nu\partial^\mu = \eta^{\mu\nu}$

Calculus
$d\mathbf{R}\cdot\partial = dR^\mu\partial_\mu$
Total Derivative
Chain Rule

[m]

4-Position
$\mathbf{R} = R^\mu = (ct, \mathbf{r}) \in$ <Event>
$\rightarrow (ct, x, y, z)$
$= (c^2\tau)\partial[\tau] = (\mathbf{R}\cdot\mathbf{U})\partial[\tau]$
$= \{(\mathbf{R}\cdot\mathbf{U})/(\mathbf{U}\cdot\mathbf{U})\}\mathbf{U} = \tau\mathbf{U} = \tau\gamma(c,\mathbf{u})$

Invariant d'Alembertian
$\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = (\partial/c\partial\tau)^2$
$= (\partial_\tau/c)^2$
Wave Equation

Invariant Interval
$\mathbf{R}\cdot\mathbf{R} = (ct)^2 - \mathbf{r}\cdot\mathbf{r}$
$= (c\tau)^2 = -\mathbf{r}_o\cdot\mathbf{r}_o$
$= (i|\mathbf{r}_o|)^2$

$(\partial\cdot\partial)[\mathbf{R}] = \partial_\mu\eta^{\mu\nu}$
$= 0^\nu = (0,\mathbf{0}) = \mathbf{Z}$

$(\partial\cdot\partial)[\mathbf{R}\cdot\mathbf{R}]$
$= \partial\cdot[\partial[\mathbf{R}\cdot\mathbf{R}]] = \partial\cdot[2\mathbf{R}] = 2\partial\cdot[\mathbf{R}] = 2(4) = 8$
$= (\partial\cdot\partial)[(c\tau)^2]$

$\partial[\mathbf{R}\cdot\mathbf{V}] = \partial^\mu[R^\sigma V_\sigma]$
$= V_\sigma\partial^\mu R^\sigma + R^\sigma\partial^\mu V_\sigma$
$= V_\sigma\eta^{\mu\sigma} + 0^\mu$
$= V^\mu = \mathbf{V}$

assumes $\mathbf{V} \neq f[\mathbf{R}]$

$\partial[\mathbf{R}\cdot\mathbf{R}] = 2R_\nu\partial^\mu[R^\nu] = 2R_\nu\eta^{\mu\nu} = 2R^\mu = 2\mathbf{R}$
$= \partial[(c\tau)^2] = (c^2)\partial[(\tau)^2] = 2(c^2\tau)\partial[\tau]$
$= \partial[(ct)^2 - \mathbf{r}\cdot\mathbf{r}] = (\partial[(ct)^2 - \mathbf{r}\cdot\mathbf{r}]/\partial ct, -\nabla[(ct)^2 - \mathbf{r}\cdot\mathbf{r}])$
$= (\partial[(ct)^2]/\partial ct, \nabla[\mathbf{r}\cdot\mathbf{r}]) = 2(ct,\mathbf{r}) = 2\mathbf{R}$

$\partial[\tau] = \partial[\mathbf{R}\cdot\mathbf{U}/\mathbf{U}\cdot\mathbf{U}] = \partial[\mathbf{R}\cdot\mathbf{U}/c^2]$
$= \mathbf{U}/c^2 = \mathbf{U}/(\mathbf{U}\cdot\mathbf{U}) = \mathbf{T}/c$
$= \mathbf{R}/(\mathbf{R}\cdot\mathbf{U}) = \mathbf{R}/(c^2\tau)$

Invariant ProperTime
$\mathbf{R}\cdot\mathbf{U}/\mathbf{U}\cdot\mathbf{U} = (c^2t_o)/c^2$
$= \mathbf{R}\cdot\mathbf{R}/\mathbf{R}\cdot\mathbf{U} = (ct_o)^2/(c^2t_o)$
$= t_o = \tau$

$\mathbf{R}\cdot\mathbf{U}$
$= (ct,\mathbf{r})\cdot\gamma(c,\mathbf{u})$
$= \gamma(c^2t - \mathbf{r}\cdot\mathbf{u})$
$= (c^2t_o)$
$= (c^2\tau)$

Invariant LightSpeed
$\mathbf{U}\cdot\mathbf{U} = \gamma^2(c^2 - \mathbf{u}\cdot\mathbf{u})$
$= c^2\gamma^2(1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}) = c^2$

$\mathbf{U}\cdot\partial[\tau] = d/d\tau[\tau] = d\tau/d\tau$
$= \mathbf{U}\cdot\mathbf{U}/c^2 = c^2/c^2 = 1$
$\mathbf{U}\cdot\partial[\tau] = \mathbf{U}\cdot\mathbf{R}/(\mathbf{R}\cdot\mathbf{U}) = 1$

Invariant ProperTime Derivative
$\mathbf{U}\cdot\partial = \gamma(c,\mathbf{u})\cdot(\partial_t/c, -\nabla) = \gamma(\partial_t + \mathbf{u}\cdot\nabla)$
$= \gamma d/dt = d/d\tau$

4-Velocity
$\mathbf{U} = U^\mu = \gamma(c,\mathbf{u}) = d\mathbf{R}/d\tau$
$= dR^\mu/d\tau = (\mathbf{U}\cdot\partial)\mathbf{R}$
$= d\{(c^2\tau)\partial[\tau]\}/d\tau = (c^2)\partial[\tau]$

[m/s]

$\mathbf{U}\cdot\partial\,[..]$
$d/d\tau\,[..] = d\,[..]/d\tau$
$\gamma d/dt\,[..]$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Lorentz Factor:
Relativistic Gamma $\gamma = 1/\sqrt{[1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: $\boldsymbol{\beta} = \mathbf{u}/c$
$\gamma' = d\gamma/dt = \gamma^3\boldsymbol{\beta}\cdot\mathbf{a}/c = \gamma^3\mathbf{u}\cdot\mathbf{a}/c^2$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: Physical 4-Vectors Some 4-Velocity U^μ Relations

4-Vector SRQM Interpretation of QM

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John B. Wilson
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f = f [t,x,y,z] = a math function of SpaceTime coords.

$df = dt(\partial f/\partial t) + dx(\partial f/\partial x) + dy(\partial f/\partial y) + dz(\partial f/\partial z) = (d\mathbf{R}\cdot\partial)[f]$

$df/dt =$
$= dt/dt\,(\partial f/\partial t) + dx/dt\,(\partial f/\partial x) + dy/dt\,(\partial f/\partial y) + dz/dt\,(\partial f/\partial z)$
$= (\partial f/\partial t) + u^x(\partial f/\partial x) + u^y(\partial f/\partial y) + u^z(\partial f/\partial z)$
$= (\partial/\partial t)[f] + \mathbf{u}\cdot\nabla[f]$

$d/dt = (\partial/\partial t) + \mathbf{u}\cdot\nabla = (\partial_t + \mathbf{u}\cdot\nabla)$

Lorentz Scalar Invariant

$\gamma(d/dt) = \gamma(\partial_t + \mathbf{u}\cdot\nabla) = (d/d\tau) = (\mathbf{U}\cdot\partial) = (d\mathbf{R}/d\tau\cdot\partial)$

[m]

4-Position
$\mathbf{R}=R^\mu=(ct,\mathbf{r})\in$<Event>
$\rightarrow(ct,x,y,z)$
alt. notation $\mathbf{X}=X^\mu$

[1/m]

4-Gradient
$\partial=\partial_R=\partial_X=\partial^\mu=(\partial_t/c,-\nabla)$
$=\partial/\partial R_\mu \rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

$\mathbf{U}\cdot\partial$
$= \gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)$
$= \gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$= \gamma(\partial/\partial t+d\mathbf{r}/dt\cdot\partial/\partial\mathbf{r})$
$= \gamma d/dt$
$= d/d\tau$

ProperTime Derivative

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

[m/s]

4-Velocity
$\mathbf{U}=U^\mu=\gamma(c,\mathbf{u})$
$=c\bar{\mathbf{T}}=c^2\partial[\tau]=d\mathbf{R}/d\tau$

$\mathbf{U}\cdot\mathbf{U}=c^2$

Invariant LightSpeed (c)

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

[m/s²]

4-Acceleration
$\mathbf{A}=A^\mu=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$
$=d\mathbf{U}/d\tau=d^2\mathbf{R}/d\tau^2$: { $\gamma'=d\gamma/dt$ }

Relativistic Gamma (γ) combines with Invariant Rest-Scalars to create Relativistic components. This allows the 4-Velocity **U** to create many other standard SR 4-Vectors. ex. with 4-Momentum
$\mathbf{P} = (mc,\mathbf{p}) = (mc,m\mathbf{u}) = m(c,\mathbf{u}) = m_o\gamma(c,\mathbf{u}) = m_o\mathbf{U}$
$m = \gamma m_o$
Relativistic mass (m) = (γ)*Invariant mass (m_o)

Rest Mass:Energy/c² [kg] — m_o, E_o/c^2

4-Momentum [kg·m/s=N·s]
$\mathbf{P}=P^\mu=(mc,m\mathbf{u})=m_o\gamma(c,\mathbf{u})=m_o\mathbf{U}= -\partial[S]$
$=(E/c,\mathbf{p})=(E_o/c^2)\gamma(c,\mathbf{u})=(E_o/c^2)\mathbf{U}=(E_o/c)\bar{\mathbf{T}}$

Rest Ang. Frequency/c² [rad·s/m²] — ω_o/c^2

4-WaveVector [rad/m]
$\mathbf{K}=K^\mu=(\omega/c=2\pi\nu/c,\mathbf{k})=(\omega_o/c^2)\mathbf{U}= -\partial[\Phi]$
$=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})=(1/c\bar{\mathbf{T}},\hat{\mathbf{n}}/\lambda\!\!\!^{-})=(\omega_o/c)\bar{\mathbf{T}}$

Rest EM Potential/c² Rest Voltage/c² [kg/C] — φ_o/c^2

4-(EM)VectorPotential [kg·m/C·s=N·s/C=T·m]
$\mathbf{A}=A^\mu=(\varphi/c,\mathbf{a})=(\varphi_o/c^2)\mathbf{U}$
$\mathbf{A_{EM}}=A_{EM}{}^\mu=(\varphi_{EM}/c,\mathbf{a_{EM}})$

(EM) Charge q

Rest (EM) Potential Energy/c² [kg] — $V_o/c^2=q\varphi_o/c^2$

4-(EM)VectorPotentialMomentum [kg·m/s=N·s]
$\mathbf{Q}=Q^\mu=(q\varphi/c,q\mathbf{a})=(V/c,\mathbf{q})$
$=q\mathbf{A}=(q\varphi_o/c^2)\mathbf{U}=(V_o/c^2)\mathbf{U}$

Rest Charge Density [C/m³] — $\rho_o=qn_o$

4-ChargeFlux : 4-CurrentDensity [C/m²·s]
$\mathbf{J}=J^\mu=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})=\rho_o\gamma(c,\mathbf{u})=\rho_o\mathbf{U}$
$=qn_o\mathbf{U}=q\mathbf{N}$

q

Rest Number Density [#/m³] — n_o

4-(Particle:Dust:Number)Flux [#/m²·s]
$\mathbf{N}=N^\mu=(nc,\mathbf{n})=n(c,\mathbf{u})=n_o\gamma(c,\mathbf{u})=n_o\mathbf{U}$

Rest Inv. Thermal Energy [1/J] — $1/k_BT_o$

4-ThermalVector [m/J·s=1/N·s]
4-InverseTemperatureMomentum
$\mathbf{\Theta}=\Theta^\mu=(\theta^0,\boldsymbol{\theta})=(c/k_BT,\mathbf{u}/k_BT)=(\theta_o/c)\mathbf{U}$
$=(1/k_BT)(c,\mathbf{u})=(1/k_B\gamma T)\mathbf{U}=(1/k_BT_o)\mathbf{U}$

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SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

4-Vector V = $V^\mu = (v^\mu) = (v^0,v^i) = (v^0,\mathbf{v})$
SR 4-Vector V = V^μ = (scalar * $c^{\pm1}$,3-vector)

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

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4-Gradient

$\partial=\partial_R=\partial_X=\partial^\mu=(\partial_t/c,-\nabla)$

$=\partial/\partial R_\mu \rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

$=(\partial/_{c\partial t},-\partial/_{\partial x},-\partial/_{\partial y},-\partial/_{\partial z})$

[1/m]

4-Position

$\mathbf{R}=R^\mu=(ct,\mathbf{r})\in$<Event>

$\rightarrow(ct,x,y,z)$

alt. notation $\mathbf{X}=X^\mu$

[m]

Invariant Interval

$\mathbf{R}\cdot\mathbf{R} = (ct)^2-\mathbf{r}\cdot\mathbf{r} = (c\tau)^2$

Invariant d'Alembert

$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla=(\partial_\tau/c)^2$

Wave Eqn

$\mathbf{U}\cdot\partial\,[..]$

$\gamma d/dt\,[..]$

$d/d\tau\,[..]$

Invariant ProperTime

$\mathbf{R}\cdot\mathbf{U} = c^2\tau$ *from* $d/d\tau(\mathbf{R}\cdot\mathbf{R}) = d/d\tau((c\tau)^2) = 2c^2\tau$

$=(\mathbf{R}\cdot\mathbf{U})+(\mathbf{U}\cdot\mathbf{R})=2(\mathbf{R}\cdot\mathbf{U})$

ProperTime Derivative is a Lorentz Scalar Invariant

$\mathbf{U}\cdot\partial$

$= \gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)$

$= \gamma(\partial_t+\mathbf{u}\cdot\nabla)$

$= \gamma(\partial/_{\partial t}+{}^{d\mathbf{r}}/_{dt}\cdot\partial/_{\partial\mathbf{r}})$

$= \gamma d/dt$

$= d/d\tau$

[m/s]

4-Velocity

$\mathbf{U}=U^\mu=\gamma(c,\mathbf{u})$

$=d\mathbf{R}/d\tau=c\bar{\mathbf{T}}$

Invariant LightSpeed (c)

Invariant LightSpeed

$\mathbf{U}\cdot\mathbf{U} = \gamma^2(c^2-\mathbf{u}\cdot\mathbf{u}) = c^2$

$\mathbf{U}\cdot\partial\,[..]$

$\gamma d/dt\,[..]$

$d/d\tau\,[..]$

Invariant Accel:Vel Orthogonality

$\mathbf{A}\cdot\mathbf{U} = 0$ from $d/d\tau(\mathbf{U}\cdot\mathbf{U}) = d/d\tau(c^2) = 0 \leftrightarrow (\mathbf{A}\perp\mathbf{U})$

{$\mathbf{U}$=Time-like, $\mathbf{A}=\mathbf{U}'$=Space-like}

4-Acceleration

$\mathbf{A}=A^\mu=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

$=\gamma^2(\gamma^2(\mathbf{u}\cdot\mathbf{a})/c,\gamma^2(\mathbf{u}\cdot\mathbf{a})\mathbf{u}/c^2+\mathbf{a})$

$=\gamma^4((\mathbf{u}\cdot\mathbf{a})/c,(\mathbf{u}\times(\mathbf{u}\times\mathbf{a}))/c^2+\mathbf{a})$

$=d\mathbf{U}/d\tau=d^2\mathbf{R}/d\tau^2$: $\{\gamma'=d\gamma/dt\}$

[m/s²]

Invariant Acceleration $\mathbf{A}\cdot\mathbf{A}$

$= \mathbf{A}_o\cdot\mathbf{A}_o = (0,\mathbf{a}_o)\cdot(0,\mathbf{a}_o) = -\mathbf{a}_o\cdot\mathbf{a}_o = -|\mathbf{a}_o|^2 = -|\boldsymbol{\alpha}|^2$

$= -\gamma^6(\mathbf{u}\cdot\mathbf{a})^2/c^2 - \gamma^4(\mathbf{a}\cdot\mathbf{a}) \rightarrow -\gamma^4(\mathbf{a}\cdot\mathbf{a})$: if ($\mathbf{u}\perp\mathbf{a}$)

$= \gamma^6(\mathbf{u}\times\mathbf{a})^2/c^2 - \gamma^6(\mathbf{a}\cdot\mathbf{a}) \rightarrow -\gamma^6(\mathbf{a}\cdot\mathbf{a})$: if ($\mathbf{u}\parallel\mathbf{a}$)

$= -\gamma^4[(\gamma(\mathbf{u}\cdot\mathbf{a})/c)^2 + a^2]$

The tedious algebra

Invariant Acceleration $\mathbf{A}\cdot\mathbf{A}$

$= \mathbf{A}_o\cdot\mathbf{A}_o = (0,\mathbf{a}_o)\cdot(0,\mathbf{a}_o) = -\mathbf{a}_o\cdot\mathbf{a}_o = -|\mathbf{a}_o|^2$

$= \gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})\cdot\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

$= \gamma^2[\,(c\gamma')^2 - (\gamma'^2)(\mathbf{u}\cdot\mathbf{u}) - 2(\gamma'\gamma)(\mathbf{u}\cdot\mathbf{a}) - (\gamma)^2(\mathbf{a}\cdot\mathbf{a})\,]$

$= \gamma^2[\,(c\gamma')^2[\,1 - (\mathbf{u}\cdot\mathbf{u})/c^2] - 2(\gamma'\gamma)(\mathbf{u}\cdot\mathbf{a}) - (\gamma)^2(\mathbf{a}\cdot\mathbf{a})\,]$

$= \gamma^2[\,(c\gamma')^2/\gamma^2 - 2(\gamma'\gamma)(\mathbf{u}\cdot\mathbf{a}) - (\gamma)^2(\mathbf{a}\cdot\mathbf{a})\,]$

$= \gamma^2[\,\gamma'\gamma'c^2/\gamma^2 - 2(\gamma'\gamma)(\mathbf{u}\cdot\mathbf{a}) - (\gamma)^2(\mathbf{a}\cdot\mathbf{a})\,]$

$= \gamma^2[\,\gamma'(\gamma^3(\mathbf{u}\cdot\mathbf{a})/c^2)c^2/\gamma^2 - 2(\gamma'\gamma)(\mathbf{u}\cdot\mathbf{a}) - (\gamma)^2(\mathbf{a}\cdot\mathbf{a})\,]$

$= \gamma^2[\,\gamma'(\gamma(\mathbf{u}\cdot\mathbf{a})) - 2(\gamma'\gamma)(\mathbf{u}\cdot\mathbf{a}) - (\gamma)^2(\mathbf{a}\cdot\mathbf{a})\,]$

$= \gamma^2[\,-(\gamma'\gamma)(\mathbf{u}\cdot\mathbf{a}) - (\gamma)^2(\mathbf{a}\cdot\mathbf{a})\,]$

$= \gamma^2[\,-(\gamma^3(\mathbf{u}\cdot\mathbf{a})/c^2)\gamma(\mathbf{u}\cdot\mathbf{a}) - (\gamma)^2(\mathbf{a}\cdot\mathbf{a})\,]$

$= \gamma^2[\,-(\gamma^4(\mathbf{u}\cdot\mathbf{a})^2/c^2) - (\gamma)^2(\mathbf{a}\cdot\mathbf{a})\,]$

$= -\gamma^6(\mathbf{u}\cdot\mathbf{a})^2/c^2 - \gamma^4(\mathbf{a}\cdot\mathbf{a})$

$= -\gamma^6[\mathbf{u}^2\mathbf{a}^2 - (\mathbf{u}\times\mathbf{a})^2]/c^2 - \gamma^4(\mathbf{a}\cdot\mathbf{a})$

$= -\gamma^6(\mathbf{u}^2\mathbf{a}^2)/c^2 + \gamma^6(\mathbf{u}\times\mathbf{a})^2/c^2 - \gamma^4(\mathbf{a}\cdot\mathbf{a})$

$= \gamma^6(\mathbf{u}\times\mathbf{a})^2/c^2 - \gamma^4(\mathbf{a}\cdot\mathbf{a})[\gamma^2(\mathbf{u}^2)/c^2 + 1]$

$= \gamma^6(\mathbf{u}\times\mathbf{a})^2/c^2 - \gamma^4(\mathbf{a}\cdot\mathbf{a})[\gamma^2\boldsymbol{\beta}^2 + 1]$

$= \gamma^6(\mathbf{u}\times\mathbf{a})^2/c^2 - \gamma^4(\mathbf{a}\cdot\mathbf{a})[\gamma^2 - 1 + 1]$

$= \gamma^6(\mathbf{u}\times\mathbf{a})^2/c^2 - \gamma^4(\mathbf{a}\cdot\mathbf{a})[\gamma^2]$

$= \gamma^6[(\mathbf{u}\times\mathbf{a})^2/c^2 - (\mathbf{a}\cdot\mathbf{a})]$

$= -\gamma^4[(\gamma(\mathbf{u}\cdot\mathbf{a})/c)^2 + a^2]$

$(|\mathbf{u}\times\mathbf{a}|)^2 + (\mathbf{u}\cdot\mathbf{a})^2 = u^2a^2$

$(|\mathbf{u}||\mathbf{a}|\sin)^2 + (|\mathbf{u}||\mathbf{a}|\cos)^2 = (|\mathbf{u}||\mathbf{a}|)^2$

$\sin^2 + \cos^2 = 1$

$\gamma = 1/\sqrt{[1-(\mathbf{u}\cdot\mathbf{u})/c^2]} = 1/\sqrt{[1-(\boldsymbol{\beta}\cdot\boldsymbol{\beta})]}$

$\gamma' = d\gamma/dt = \gamma^3(\mathbf{u}\cdot\mathbf{a})/c^2$

$(\gamma\beta)^2 = \gamma^2 - 1$

$\boldsymbol{\beta} = \mathbf{u}/c$

Relativistic Hyperbolic Motion:

fully tensorial eqn.

if { $d/d\tau[\mathbf{A}] = (k)\mathbf{U}$ }

then $k = (a_o/c)^2 = (\alpha/c)^2$ = invariant

from $d/d\tau(\mathbf{A}\cdot\mathbf{U}) = 0 = d/d\tau(\mathbf{A})\cdot\mathbf{U}+\mathbf{A}\cdot\mathbf{A}$

thus $d/d\tau(\mathbf{A})\cdot\mathbf{U} = (a_o)^2 = (\alpha)^2$

implies $d/dt[\gamma^3\mathbf{a}] = \mathbf{0}$

SR 4-Tensor

(2,0)-Tensor $T^{\mu\nu}$

(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$

(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector

(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$

SR 4-CoVector:OneForm

(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar

(0,0)-Tensor S or S_o

Lorentz Scalar

4-Acceleration$_{\{MCRF\}}$

$\mathbf{A}_o=A_o{}^\mu=(0,\mathbf{a}_o)=(0,\boldsymbol{\alpha})$

MCRF =

Momentarily Co-Moving Reference Frame

$\mathbf{u} \rightarrow \mathbf{0}$

$\gamma \rightarrow 1$: $\gamma' \rightarrow 0$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$

$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$

= Lorentz Scalar Invariant

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A Tensor Study of Physical 4-Vectors

SRQM Study: Physical 4-Vectors
Some 4-Gradient ∂^μ Relations

The relations below are for the 4-(Position)Gradient ∂_R,
4-Gradients wrt. other 4-Vector variables exist also… ex. 4-WaveGradient ∂_K

4-Vector SRQM Interpretation of QM

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{ These combos between ∂ & **R** are all unit-dimensionless = [1/m]*[m] = [1] }

4-Gradient: 4-(Position)Gradient
$\partial=\partial_R=\partial_X=\partial^\mu=(\partial_t/c,-\nabla=-\partial_r)$
$=\partial/\partial R_\mu\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$
[1/m]

Minkowski
$\partial[\mathbf{R}]=\partial^\mu[R^\nu]=\eta^{\mu\nu}$
Metric

Lorentz
$\partial_\nu[R^{\mu'}]=\Lambda^{\mu'}{}_\nu$
Transform

SpaceTime
$\partial\cdot\mathbf{R}=\partial_\mu R^\mu=4$
Dimension

SRQM Non-Zero Commutation
$[\partial,\mathbf{R}]=[\partial^\mu,R^\nu]$
$=\partial^\mu R^\nu-R^\nu\partial^\mu=\eta^{\mu\nu}$

Calculus
$d\mathbf{R}\cdot\partial=dR^\mu\partial_\mu$
Total Derivative
Chain Rule

4-Position:4-Differential
$\mathbf{R}=R^\mu=(ct,\mathbf{r})\in$<Event>
$\rightarrow(ct,x,y,z)$
$d\mathbf{R}=dR^\mu=(cdt,d\mathbf{r})$
[m]

$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$
$=(\partial_\tau/c)^2$
Invariant d'Alembertian Wave Eqn.

ProperTime Derivative
$\mathbf{U}\cdot\partial$
$=\gamma d/dt$
$=d/d\tau$

Math/Phys Wave Solution of d'Alembertian

$-\partial[S_{action,free}]=\mathbf{P}$

Phase (Φ) & Action (S) Lorentz Scalars

$-\partial[\Phi_{phase,free}]=\mathbf{K}$

4-Momentum
$\mathbf{P}=P^\mu=(mc,m\mathbf{u})=m_o\gamma(c,\mathbf{u})=m_o\mathbf{U}=-\partial[S]$
$=(E/c,\mathbf{p})=(E_o/c^2)\gamma(c,\mathbf{u})=(E_o/c^2)\mathbf{U}=(E_o/c)\bar{\mathbf{T}}$

4-WaveVector
$\mathbf{K}=K^\mu=(\omega/c=2\pi\nu/c,\mathbf{k})=(\omega_o/c^2)\mathbf{U}=-\partial[\Phi]$
$=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})=(1/c\mathrm{T},\hat{\mathbf{n}}/\lambda\!\!\!^{-})=(\omega_o/c)\bar{\mathbf{T}}$

Faraday EM Tensor
$F^{\alpha\beta}=\partial^\alpha A^\beta-\partial^\beta A^\alpha=\partial\wedge\mathbf{A}$

0	$-e^{0j}/c$
$+e^{i0}/c$	$-\varepsilon^{ij}{}_k b^k$

[T]

Conservation of EM Potential
Lorenz Gauge
$\partial\cdot\mathbf{A}=0$

$(\partial\cdot\mathbf{A})=0$

4-(EM)VectorPotential
$\mathbf{A}=A^\mu=(\varphi/c,\mathbf{a})=(\varphi_o/c^2)\mathbf{U}$
$\mathbf{A_{EM}}=A_{EM}{}^\mu=(\varphi_{EM}/c,\mathbf{a_{EM}})$

[m/s]

4-Velocity
$\mathbf{U}=U^\mu=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau=c\bar{\mathbf{T}}$

4D Generalized Stokes' Theorem
Integration of:
4D Div = 4D Surface Flow

$\int_\Omega d^4\mathbf{X}(\partial_\mu V^\mu)=\int_\Omega d^4\mathbf{X}(\partial\cdot\mathbf{V})$ = $\oint_{\partial\Omega}dS(V^\mu\hat{N}_\mu)=\oint_{\partial\Omega}dS(\mathbf{V}\cdot\hat{\mathbf{N}})$

[{V}·m³] [{V}·m³]

Conservation of Charge
$\partial\cdot\mathbf{J}=0$

$(\partial\cdot\mathbf{J})=0$

4-ChargeFlux : 4-CurrentDensity
$\mathbf{J}=J^\mu=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})=\rho_o\gamma(c,\mathbf{u})=\rho_o\mathbf{U}$
$=qn_o\mathbf{U}=q\mathbf{N}$

Conservation of Particle #
$\partial\cdot\mathbf{N}=0$

$(\partial\cdot\mathbf{N})=0$

4-(Particle:Dust:Number)Flux
$\mathbf{N}=N^\mu=(nc,\mathbf{n})=n(c,\mathbf{u})=n_o\gamma(c,\mathbf{u})=n_o\mathbf{U}$

q

Ω = 4D Minkowski Region, ∂Ω = it's 3D boundary
$d^4\mathbf{X}$ = 4D Volume Element, **V** = V^μ = Arbitrary 4-Vector Field
dS = 3D Surface Element, $\hat{\mathbf{N}}$ = $\hat{N}^\mu$ = Surface Normal

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

4-Vector V = $V^\mu=(v^\mu)=(v^0,v^i)=(v^0,\mathbf{v})$
SR 4-Vector V = V^μ = (scalar * $c^{\pm1}$,3-vector)

$\mathrm{Trace}[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

4-Vector SRQM Interpretation of QM

SRQM Study: Constant |Magnitudes| Special Motion: Circular & Hyperbolic

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John B. Wilson
SciRealm@aol.com
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$(\mathbf{U}\cdot\mathbf{U}) = (c)^2$ is temporal, invariant, fundamental constant $\qquad \mathbf{R}^{n'} = d\mathbf{R}^{(n-1)'}/d\tau$

$d/d\tau[\mathbf{U}\cdot\mathbf{U}] = d/d\tau[c^2] = 0 = d/d\tau[\mathbf{U}]\cdot\mathbf{U} + \mathbf{U}\cdot d/d\tau[\mathbf{U}] = 2\mathbf{A}\cdot\mathbf{U} = 0$

$(\mathbf{A}\cdot\mathbf{U} = 0) \leftrightarrow (\mathbf{A}\perp\mathbf{U})$: 4-Acceleration (normal to worldline) is orthogonal(⊥) to 4-Velocity (tangent to worldline)

$d/d\tau[\mathbf{A}\cdot\mathbf{U}] = 0 = d/d\tau[\mathbf{A}]\cdot\mathbf{U}+\mathbf{A}\cdot d/d\tau[\mathbf{U}] = \mathbf{J}\cdot\mathbf{U}+\mathbf{A}\cdot\mathbf{A} = \mathbf{J}\cdot\mathbf{U}+ -(\alpha)^2$

$(\mathbf{J}\cdot\mathbf{U}) = (\alpha)^2 = -(\mathbf{A}\cdot\mathbf{A}) \qquad (\mathbf{J}\cdot\mathbf{J}) = (c\gamma_o'')^2 - (j_o)^2$

General Motion:

4-Position	$\mathbf{R} = R^\mu = (ct,\mathbf{r})$	$= \mathbf{R}$	$= d^0\mathbf{R}/d\tau^0$
4-Velocity	$\mathbf{U} = U^\mu = \gamma(c,\mathbf{u})$	$= d\mathbf{R}/d\tau$	$= d^1\mathbf{R}/d\tau^1$
4-Acceleration	$\mathbf{A} = A^\mu = \gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$	$= d\mathbf{U}/d\tau$	$= d^2\mathbf{R}/d\tau^2$
4-Jerk	$\mathbf{J} = J^\mu = \gamma(c(\gamma'^2+\gamma\gamma''),(\gamma'^2+\gamma\gamma'')\mathbf{u}+\gamma(3\gamma'\mathbf{a}+\gamma\mathbf{j}))$	$= d\mathbf{A}/d\tau$	$= d^3\mathbf{R}/d\tau^3$

All Lorentz Scalar Products are Invariants

$(\mathbf{R}\cdot\mathbf{R}) = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (ct_o)^2 = (c\tau)^2 = -(\mathbf{r}_o\cdot\mathbf{r}_o)$:either (±),variable

$(\mathbf{U}\cdot\mathbf{U}) = (c)^2$:temporal(+),fundamental constant

$(\mathbf{A}\cdot\mathbf{A}) = -(a_o)^2 = -(\alpha)^2 = (i\alpha)^2$:spatial(-),variable

$(\mathbf{J}\cdot\mathbf{J}) = (\mathbf{J}_o\cdot\mathbf{J}_o) = (c\gamma_o'')^2 - (j_o)^2$:either(±),variable

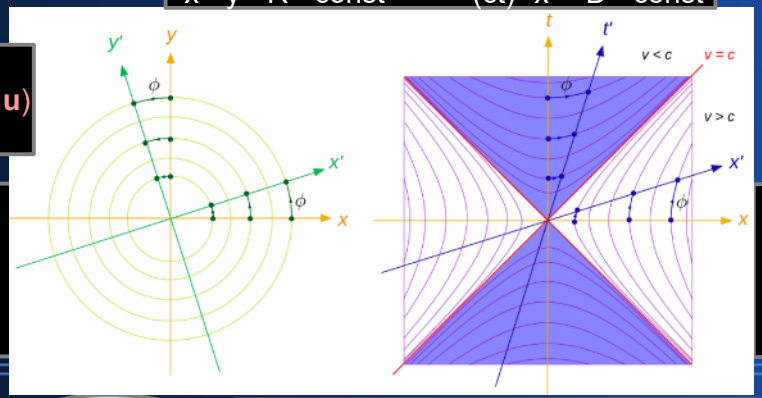
General Motion: (alt form A,J)

		w/ Spatial Orthogonality(⊥)
4-Position	$\mathbf{R} = R^\mu = (ct,\mathbf{r})$	$= (ct,\mathbf{r})$
4-Velocity	$\mathbf{U} = U^\mu = \gamma(c,\mathbf{u})$	$= \gamma(c,\mathbf{u})$
4-Acceleration	$\mathbf{A} = A^\mu = (\gamma^4(\mathbf{a}\cdot\mathbf{u})/c , \gamma^4(\mathbf{a}\cdot\mathbf{u})\mathbf{u}/c^2+\gamma^2\mathbf{a})$	$= \gamma^2(0,\mathbf{a})_\perp$ if $(\mathbf{a}\cdot\mathbf{u})=0$
4-Jerk	$\mathbf{J} = J^\mu = \gamma(c(\gamma^6(\mathbf{a}\cdot\mathbf{u})^2/c^4 + \gamma^4[3\gamma^2(\mathbf{a}\cdot\mathbf{u})^2+(\mathbf{a}\cdot\mathbf{u})']/c^2) , (\gamma^6(\mathbf{a}\cdot\mathbf{u})^2/c^4 + \gamma^4[3\gamma^2(\mathbf{a}\cdot\mathbf{u})^2+(\mathbf{a}\cdot\mathbf{u})']/c^2)\mathbf{u}+\gamma(3\gamma^3(\mathbf{a}\cdot\mathbf{u})\mathbf{a}/c^2+\gamma\mathbf{j}))$	$= \gamma^3(0,\mathbf{j})_\perp$ if $(\mathbf{a}\cdot\mathbf{u})=0$

Lorentz Trans. **Λ→R**
(R)otation = Spatial:
{ |**r**|,|**u**|,|**a**|,|**j**| } &
{ R,Ω,γ } constant
$\mathbf{a} = (-\Omega^2)\mathbf{r}$, $\mathbf{j} = (-\Omega^2)\mathbf{u}$
$(\mathbf{a}\cdot\mathbf{u}) = 0 = \gamma'$
$n_1 = \cos : n_2 = \sin$

Circular Case:
$-(\mathbf{a}\cdot\mathbf{r}) = (R\Omega)^2 = (\mathbf{u}\cdot\mathbf{u})$
$|\mathbf{a}| = |\mathbf{u}|^2/|\mathbf{r}| = R\Omega^2$

$x^2+y^2=R^2=\text{const} \qquad (ct)^2-x^2=-D^2=\text{const}$



Hyperbolic Case:
$-(\mathbf{A}\cdot\mathbf{R}) = (c)^2 = (\mathbf{U}\cdot\mathbf{U})$
$|\mathbf{A}| = |\mathbf{U}|^2/|\mathbf{R}| = \alpha$

Lorentz Trans. **Λ→B**
(B)oost = Time·Space:
{ |**R**|,|**U**|,|**A**|,|**J**| } &
{ D,c,α } constant
$\mathbf{A} = (\alpha^2/c^2)\mathbf{R}$, $\mathbf{J} = (\alpha^2/c^2)\mathbf{U}$
$(\mathbf{A}\cdot\mathbf{U}) = 0$
$\gamma = \cosh : \gamma\beta = \sinh$

Circular Motion: constants {R,Ω,γ}

4-Position	$\mathbf{R} = R^\mu = (ct,\mathbf{r} = R\hat{\mathbf{r}})$
4-Velocity	$\mathbf{U} = U^\mu = \gamma^1(c,\mathbf{u} = R\Omega\hat{\boldsymbol{\theta}})$
4-Acceleration	$\mathbf{A} = A^\mu = \gamma^2(0,\mathbf{a} = -R\Omega^2\hat{\mathbf{r}})$
4-Jerk	$\mathbf{J} = J^\mu = \gamma^3(0,\mathbf{j} = -R\Omega^3\hat{\boldsymbol{\theta}})$

Hyperbolic Motion: constant {Rindler "Dist" $D=c^2/\alpha$}

4-Position	$\mathbf{R} = R^\mu = (c^2/\alpha)(\sinh[\alpha\tau/c],\cosh[\alpha\tau/c]\,\hat{\mathbf{n}})$
4-Velocity	$\mathbf{U} = U^\mu = (c)(\cosh[\alpha\tau/c],\sinh[\alpha\tau/c]\,\hat{\mathbf{n}})$
4-Acceleration	$\mathbf{A} = A^\mu = (\alpha)(\sinh[\alpha\tau/c],\cosh[\alpha\tau/c]\,\hat{\mathbf{n}})$
4-Jerk	$\mathbf{J} = J^\mu = (\alpha^2/c)(\cosh[\alpha\tau/c],\sinh[\alpha\tau/c]\,\hat{\mathbf{n}})$

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SR 4-CoVector:OneForm
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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Special Cases still maintain: $\mathbf{R}^{n'} = d\mathbf{R}^{(n-1)'}/d\tau$: $(\mathbf{R}\cdot\mathbf{R}) = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = -(\mathbf{r}_o\cdot\mathbf{r}_o)$
$(\mathbf{U}\cdot\mathbf{U}) = c^2$: $(\mathbf{A}\cdot\mathbf{U}) = 0$: $(\mathbf{J}\cdot\mathbf{U}) = \alpha^2 = -(\mathbf{A}\cdot\mathbf{A})$: $(\mathbf{J}\cdot\mathbf{J}) = (c\gamma_o'')^2 - (j_o)^2$

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study:
Fundamental Dimensional-Units
Fundamental Physical Constants

4-Vector SRQM Interpretation of QM

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4-Gradient [1/m]

$\partial=\partial_R=\partial_X=\partial^\mu=(\partial_t/c,-\nabla)$
$=\partial/\partial R_\mu \rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

4-Position [m]

$\mathbf{R}=R^\mu=(ct,\mathbf{r})\in$ <Event>
$\rightarrow(ct,x,y,z)$
(c[time],[length])

$\mathbf{U}\cdot\partial$
$=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)$
$=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$=\gamma(\partial/\partial t+d\mathbf{r}/dt\cdot\partial/\partial\mathbf{r})$
$=\gamma d/dt$
$=d/d\tau$

$\mathbf{U}\cdot\partial$ [..]
$\gamma d/dt$ [..]
$d/d\tau$ [..]

ProperTime Derivative

4-Velocity [m/s]

$\mathbf{U}=U^\mu=\gamma(c,\mathbf{u})$
$=c\bar{\mathbf{T}}=c^2\partial[\tau]=d\mathbf{R}/d\tau$

$\mathbf{U}\cdot\mathbf{U}=c^2$

Invariant LightSpeed (c)

Fundamental Constants

Universal:

c	LightSpeed (vacuum)
ħ,h	Dirac,Planck
G	Gravitational
k_B	Boltzmann
e	Elementary Charge
ε_o,μ_o	Electric,Magnetic Constants

Depends on Particle Type:

q	Particle Charge = (n/3)*e
s_o	Particle Spin = (n/2)*ħ
m_o : E_o	Rest Mass:Energy
$\Delta\nu_o$	Rest Spectra Transition Frequency

Semi-Arbitrary:

N_A	Avogadro # = approx. # nucleons in 1 gram matter
K_{CD}	Luminous efficacy of 540 THz radiation

Rest Mass:Energy/c² [kg]: m_o [mass] → **4-Momentum** [kg·m/s=N·s]

$\mathbf{P}=P^\mu=(mc,m\mathbf{u})=m_o\gamma(c,\mathbf{u})=m_o\mathbf{U}=-\partial[S]$
$=(E/c,\mathbf{p})=(E_o/c^2)\gamma(c,\mathbf{u})=(E_o/c^2)\mathbf{U}=(E_o/c)\bar{\mathbf{T}}$

ħ

Rest Ang. Frequency/c² [rad·s/m²]: ω_o/c^2 [{angle}] → **4-WaveVector** [rad/m]

$\mathbf{K}=K^\mu=(\omega/c=2\pi\nu/c,\mathbf{k})=(\omega_o/c^2)\mathbf{U}=-\partial[\Phi]$
$=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})=(1/c\bar{T},\hat{\mathbf{n}}/\bar{\lambda})=(\omega_o/c)\bar{\mathbf{T}}$

Rest Charge Density [C/m³]: $\rho_o=qn_o$ [EM charge] → **4-ChargeFlux : 4-CurrentDensity** [C/m²·s]

$\mathbf{J}=J^\mu=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})=\rho_o\gamma(c,\mathbf{u})=\rho_o\mathbf{U}$
$=qn_o\mathbf{U}=q\mathbf{N}$

q ~ e

Rest Number Density [#/m³]: $n_o=N/V_o$ [amount of subtance] [luminous intensity] → **4-(Particle:Dust:Number)Flux** [#/m²·s]

$\mathbf{N}=N^\mu=(nc,\mathbf{n})=n(c,\mathbf{u})=n_o\gamma(c,\mathbf{u})=n_o\mathbf{U}$

Rest Inv. Thermal Energy [1/J]: $1/k_BT_o$ [temperature] → **4-ThermalVector** [m/J·s=1/N·s]

4-InverseTemperatureMomentum
$\boldsymbol{\Theta}=\Theta^\mu=(\theta^0,\boldsymbol{\theta})=(c/k_BT,\mathbf{u}/k_BT)=(\theta_o/c)\mathbf{U}$
$=(1/k_BT)(c,\mathbf{u})=(1/k_B\gamma T)\mathbf{U}=(1/k_BT_o)\mathbf{U}$

SI Units

Symbol	Name	Dimension Type
[s]	second	[T] = [time] = extent in the temporal spacetime direction, temporal displacement Δt
[m]	meter	[L] = [length] = extent in one or more spatial spacetime directions, spatial displacement |Δr|
[kg]	kilogram	[M] = [mass] = count of material stuff (m) using Gravitational force, Poincaré Casimir Invariant
[C]	coulomb	[Q] = [EM charge] = count of material stuff (q) using ElectroMagnetic force, CPT Symmetry
[K] or [°K]	kelvin	[Θ] = [thermodynamic temperature] = statistical count of information
[mol]	mole	[N] = [amount of substance] = count of fermionic matter-particle stuff: typically "atoms/molecules"
[cd]	candela	[J] = [luminous intensity] = count of bosonic force-particle stuff : typically "photons"

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Lorentz Scalar

4-Vector V = $V^\mu=(v^\mu)=(v^0,v^i)=(v^0,\mathbf{v})$
SR 4-Vector V = V^μ = (scalar * $c^{\pm1}$,3-vector)

$\mathrm{Trace}[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4-Vector SRQM Interpretation of QM

SRQM Study: Physical 4-Tensors

Some SR 4-Tensors and Symbols

A Tensor Study of Physical 4-Vectors

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John B. Wilson
SciRealm@aol.com
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←Discrete Continuous→
SR:Lorentz Transforms

Lorentz Identity Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow \eta^{\mu'}{}_{\nu} = I_{(4)} = \delta^{\mu'}{}_{\nu}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & 1 & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array} = \begin{matrix} 1 & 0_j \\ 0^i & \delta^i{}_j \end{matrix}$$

Lorentz x-Boost Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow B^{\mu'}{}_{\nu} =$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \gamma & -\beta\gamma & 0 & 0 \\ x & -\beta\gamma & \gamma & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array} = \begin{array}{c|cccc} & t & x & y & z \\ \hline t & \cosh[w] & -\sinh[w] & 0 & 0 \\ x & -\sinh[w] & \cosh[w] & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array}$$

General Time-Space Boost

$$\begin{matrix} \gamma & -\gamma\beta_j \\ -\gamma\beta^i & (\gamma-1)\beta^i\beta_j/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\delta^i{}_j \end{matrix}$$

Symmetric Mixed 4-Tensor
$B^T = B \;:\; B^{-1} = B[-\beta]$

Lorentz Time-Reverse Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow T^{\mu'}{}_{\nu} =$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & -1 & 0 & 0 & 0 \\ x & 0 & 1 & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array} = \begin{matrix} -1 & 0_j \\ 0^i & \delta^i{}_j \end{matrix}$$

SR:Minkowski Metric
$\partial[\mathbf{R}] = \partial^{\mu}R^{\nu} = \eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu}$
$\rightarrow \mathrm{Diag}[1,-I_{(3)}] = \mathrm{Diag}[1,-\delta^{ij}]$
{Cartesian/rectangular basis}

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & -1 \end{array} \qquad \begin{matrix} 1 & 0^{0j} \\ 0^{i0} & -\delta^{ij} \end{matrix}$$

Particle Physics" Convention
4-Tensor [Unit Dimensionless]
Symmetric, Spatial Isotropic

Lorentz z-Rotation Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow R^{\mu'}{}_{\nu} =$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & \cos[\theta] & -\sin[\theta] & 0 \\ y & 0 & \sin[\theta] & \cos[\theta] & 0 \\ z & 0 & 0 & 0 & 1 \end{array}$$

General Space-Space Rotation

$$\begin{matrix} 1 & 0_j \\ 0^i & (\delta^i{}_j - n^i n_j)\cos(\theta) - (\varepsilon^i{}_{jk} n^k)\sin(\theta) + n^i n_j \end{matrix}$$

Non-symmetric Mixed 4-Tensor
Orthogonal $R^T = R^{-1} \;:\; R^{-1} = R[-\theta]$

Lorentz Space-Reverse (Parity Inverse) Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow P^{\mu'}{}_{\nu} =$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & -1 \end{array} = \begin{matrix} 1 & 0_j \\ 0^i & -\delta^i{}_j \end{matrix}$$

SpaceTime
$\partial\cdot\mathbf{R} = \partial_{\mu}R^{\mu} = \mathrm{Tr}[\eta^{\mu\nu}] = 4$
Dimension

SpaceTime
$\Lambda_{\mu\nu}\Lambda^{\mu\nu} = 4 = \Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$
Dimension

Lorentz Transform $\partial_{\nu}[R^{\mu'}] = \Lambda^{\mu'}{}_{\nu}$
$[\Lambda^{0'}{}_{0}, \Lambda^{0'}{}_{j}]$ temporal-spatial-mixed
$[\Lambda^{i'}{}_{0}, \Lambda^{i'}{}_{j}]$ components
4-Tensor [Unit Dimensionless=1]

Lorentz ComboPT Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow (PT)^{\mu'}{}_{\nu} = -I_{(4)} = -\delta^{\mu'}{}_{\nu}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & -1 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & -1 \end{array} = \begin{matrix} -1 & 0_j \\ 0^i & -\delta^i{}_j \end{matrix}$$

Perfect Fluid
$T^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu}$
$\rightarrow \mathrm{Diag}[\rho_e, p\delta^{ij}]$ {rectangular basis}{MCRF}

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & p & 0 & 0 \\ y & 0 & 0 & p & 0 \\ z & 0 & 0 & 0 & p \end{array} \qquad \begin{matrix} \rho_e = \rho_m c^2 & 0^{0j} \\ 0^{i0} & p\delta^{ij} \end{matrix}$$

[J/m³]
[N/m²]
[Pa]

4-Tensor
Symmetric, Spatial Isotropic

Faraday EM
$F^{\alpha\beta} = \partial^{\alpha}A^{\beta} - \partial^{\beta}A^{\alpha} = \boldsymbol{\partial} \wedge \mathbf{A}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & -e^x/c & -e^y/c & -e^z/c \\ x & +e^x/c & 0 & -b^z & +b^y \\ y & +e^y/c & +b^z & 0 & -b^x \\ z & +e^z/c & -b^y & +b^x & 0 \end{array} \qquad \begin{matrix} 0 & -e^{0j}/c \\ +e^{i0}/c & -\varepsilon^{ij}{}_k b^k \end{matrix} \qquad \begin{matrix} 0 & -\mathbf{e}/c \\ +\mathbf{e}^T/c & -\nabla\wedge\mathbf{a} \end{matrix}$$

4-Tensor [SI Unit=T]
Anti-symmetric (skew)

4-AngularMomentum
$M^{\alpha\beta} = X^{\alpha}P^{\beta} - X^{\beta}P^{\alpha} = \mathbf{X} \wedge \mathbf{P}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & -cn^x & -cn^y & -cn^z \\ x & +cn^x & 0 & +l^z & -l^y \\ y & +cn^y & -l^z & 0 & +l^x \\ z & +cn^z & +l^y & -l^x & 0 \end{array} \qquad \begin{matrix} 0 & -cn^{0j} \\ +cn^{i0} & \varepsilon^{ij}{}_k l^k \end{matrix} \qquad \begin{matrix} 0 & -c\mathbf{n} \\ +c\mathbf{n}^T & \mathbf{x}\wedge\mathbf{p} \end{matrix}$$

4-Tensor [SI Unit=J·s=Action]
Anti-symmetric (skew)

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(0,0)-Tensor S or S_o
Lorentz Scalar

Note that all the Lorentz Transforms and the Minkowski Metric are [unit dimensionless=1].
The Perfect Fluid has units of
[energy density = pressure = Pa = J/m³ = N/m² = kg/m·s²]

w = Rapidity = Ln[$\gamma(1+\beta)$]
$\gamma = \cosh(w) = 1/\sqrt{[1-\beta^2]}$
$\beta = \tanh(w) = (v/c)$
$\gamma\beta = \sinh(w)$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

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SRQM Study: Physical 4-Tensors

Projection 4-Tensors $\{P^{\mu\nu} : P^{\mu}{}_{\nu} : P_{\mu\nu}\}$

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$Tr[V^{\mu\nu}]=1$

Temporal "(V)ertical" Projection (2,0)-Tensor

$P^{\mu\nu} \rightarrow V^{\mu\nu} = \|^{\mu\nu} = \bar{T}^{\mu}\bar{T}^{\nu}$

$\rightarrow Diag[1,0^{0j}]_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array} \qquad \begin{matrix} 1 & 0^{0j} \\ 0^{i0} & 0^{ij} \end{matrix}$$

4-Tensor

Symmetric, Spatial Isotropic

[Unit Dimensionless]

$Tr[V^{\mu}{}_{\nu}]=1$

Temporal "(V)ertical" Projection (1,1)-Tensor

$P^{\mu}{}_{\nu} \rightarrow V^{\mu}{}_{\nu} = \|^{\mu}{}_{\nu} = \bar{T}^{\mu}\bar{T}_{\nu}$

$\rightarrow Diag[1,0^{i}{}_{j}]_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array} \qquad \begin{matrix} 1 & 0^{0}{}_{j} \\ 0^{i}{}_{0} & 0^{i}{}_{j} \end{matrix}$$

4-Tensor

Symmetric, Spatial Isotropic

[Unit Dimensionless]

$Tr[V_{\mu\nu}]=1$

Temporal "(V)ertical" Projection (0,2)-Tensor

$P_{\mu\nu} \rightarrow V_{\mu\nu} = \|_{\mu\nu} = \bar{T}_{\mu}\bar{T}_{\nu}$

$\rightarrow Diag[1,0_{ij}]_{\{MCRF\}}$

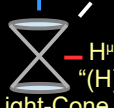
$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array} \qquad \begin{matrix} 1 & 0_{0j} \\ 0_{i0} & 0_{ij} \end{matrix}$$

4-Tensor

Symmetric, Spatial Isotropic

[Unit Dimensionless]

"(V)ertical"
$V^{\mu\nu}$ Temporal



$H^{\mu\nu}$ Spatial "(H)orizontal"

Light-Cone

$P^{\mu}{}_{\nu} = P^{\mu\alpha}\eta_{\alpha\nu}$

$P_{\mu\nu} = P^{\alpha\beta}\eta_{\alpha\mu}\eta_{\beta\nu}$

SR Perfect Fluid Stress-Energy 4-Tensor

$T_{perfectfluid}{}^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} \rightarrow_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e=\rho_m c^2 & 0 & 0 & 0 \\ x & 0 & p & 0 & 0 \\ y & 0 & 0 & p & 0 \\ z & 0 & 0 & 0 & p \end{array} \qquad \begin{matrix} \rho_e=\rho_m c^2 & 0^{0j} \\ 0^{i0} & p\delta^{ij} \end{matrix}$$

Units of [EnergyDensity=Pressure=Pa]

Symmetric

$EoS[T^{\mu\nu}]=w=p_o/\rho_{eo}$

$Tr[T^{\mu\nu}]=\rho_{eo}-3p_o$

The projection tensors can work on 4-Vectors to give a new 4-Vector, or on 4-Tensors to give either a 4-Scalar component or a new 4-Tensor.

4-UnitTemporal $\bar{T}^{\mu} = \gamma(1,\beta)$

4-Generic $A^{\nu} = (a^0,a) = (a^0,a^1,a^2,a^3)$

4-UnitTemporal $\bar{T}=T^{\mu}=\gamma(1,\beta)$ $=\gamma(1,u/c)=U/c$ $\rightarrow(1,0)_{\{RestFrame\}}$

4-UnitSpatial $\bar{S}=S^{\mu}=\gamma_{\hat{n}}(\beta\cdot\hat{n},\hat{n})$ from $\bar{T}\cdot\bar{S}=0$ $\rightarrow(0,\hat{n})_{\{RestFrame\}}$

$\bar{T}\cdot\bar{T}=+1$ $\bar{T}\cdot\bar{S}=0$ $\bar{S}\cdot\bar{S}=-1$

$$\begin{aligned} V^{\mu}{}_{\nu}A^{\nu} = (&1\cdot a^0+0\cdot a^1+0\cdot a^2+0\cdot a^3, \\ &0\cdot a^0+0\cdot a^1+0\cdot a^2+0\cdot a^3, \\ &0\cdot a^0+0\cdot a^1+0\cdot a^2+0\cdot a^3, \\ &0\cdot a^0+0\cdot a^1+0\cdot a^2+0\cdot a^3) = (a^0,0,0,0) = (a^0,\mathbf{0}) \end{aligned}$$

: Temporal Projection

$$\begin{aligned} H^{\mu}{}_{\nu}A^{\nu} = (&0\cdot a^0+0\cdot a^1+0\cdot a^2+0\cdot a^3, \\ &0\cdot a^0+1\cdot a^1+0\cdot a^2+0\cdot a^3, \\ &0\cdot a^0+0\cdot a^1+1\cdot a^2+0\cdot a^3, \\ &0\cdot a^0+0\cdot a^1+0\cdot a^2+1\cdot a^3) = (0,a^1,a^2,a^3) = (0,\mathbf{a}) \end{aligned}$$

: Spatial Projection

$V_{\mu\nu}H^{\mu\nu}=0$

Minkowski $\partial[\mathbf{R}]=\partial^{\mu}[R^{\nu}]=\eta^{\mu\nu}=V^{\mu\nu}+H^{\mu\nu}$ Metric

$$\begin{aligned} V_{\mu\nu}T^{\mu\nu} &= V_{\mu\nu}[(\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu}] = (\rho_{eo})V_{\mu\nu}V^{\mu\nu} + (0) = (\rho_{eo}) \quad : (\rho_{eo}) = V_{\mu\nu}T^{\mu\nu} \\ H_{\mu\nu}T^{\mu\nu} &= H_{\mu\nu}[(\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu}] = (0)+(-p_o)H_{\mu\nu}H^{\mu\nu} = (-3p_o) : (p_o) = (-1/3)H_{\mu\nu}T^{\mu\nu} \\ Trace[T^{\mu\nu}] &= \eta_{\mu\nu}T^{\mu\nu} = V_{\mu\nu}T^{\mu\nu} + H_{\mu\nu}T^{\mu\nu} = (\rho_{eo})+(-3p_o) = (\rho_{eo} - 3p_o) \end{aligned}$$

$$\begin{aligned} V^{\mu}{}_{\alpha}T^{\alpha\nu} &= V^{\mu}{}_{\alpha}[(\rho_{eo})V^{\alpha\nu} + (-p_o)H^{\alpha\nu}] = (\rho_{eo})V^{\mu}{}_{\alpha}V^{\alpha\nu}+(0^{\mu\nu}) = (\rho_{eo})V^{\mu\nu} \rightarrow Diag[\rho_e,0,0,0] \\ H^{\mu}{}_{\alpha}T^{\alpha\nu} &= H^{\mu}{}_{\alpha}[(\rho_{eo})V^{\alpha\nu} + (-p_o)H^{\alpha\nu}] = (0^{\mu\nu})+(-p_o)H^{\mu}{}_{\alpha}H^{\alpha\nu} = (-p_o)H^{\mu\nu} \rightarrow Diag[0,p,p,p] \end{aligned}$$

$Tr[H^{\mu\nu}]=3$

Spatial "(H)orizontal" Projection (2,0)-Tensor

$P^{\mu\nu} \rightarrow H^{\mu\nu} = \perp^{\mu\nu} = \eta^{\mu\nu} - \bar{T}^{\mu}\bar{T}^{\nu}$

$\rightarrow Diag[0,-\mathbf{I}_{(3)}]=Diag[0,-\delta^{ij}]_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & -1 \end{array} \qquad \begin{matrix} 0 & 0^{0j} \\ 0^{i0} & -\delta^{ij} \end{matrix}$$

4-Tensor

Symmetric, Spatial Isotropic

[Unit Dimensionless]

$Tr[H^{\mu}{}_{\nu}]=3$

Spatial "(H)orizontal" Projection (1,1)-Tensor

$P^{\mu}{}_{\nu} \rightarrow H^{\mu}{}_{\nu} = \perp^{\mu}{}_{\nu} = \eta^{\mu}{}_{\nu} - \bar{T}^{\mu}\bar{T}_{\nu}$

$\rightarrow Diag[0,\mathbf{I}_{(3)}]=Diag[0,\delta^{i}{}_{j}]_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & 0 & 0 & 0 \\ x & 0 & 1 & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array} \qquad \begin{matrix} 0 & 0^{0}{}_{j} \\ 0^{i}{}_{0} & \delta^{i}{}_{j} \end{matrix}$$

4-Tensor

Symmetric, Spatial Isotropic

[Unit Dimensionless]

Note Sign-flip

$Tr[H_{\mu\nu}]=3$

Spatial "(H)orizontal" Projection (0,2)-Tensor

$P_{\mu\nu} \rightarrow H_{\mu\nu} = \perp_{\mu\nu} = \eta_{\mu\nu} - \bar{T}_{\mu}\bar{T}_{\nu}$

$\rightarrow Diag[0,-\mathbf{I}_{(3)}]=Diag[0,-\delta_{ij}]_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & -1 \end{array} \qquad \begin{matrix} 0 & 0_{0j} \\ 0_{i0} & -\delta_{ij} \end{matrix}$$

4-Tensor

Symmetric, Spatial Isotropic

[Unit Dimensionless]

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Note that the Projection Tensors are [Unit Dimensionless = 1]: the object projected retains its own dimensional measurement units
Note that the (2,0)- & (0,2)- Spatial Projectors have opposite signs from the mixed (1,1)- Spatial due to minuses in the Minkowski Metric.

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: SR 4-Vector Properties
General T^μ → Temporal:Spatial
(1+3) Decomposition via Projection

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Any SR (1-index) Tensor { T^μ , T_μ } can be decomposed into = { Temporal , Spatial } parts, by using combinations of (V)ertical & (H)orizontal Projection Tensors, with ($\eta^\mu{}_\nu = V^\mu{}_\nu + H^\mu{}_\nu$) and contraction orthogonality ($V^\mu{}_\alpha H^\alpha{}_\nu$) = ($V^\mu{}_\alpha (\eta^\alpha{}_\nu - V^\alpha{}_\nu)$) = ($V^\mu{}_\alpha\eta^\alpha{}_\nu - V^\mu{}_\alpha V^\alpha{}_\nu$) = ($V^\mu{}_\nu - V^\mu{}_\nu$) = $0^\mu{}_\nu$

$\text{Temporal}^\mu = V^\mu{}_\alpha (T^\alpha)$: $\text{LSP}[\text{Temporal}^\mu] = \eta_{\mu\nu}V^\mu{}_\alpha(T^\alpha)V^\nu{}_\beta(T^\beta) = (T^0)^2$

$\text{Spatial}^\mu = H^\mu{}_\alpha (T^\alpha)$: $\text{LSP}[\text{Spatial}^\mu] = \eta_{\mu\nu}H^\mu{}_\alpha(T^\alpha)H^\nu{}_\beta(T^\beta) = -[(T^1)^2+(T^2)^2+(T^3)^2]$

$\text{FullSpaceTime}^\mu = \eta^\mu{}_\alpha (T^\alpha) = T^\mu$: $\text{LSP}[\text{FullSpaceTime}^\mu] = \eta_{\mu\nu}\eta^\mu{}_\alpha(T^\alpha)\eta^\nu{}_\beta(T^\beta) = (T^0)^2-[(T^1)^2+(T^2)^2+(T^3)^2]$

$T^\mu = (\text{Temporal}^\mu + \text{Spatial}^\mu) = (\text{FullSpaceTime}^\mu)$
$T^\mu = V^\mu{}_\alpha (T^\alpha) + H^\mu{}_\alpha (T^\alpha)$
$T^\mu = (V^\mu{}_\alpha + H^\mu{}_\alpha)(T^\alpha)$
$T^\mu = [(\eta^\mu{}_\alpha)](T^\alpha) = [(\delta^\mu{}_\alpha)](T^\alpha)$

SR:Minkowski Metric (1+3) Splitting
$\eta^\mu{}_\nu = V^\mu{}_\nu + H^\mu{}_\nu$
(1,1)-Tensor version

$$\begin{matrix} 1 & 0^0{}_j \\ 0^i{}_0 & \delta^i{}_j \end{matrix}$$

Temporal:Spatial Contraction Orthogonality
$(V^\mu{}_\alpha H^\alpha{}_\nu) = (V^\alpha{}_\nu H^\mu{}_\alpha) = 0^\mu{}_\nu$
$(\bar{T}_\alpha H^\alpha{}_\nu)=0_\nu (\bar{T}^\alpha H^\mu{}_\alpha)=0^\mu$

General
4-Vector
4D (1,0)-Tensor
$T^\alpha = (T^0,T^1,T^2,T^3)$
max $4^1 = 4$ components

"(V)ertical" $V^{\mu\nu}$ Temporal
$H^{\mu\nu}$ Spatial "(H)orizontal"
Light-Cone

4-UnitTemporal $\bar{T}\cdot\bar{T}=+1$
$\bar{T}=\bar{T}^\mu=\gamma(1,\beta)=U/c\rightarrow(1,0)_{\{Rest\}}$

Temporal "(V)ertical" Projection (1,1)-Tensor
$P^\mu{}_\nu \rightarrow V^\mu{}_\nu = \|^\mu{}_\nu = \bar{T}^\mu\bar{T}_\nu$
$\rightarrow \text{Diag}[1,0_i]_{\{MCRF\}}$

$$\begin{matrix} & t & x & y & z \\ t & [1 & 0 & 0 & 0] \\ x & [0 & 0 & 0 & 0] \\ y & [0 & 0 & 0 & 0] \\ z & [0 & 0 & 0 & 0] \end{matrix} \qquad \begin{matrix} 1 & 0^0{}_j \\ 0^i{}_0 & 0^i{}_j \end{matrix}$$

4-Tensor
Symmetric,Spatial Isotropic
[Unit Dimensionless]

Spatial "(H)orizontal" Projection (1,1)-Tensor
$P^\mu{}_\nu \rightarrow H^\mu{}_\nu = \perp^\mu{}_\nu = \eta^\mu{}_\nu - \bar{T}^\mu\bar{T}_\nu$
$\rightarrow \text{Diag}[0,\mathbf{I}_{(3)}]=\text{Diag}[0,\delta^i{}_j]_{\{MCRF\}}$

$$\begin{matrix} & t & x & y & z \\ t & [0 & 0 & 0 & 0] \\ x & [0 & 1 & 0 & 0] \\ y & [0 & 0 & 1 & 0] \\ z & [0 & 0 & 0 & 1] \end{matrix} \qquad \begin{matrix} 0 & 0^0{}_j \\ 0^i{}_0 & \delta^i{}_j \end{matrix}$$

4-Tensor
Symmetric,Spatial Isotropic
[Unit Dimensionless]

SR:Minkowski Metric
$\partial[\mathbf{R}] = \partial^\mu R^\nu = \eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu}$
$\rightarrow \text{Diag}[1,-\mathbf{I}_{(3)}]=\text{Diag}[1,-\delta^{ij}]$
{Cartesian/rectangular basis}

$$\begin{matrix} & t & x & y & z \\ t & [1 & 0 & 0 & 0] \\ x & [0 & -1 & 0 & 0] \\ y & [0 & 0 & -1 & 0] \\ z & [0 & 0 & 0 & -1] \end{matrix} \qquad \begin{matrix} 1 & 0^{0j} \\ 0^{i0} & -\delta^{ij} \end{matrix}$$

4-Tensor
Symmetric, Spatial Isotropic
[Unit Dimensionless]

$V^\mu{}_\alpha$ → **Temporal** 4-Vector 4D (1,0)-Tensor
$T^\mu = (T^0,0,0,0) = V^\mu{}_\alpha T^\alpha$
max 1 component

+

$H^\mu{}_\alpha$ → **Spatial** 4-Vector 4D (1,0)-Tensor
$T^\mu = (0,T^1,T^2,T^3) = H^\mu{}_\alpha T^\alpha$
max 3 components

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Note that in this example:
$\bar{T} = \bar{T}^\mu = \gamma(1,\beta)$ is the 4-UnitTemporal
$T^\alpha = (T^0,T^1,T^2,T^3)$ is a general 4-Vector = 4D (1,0)-Tensor T
It can be made into the 4-UnitTemporal via Temporal Projection, followed by a Lorentz Boost

(+,-,-,-) SR → QM Physics

4

A Tensor Study of Physical 4-Vectors

SRQM Study: SR 4-Tensor Properties

General $T^{\mu\nu}$ → Temporal:Mixed:Spatial

(1+3) Decomposition via Projection

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
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Any SR (2-index) Tensor { $T^{\mu\nu}$, $T^{\mu}{}_{\nu}$, $T_{\mu\nu}$ } can be decomposed into = { Temporal , Mixed , Spatial } parts, by using combinations of (V)ertical & (H)orizontal Projection Tensors, with ($\eta^{\mu}{}_{\nu} = V^{\mu}{}_{\nu} + H^{\mu}{}_{\nu}$) and contraction orthogonality ($V^{\mu}{}_{\alpha} H^{\alpha}{}_{\nu}$) = ($V^{\mu}{}_{\alpha} (\eta^{\alpha}{}_{\nu} - V^{\alpha}{}_{\nu})$) = ($V^{\mu}{}_{\alpha}\eta^{\alpha}{}_{\nu} - V^{\mu}{}_{\alpha}V^{\alpha}{}_{\nu}$) = ($V^{\mu}{}_{\nu} - V^{\mu}{}_{\nu}$) = $0^{\mu}{}_{\nu}$

$\text{Temporal}^{\mu\nu} = V^{\mu}{}_{\alpha}V^{\nu}{}_{\beta} (T^{\alpha\beta})$: $\text{Tr}[\text{Temporal}^{\mu\nu}] = \eta_{\mu\nu}V^{\mu}{}_{\alpha}V^{\nu}{}_{\beta}(T^{\alpha\beta}) = V_{\nu\alpha}V^{\nu}{}_{\beta}(T^{\alpha\beta}) = V_{\alpha\beta}(T^{\alpha\beta}) = \{T^{00}\}$

$\text{Mixed}^{\mu\nu} = V^{\mu}{}_{\alpha}H^{\nu}{}_{\beta} (T^{\alpha\beta}) + H^{\mu}{}_{\alpha}V^{\nu}{}_{\beta} (T^{\alpha\beta})$: $\text{Tr}[\text{Mixed}^{\mu\nu}] = \eta_{\mu\nu}(V^{\mu}{}_{\alpha}H^{\nu}{}_{\beta}(T^{\alpha\beta})+H^{\mu}{}_{\alpha}V^{\nu}{}_{\beta}(T^{\alpha\beta})) = (V_{\nu\alpha}H^{\nu}{}_{\beta}+H_{\nu\alpha}V^{\nu}{}_{\beta})(T^{\alpha\beta}) = \{0\}$

$\text{Spatial}^{\mu\nu} = H^{\mu}{}_{\alpha}H^{\nu}{}_{\beta} (T^{\alpha\beta})$: $\text{Tr}[\text{Spatial}^{\mu\nu}] = \eta_{\mu\nu}H^{\mu}{}_{\alpha}H^{\nu}{}_{\beta}(T^{\alpha\beta}) = H_{\nu\alpha}H^{\nu}{}_{\beta}(T^{\alpha\beta}) = H_{\alpha\beta}(T^{\alpha\beta}) = \{T^{11}+T^{22}+T^{33}\}$

$T^{\mu\nu} = (\text{Temporal}^{\mu\nu} + \text{Mixed}^{\mu\nu} + \text{Spatial}^{\mu\nu}) = (\text{FullSpaceTime}^{\mu\nu})$

$T^{\mu\nu} = V^{\mu}{}_{\alpha}V^{\nu}{}_{\beta} (T^{\alpha\beta}) + [V^{\mu}{}_{\alpha}H^{\nu}{}_{\beta} (T^{\alpha\beta}) + H^{\mu}{}_{\alpha}V^{\nu}{}_{\beta} (T^{\alpha\beta})] + H^{\mu}{}_{\alpha}H^{\nu}{}_{\beta} (T^{\alpha\beta})$

$T^{\mu\nu} = [V^{\mu}{}_{\alpha}V^{\nu}{}_{\beta} + V^{\mu}{}_{\alpha}H^{\nu}{}_{\beta} + H^{\mu}{}_{\alpha}V^{\nu}{}_{\beta} + H^{\mu}{}_{\alpha}H^{\nu}{}_{\beta}](T^{\alpha\beta})$

$T^{\mu\nu} = [V^{\mu}{}_{\alpha}(V^{\nu}{}_{\beta} + H^{\nu}{}_{\beta}) + H^{\mu}{}_{\alpha}(V^{\nu}{}_{\beta} + H^{\nu}{}_{\beta})](T^{\alpha\beta})$

$T^{\mu\nu} = [(V^{\mu}{}_{\alpha} + H^{\mu}{}_{\alpha}) (V^{\nu}{}_{\beta} + H^{\nu}{}_{\beta})](T^{\alpha\beta})$

$T^{\mu\nu} = [(\eta^{\mu}{}_{\alpha}) (\eta^{\nu}{}_{\beta})](T^{\alpha\beta}) = [(\delta^{\mu}{}_{\alpha}) (\delta^{\nu}{}_{\beta})](T^{\alpha\beta})$

4-UnitTemporal $\bar{T}\cdot\bar{T}=+1$

$\bar{T}=\bar{T}^{\mu}=\gamma(1,\beta)=U/c\rightarrow(1,0)_{\{Rest\}}$

Temporal "(V)ertical" Projection (1,1)-Tensor

$P^{\mu}{}_{\nu} \rightarrow V^{\mu}{}_{\nu} = \|^{\mu}{}_{\nu} = \bar{T}^{\mu}\bar{T}_{\nu}$

$\rightarrow \text{Diag}[1,0^{i}]_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array} \qquad \begin{array}{cc} 1 & 0^{0}{}_{j} \\ 0^{i}{}_{0} & 0^{i}{}_{j} \end{array}$$

4-Tensor

Symmetric,Spatial Isotropic

[Unit Dimensionless]

Spatial "(H)orizontal" Projection (1,1)-Tensor

$P^{\mu}{}_{\nu} \rightarrow H^{\mu}{}_{\nu} = \perp^{\mu}{}_{\nu} = \eta^{\mu}{}_{\nu} - \bar{T}^{\mu}\bar{T}_{\nu}$

$\rightarrow \text{Diag}[0,\mathbf{I}_{(3)}]=\text{Diag}[0,\delta^{i}{}_{j}]_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & 0 & 0 & 0 \\ x & 0 & 1 & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array} \qquad \begin{array}{cc} 0 & 0^{0}{}_{j} \\ 0^{i}{}_{0} & \delta^{i}{}_{j} \end{array}$$

4-Tensor

Symmetric,Spatial Isotropic

[Unit Dimensionless]

SR:Minkowski Metric (1+3) Splitting

$\eta^{\mu}{}_{\nu} = V^{\mu}{}_{\nu} + H^{\mu}{}_{\nu}$

(1,1)-Tensor version

$$\begin{array}{cc} 1 & 0^{0}{}_{j} \\ 0^{i}{}_{0} & \delta^{i}{}_{j} \end{array}$$

Temporal:Spatial Contraction Orthogonality

($V^{\mu}{}_{\alpha} H^{\alpha}{}_{\nu}$) = ($V^{\alpha}{}_{\nu} H^{\mu}{}_{\alpha}$) = $0^{\mu}{}_{\nu}$

($\bar{T}_{\alpha} H^{\alpha}{}_{\nu}$)=0_{ν} ($\bar{T}^{\alpha} H^{\mu}{}_{\alpha}$)=0^{μ}

SR:Minkowski Metric

$\partial[\mathbf{R}] = \partial^{\mu}R^{\nu} = \eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu}$

$\rightarrow \text{Diag}[1,-\mathbf{I}_{(3)}]=\text{Diag}[1,-\delta^{ij}]$

{Cartesian/rectangular basis}

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & -1 \end{array} \qquad \begin{array}{cc} 1 & 0^{0j} \\ 0^{i0} & -\delta^{ij} \end{array}$$

4-Tensor

Symmetric, Spatial Isotropic

[Unit Dimensionless]

General 4-Tensor

$T^{\alpha\beta}$ =

$[T^{00},T^{01},T^{02},T^{03}]$
$[T^{10},T^{11},T^{12},T^{13}]$
$[T^{20},T^{21},T^{22},T^{23}]$
$[T^{30},T^{31},T^{32},T^{33}]$

max 4^2 = 16 components

"(V)ertical"
$V^{\mu\nu}$ Temporal

$H^{\mu\nu}$ Spatial
"(H)orizontal"

Light-Cone

$V^{\mu}{}_{\alpha}$ $V^{\nu}{}_{\beta}$ + $V^{\mu}{}_{\alpha}$ $H^{\nu}{}_{\beta}$ + $H^{\mu}{}_{\alpha}$ $V^{\nu}{}_{\beta}$ + $H^{\mu}{}_{\alpha}$ $H^{\nu}{}_{\beta}$

Temporal 4-Tensor

$\text{Temporal}^{\mu\nu}$ =

$[T^{00},0,0,0]$
$[0,0,0,0]$
$[0,0,0,0]$
$[0,0,0,0]$

max 1 component

+

Mixed 4-Tensor

$\text{Mixed}^{\mu\nu}$ =

$[0, T^{01},T^{02},T^{03}]$
$[T^{10},0, 0, 0]$
$[T^{20},0, 0, 0]$
$[T^{30},0, 0, 0]$

max 3+3 = 6 components

+

Spatial 4-Tensor

$\text{Spatial}^{\mu\nu}$ =

$[0, 0, 0, 0]$
$[0,T^{11},T^{12},T^{13}]$
$[0,T^{21},T^{22},T^{23}]$
$[0,T^{31},T^{32},T^{33}]$

max 9 components

SR 4-Tensor

(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector

(1,0)-Tensor V^{μ} = **V** = (v^{0},**v**)
SR 4-CoVector:OneForm
(0,1)-Tensor V_{μ} = (v_{0},-**v**)

SR 4-Scalar

(0,0)-Tensor S or S_{o}
Lorentz Scalar

$\text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$

$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^{0})^{2} - \mathbf{v}\cdot\mathbf{v}] = (v^{0}{}_{o})^{2}$

= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4-Vector SRQM Interpretation of QM

SRQM Study: Physical 4-Tensors

Relativisitic Fluid 4-Tensor

A Tensor Study of Physical 4-Vectors

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John B. Wilson
SciRealm@aol.com
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[Unit Dimensionless]

4-UnitTemporal
$\bar{T}=T^\mu=\gamma(1,\beta)$
$=\gamma(1,u/c)=U/c$

Temporal "(V)ertical" Projection (2,0)-Tensor
$P^{\mu\nu} \rightarrow V^{\mu\nu} = \|^{\mu\nu} = \bar{T}^\mu\bar{T}^\nu = U^\mu U^\nu/c^2$
$\rightarrow \mathrm{Diag}[1,0^{ij}]_{\{MCRF\}}$

	t	x	y	z
t	1	0	0	0
x	0	0	0	0
y	0	0	0	0
z	0	0	0	0

$1 \quad 0^{0j}$
$0^{i0} \quad 0^{ij}$

$\mathrm{Tr}[V^{\mu\nu}]=1$

4-Tensor
Symmetric
Spatial Isotropic
[Unit Dimensionless]

4-HeatEnergyFlux
$\mathbf{Q}=Q^\mu=(\rho_e c,\mathbf{q})=\rho_e(c,\mathbf{u})$
$=(\boldsymbol{\beta}\cdot\mathbf{q},\mathbf{q})$ w/ $Q^\mu\bar{T}_\mu=0$
$=E_o\mathbf{N}=n_oE_o\mathbf{U}=c^2\mathbf{G}$

$Q^\mu\bar{T}_\mu=0$

[J/m²·s]
[W/m²]

1/c

ViscousShear Spatial (3D) 4-Tensor $\Pi^{\mu\nu}$
$\rightarrow_{\{MCRF\}}$
w/ $\Pi^{\mu\nu}\bar{T}_\mu = 0$
Symmetric, Anisotropic=Traceless

	t	x	y	z
t	0	0	0	0
x	0	π^{11}	π^{12}	π^{13}
y	0	π^{21}	π^{22}	π^{23}
z	0	π^{31}	π^{32}	π^{33}

[J/m³]
[N/m²]
[Pa]

$\Pi^{\mu\nu}\bar{T}_\mu = 0$

$\mathrm{Tr}[\Pi^{\mu\nu}]=0$

Maybe the Cauchy Stress Tensor?

$Q^\mu = 0$
$\Pi^{\mu\nu} = 0^{\mu\nu}$

No Heat Flux
No Viscosity

Perfect Fluid Stress-Energy(Density)
$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} \rightarrow_{\{MCRF\}}$

[J/m³]
[N/m²]
[Pa]

	t	x	y	z
t	ρ_e	0	0	0
x	0	p	0	0
y	0	0	p	0
z	0	0	0	p

$\rho_e=\rho_m c^2 \quad 0^{0j}$
$0^{i0} \quad p\delta^{ij}$

4-Tensor
Symmetric,
Spatial Isotropic

2
[DoF]

$\mathrm{EoS}[T^{\mu\nu}]=w=p_o/\rho_{eo}$

$\mathrm{Tr}[T^{\mu\nu}]=\rho_{eo}-3p_o$

ρ_{eo}

energy density $_{MCRF}$ [J/m³] [N/m²] [Pa]

negative pressure $_{MCRF}$

$-p_o$

x

+

Full Relativistic Fluid Stress-Energy(Density)
$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} + (\bar{T}^\mu H^\nu{}_\sigma Q^\sigma + Q^\sigma H^\mu{}_\sigma \bar{T}^\nu)/c + \Pi^{\mu\nu}$
$\rightarrow_{\{MCRF\}}$

$\rho_e=\rho_m c^2 \quad Q^{0j}/c$
$Q^{i0}/c \quad p\delta^{ij} + \Pi^{ij}$

(ρ_{eo}) = (Temporal) EnergyDensity 4-Scalar — 1 independent component
(p_o) = (Spatial) Isotropic Pressure 4-Scalar — 1 independent component
$(\bar{T}^\mu)$ = UnitTemporal 4-Vector
(Q^μ) = HeatEnergyFlux 4-Vector w/ $Q^\mu\bar{T}_\mu=0$ — 3 independent components
aka MomentumDensity
$(\Pi^{\mu\nu})$ = ViscousShear 4-Tensor w/ $\Pi^{\mu\nu}\bar{T}_\mu = 0$ — 5 independent components
aka. Anisotropic Stress (traceless $\mathrm{Tr}[\Pi^{\mu\nu}]=0$ and $\Pi^{\mu\nu}=H^\mu{}_\rho H^\nu{}_\sigma \Pi^{\sigma\rho}$)
$(V^{\mu\nu})$ = (Temporal) (V)ertical Projection 4-Tensor
$(H^{\mu\nu})$ = (Spatial) (H)orizontal Projection 4-Tensor

10 independent components
Symmetric 4D-Tensor $T^{\mu\nu}$

4-Tensor
Symmetric

[J/m³]
[N/m²]
[Pa]

10
[DoF]

$\mathrm{Tr}[T^{\mu\nu}]=\rho_{eo}-3p_o$
because ViscousShear is traceless

Spatial "(H)orizontal" Projection (2,0)-Tensor
$P^{\mu\nu} \rightarrow H^{\mu\nu} = \perp^{\mu\nu} = \eta^{\mu\nu} - \bar{T}^\mu\bar{T}^\nu$
$\rightarrow \mathrm{Diag}[0,-I_{(3)}]=\mathrm{Diag}[0,-\delta^{ij}]_{\{MCRF\}}$

	t	x	y	z
t	0	0	0	0
x	0	-1	0	0
y	0	0	-1	0
z	0	0	0	-1

$0 \quad 0^{0j}$
$0^{i0} \quad -\delta^{ij}$

$\mathrm{Tr}[H^{\mu\nu}]=3$

4-Tensor
Symmetric
Spatial Isotropic
[Unit Dimensionless]

Stress-Energy(Density)-Tensor $T^{\mu\nu}$ (symmetric)
Covariant Decomposition

Tensor Invariants: Symmetry, AntiSymmetry, Trace→Isotropy, Anisotropy

$T^{\mu\nu}$ = flux of $\hat{e}_\mu$-component of 4-Momentum along $\hat{e}_\nu$

(temporal:mixed:spatial) splitting
1 Temporal:Temporal EnergyDensity $(\rho_{eo})=V_{\mu\nu}T^{\mu\nu}$
3 Temporal:Spatial HeatEnergy Flux $(Q^\mu)=c\bar{T}_\nu T^{\mu\nu}$
1 Spatial:Spatial Isotropic Pressure $(p_o)=(-1/3)H_{\mu\nu}T^{\mu\nu}$
5 Spatial:Spatial Anisotropic Stress $(\Pi^{\mu\nu})=H^\mu{}_\alpha H^\nu{}_\beta T^{\alpha\beta} + (p_o)H^{\mu\nu}$
10 Total Independent components

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Note that the Projection Tensors & the Minkowski Metric are [unit dimensionless=1]. EnergyDensity (temporal) & Pressure (spatial) have the same dimensional measurement units. [Pa = J/m³ = N/m² = kg/m·s²]

Equation of State
$\mathrm{EoS}[T^{\mu\nu}] = w = p_o/\rho_{eo}$
4-Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4-Vector SRQM Interpretation of QM

SRQM Study: Physical 4-Tensors

Relativisitic Fluid 4-Tensor

A Tensor Study of Physical 4-Vectors

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Stress-Energy(Density)-Tensor $T^{\mu\nu}$ (Symmetric)

Covariant Decomposition

Tensor Invariants: { Symmetry+AntiSymmetry, Trace→Isotropy+Anisotropy, 4D (1+3) Splitting }

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} + (\bar{T}^\mu H^\nu{}_\sigma Q^\sigma + Q^\sigma H^\mu{}_\sigma \bar{T}^\nu)/c + H^\mu{}_\rho H^\nu{}_\sigma \Pi^{\sigma\rho}$

$V_{\mu\nu}T^{\mu\nu} \rightarrow (\rho_{eo})V_{\mu\nu}V^{\mu\nu} + (-p_o)V_{\mu\nu}H^{\mu\nu} + (\bar{T}^\mu V_{\mu\nu}H^\nu{}_\sigma Q^\sigma + Q^\sigma V_{\mu\nu}H^\mu{}_\sigma \bar{T}^\nu)/c + V_{\mu\nu}H^\mu{}_\rho H^\nu{}_\sigma \Pi^{\sigma\rho}$

$V_{\mu\nu}T^{\mu\nu} \rightarrow (\rho_{eo})(1) + (-p_o)(0) + (\bar{T}^\mu(0_{\mu\sigma})Q^\sigma + Q^\sigma(0_{\nu\sigma})\bar{T}^\nu)/c + (0)$

$V_{\mu\nu}T^{\mu\nu} \rightarrow (\rho_{eo})$ EnergyDensity $(\rho_{eo})=V_{\mu\nu}T^{\mu\nu}$

$H_{\mu\nu}T^{\mu\nu} \rightarrow (\rho_{eo})H_{\mu\nu}V^{\mu\nu} + (-p_o)H_{\mu\nu}H^{\mu\nu} + (\bar{T}^\mu H_{\mu\nu}H^\nu{}_\sigma Q^\sigma + Q^\sigma H_{\mu\nu}H^\mu{}_\sigma \bar{T}^\nu)/c + H_{\mu\nu}H^\mu{}_\rho H^\nu{}_\sigma \Pi^{\sigma\rho}$

$H_{\mu\nu}T^{\mu\nu} \rightarrow (\rho_{eo})(0) + (-p_o)(3) + (\bar{T}^\mu H_{\mu\sigma}Q^\sigma + Q^\sigma H_{\nu\sigma}\bar{T}^\nu)/c + (0$ bc traceless$)$

$H_{\mu\nu}T^{\mu\nu} \rightarrow (\rho_{eo})(0) + (-p_o)(3) + [(0_\sigma)Q^\sigma + Q^\sigma(0_\sigma)]/c + (0$ bc traceless$)$

$H_{\mu\nu}T^{\mu\nu} \rightarrow (-p_o)(3)$ (Isotropic) Pressure $(p_o)=(-1/3)H_{\mu\nu}T^{\mu\nu}$

$\bar{T}_\mu T^{\mu\nu} \rightarrow (\rho_{eo})\bar{T}_\mu V^{\mu\nu} + (-p_o)\bar{T}_\mu H^{\mu\nu} + (\bar{T}_\mu \bar{T}^\mu H^\nu{}_\sigma Q^\sigma + \bar{T}_\mu Q^\sigma H^\mu{}_\sigma \bar{T}^\nu)/c + \bar{T}_\mu H^\mu{}_\rho H^\nu{}_\sigma \Pi^{\sigma\rho}$

$\bar{T}_\mu T^{\mu\nu} \rightarrow (\rho_{eo})\bar{T}^\nu + (-p_o)(0^\nu) + [(1)H^\nu{}_\sigma Q^\sigma + Q^\sigma(0_\sigma)\bar{T}^\nu]/c + (0^\nu)$

$\bar{T}_\mu T^{\mu\nu} \rightarrow (\rho_{eo})\bar{T}^\nu + [Q^j + 0^\nu]/c$

$\bar{T}_\mu T^{\mu\nu} \rightarrow (\rho_{eo})\bar{T}^\nu + [Q^j]/c$

$\bar{T}_\mu T^{\mu\nu} \rightarrow (\rho_e,\mathbf{q}/c) = Q^\nu/c$

$c\bar{T}_\mu T^{\mu\nu} \rightarrow c(\rho_e,\mathbf{q}/c) = Q^\nu = (\rho_e/c,\mathbf{q})$: $c\bar{T}_\nu T^{\mu\nu} \rightarrow c(\rho_e,\mathbf{q}/c) =$ HeatFlux $Q^\mu = (\rho_e/c,\mathbf{q})$

$H_\mu{}^\alpha H_\nu{}^\beta T^{\mu\nu} \rightarrow (\rho_{eo})H_\mu{}^\alpha H_\nu{}^\beta V^{\mu\nu} + (-p_o)H_\mu{}^\alpha H_\nu{}^\beta H^{\mu\nu} + (\bar{T}^\mu H_\mu{}^\alpha H_\nu{}^\beta H^\nu{}_\sigma Q^\sigma + Q^\sigma H_\mu{}^\alpha H_\nu{}^\beta H^\mu{}_\sigma \bar{T}^\nu)/c + H_\mu{}^\alpha H_\nu{}^\beta \Pi^{\mu\nu}$

$H_\mu{}^\alpha H_\nu{}^\beta T^{\mu\nu} \rightarrow (\rho_{eo})(0^{\alpha\beta}) + (-p_o)H^{\alpha\beta} + [(0^{\alpha\beta}) + (0^{\alpha\beta})]/c + H^\mu{}_\rho H^\nu{}_\sigma \Pi^{\sigma\rho}$

$H_\mu{}^\alpha H_\nu{}^\beta T^{\mu\nu} \rightarrow (-p_o)H^{\alpha\beta} + \Pi^{\alpha\beta}$

$(p_o) = (-1/3)H_{\alpha\beta}H_\mu{}^\alpha H_\nu{}^\beta T^{\mu\nu}$: (Anisotropic) ViscousShear $\Pi^{\alpha\beta} = H_\mu{}^\alpha H_\nu{}^\beta T^{\mu\nu} + (p_o)H^{\alpha\beta}$

Full Relativistic Fluid Stress-Energy(Density)

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} + (\bar{T}^\mu H^\nu{}_\sigma Q^\sigma + Q^\sigma H^\mu{}_\sigma \bar{T}^\nu)/c + \Pi^{\mu\nu}$

$\rho_e=\rho_m c^2$ Q^{0j}/c

$\rightarrow_{\{MCRF\}}$ Q^{i0}/c $p\delta^{ij} + \Pi^{ij}$

(ρ_{eo}) = (Temporal) EnergyDensity 4-Scalar	1 independent component
(p_o) = (Spatial) Isotropic Pressure 4-Scalar	1 independent component
$(\bar{T}^\mu)$ = UnitTemporal 4-Vector	
(Q^μ) = HeatEnergyFlux 4-Vector w/ $Q^\mu\bar{T}_\mu=0$	3 independent components
aka MomentumDensity	
$(\Pi^{\mu\nu})$ = ViscousShear 4-Tensor w/ $\Pi^{\mu\nu}\bar{T}_\mu = 0$	5 independent components
aka. Anisotropic Stress (traceless $Tr[\Pi^{\mu\nu}]=0$ and $\Pi^{\mu\nu}=H^\mu{}_\rho H^\nu{}_\sigma \Pi^{\sigma\rho}$)	
$(V^{\mu\nu})$ = (Temporal) (V)ertical Projection 4-Tensor	
$(H^{\mu\nu})$ = (Spatial) (H)orizontal Projection 4-Tensor	
	10 independent components
	Symmetric 4D-Tensor $T^{\mu\nu}$

4-Tensor Symmetric [J/m³] [N/m²] [Pa] 10 [DoF]

$Tr[T^{\mu\nu}]=\rho_{eo}-3p_o$ because ViscousShear is traceless

The combo ($p\delta^{ij} + \Pi^{ij}$) appears to be (σ^{ij}), the Cauchy Stress Tensor = True Stress Tensor = Stress Tensor

The ($p\delta^{ij}$) is the mean hydrostatic stress tensor = volumetric stress tensor = "pressure" tensor

The (Π^{ij}) is the stress deviator tensor

[Time·Space] Stress-EnergyDensity Tensor:
(temporal) EnergyDensity = MassDensity * c^2
(mixed) HeatFlux / c
(spatial) Pressure & ViscousShear
all have the same dimensional measurement units:
[Pa = J/m³ = N/m² = kg/m·s²]

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Equation of State
$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$
4-Scalar

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: Physical 4-Tensors
SR Perfect Fluid 4-Tensor
Special-Cases based on "velocity" (β)

4-Vector SRQM Interpretation of QM

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4-ForceDensity [N/m³ = Pa/m]
$\mathbf{F}_{den} = F_{den}{}^{\nu} = -\partial_\mu T^{\mu\nu} = -\partial\cdot T^{\mu\nu}$
{= **0** = 0^ν if conserved}

No Heat Flux $Q^\mu = 0^\mu$
No Viscosity $\Pi^{\mu\nu} = 0^{\mu\nu}$

Perfect Fluid Stress-Energy

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} \rightarrow_{\{MCRF\}}$

based on pressure (p)

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & p & 0 & 0 \\ y & 0 & 0 & p & 0 \\ z & 0 & 0 & 0 & p \end{array}$$

$\rho_e = \rho_m c^2$ 0^{0j}
0^{i0} $p\delta^{ij}$

2 [DoF] [Pa]

4-Tensor
Symmetric,
Spatial Isotropic

$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$

$Tr[T^{\mu\nu}] = \rho_{eo} - 3p_o$

$(p_o) = \beta^2\rho_{eo}/3 = v^2\rho_{mo}/3 = \rho_{mo}RT_o$

(Cold) Matter-Dust

$T^{\mu\nu} \rightarrow P^\mu N^\nu = m_o U^\mu n_o U^\nu = (\rho_{eo})V^{\mu\nu} \rightarrow_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array}$$

$\rho_e = \rho_m c^2$ 0^{0j}
0^{i0} 0^{ij}

[Pa] 1 [DoF]

$EoS[T^{\mu\nu}] = w = 0$

$Tr[T^{\mu\nu}] = \rho_{eo}$

Stress-Energy 4-Tensor
Symmetric, Spatial Isotropic, Pressureless

based on velocity factor (β)

Perfect Fluid Stress-Energy

in terms of velocity (β) instead of pressure (p)

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-\beta^2\rho_{eo}/3)H^{\mu\nu} \rightarrow_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & \beta^2\rho_e/3 & 0 & 0 \\ y & 0 & 0 & \beta^2\rho_e/3 & 0 \\ z & 0 & 0 & 0 & \beta^2\rho_e/3 \end{array}$$

$\rho_e = \rho_m c^2$ 0^{0j}
0^{i0} $(\beta^2\rho_e/3)\delta^{ij}$

$EoS[T^{\mu\nu}] = w = \beta^2/3$

$Tr[T^{\mu\nu}] = (1-\beta^2)\rho_{eo} = (1/\gamma^2)\rho_{eo}$

2 [DoF] [Pa]

4-Tensor
Symmetric,
Spatial Isotropic

Full Relativistic Fluid Stress-Energy(Density)

$T^{\mu\nu} \rightarrow$
$(\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} + (\bar{T}^\mu H^\nu{}_\sigma Q^\sigma + Q^\sigma H^\mu{}_\sigma \bar{T}^\nu)/c + \Pi^{\mu\nu}$
$\rightarrow_{\{MCRF\}}$

[J/m³] [N/m²] [Pa] 10 [DoF]

$\rho_e = \rho_m c^2$ Q^{0j}/c
Q^{i0}/c $p\delta^{ij} + \Pi^{ij}$

$Q^\mu \bar{T}_\mu = 0$

$\Pi^{\mu\nu}\bar{T}_\mu = 0$

4-Tensor
Symmetric
10 independent components

$Tr[T^{\mu\nu}] = \rho_{eo} - 3p_o$
because ViscousShear is traceless

(Warm) Matter-Dust

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-\beta^2\rho_{eo}/3)H^{\mu\nu}$
with {0<=β<=1} = {0<=v<=c}

Stress-Energy 4-Tensor
Symmetric, Spatial Isotropic

(β) = 0

(β) = 1

PhotonGas=RadiationFluid

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-\rho_{eo}/3)H^{\mu\nu} \rightarrow_{\{MCRF?\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & \rho_e/3 & 0 & 0 \\ y & 0 & 0 & \rho_e/3 & 0 \\ z & 0 & 0 & 0 & \rho_e/3 \end{array}$$

$\rho_e = \rho_m c^2$ 0^{0j}
0^{i0} $(\rho_e/3)\delta^{ij}$

1 [DoF]

$EoS[T^{\mu\nu}] = w = 1/3$

$Tr[T^{\mu\nu}] = 0$

Stress-Energy 4-Tensor
[Pa] Symmetric, Spatial Isotropic

[Time·Space] Stress-EnergyDensity Tensor:
(temporal) EnergyDensity = MassDensity * c^2
(mixed) HeatFlux / c
(spatial) Pressure & Viscous Shear
all have the same dimensional measurement units:
[Pa = J/m³ = N/m² = kg/m·s²]

Perfect Gas (|**v**|<c) = Warm Dust
$v = v_{thermal} = \sqrt{[3RT]} = \sqrt{[3k_BT/m_o]} = \{0..c\}$
= characteristic rms 3D thermal speed of molecules
essentially this smoothly varies from
Matter-Dust (v~0) to Radiation-Fluid (v~c)
$v^2 = [3RT] = [3k_BT/m_o]$
$\beta^2 = (v/c)^2 = [3RT/c^2] = [3k_BT/m_oc^2] = [3k_BT/E_o]$
$\beta^2\rho_e/3 = \beta^2 n_o E_o/3 = [3k_BT/E_o]\rho_e/3 = n_o k_B T$
= EnergyDensity based on temperature

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Technically, all these are 4D Stress-EnergyDensity Tensors
That they are usually called "Stress-Energy" Tensors is a
lazy abbreviation which causes confusion

Equation of State
$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$
4-Scalar

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: Physical 4-Tensors

SR Dust 4-Tensor Special-Cases

4-Vector SRQM Interpretation of QM

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(Cold) Matter-Dust

$T^{\mu\nu} \rightarrow P^{\mu}N^{\nu}=m_o U^{\mu} n_o U^{\nu}=(\rho_{eo})V^{\mu\nu} \rightarrow_{\{MCRF\}}$

	t	x	y	z
t	ρ_e	0	0	0
x	0	0	0	0
y	0	0	0	0
z	0	0	0	0

$\rho_e=\rho_m c^2$ 0^{0j}
0^{i0} 0^{ij}

[Pa]

1 [DoF]

EoS[$T^{\mu\nu}$]=w=0

Tr[$T^{\mu\nu}$]=ρ_{eo}

Stress-Energy 4-Tensor

Symmetric, Spatial Isotropic, Pressureless

	t	x	y	z
t	$\gamma^2 n_o m_o c^2$	$\gamma^2 n_o m_o c u^x$	$\gamma^2 n_o m_o c u^y$	$\gamma^2 n_o m_o c u^z$
x	$\gamma^2 n_o m_o c u^x$	$\gamma^2 n_o m_o u^x u^x$	$\gamma^2 n_o m_o u^x u^y$	$\gamma^2 n_o m_o u^x u^z$
y	$\gamma^2 n_o m_o c u^y$	$\gamma^2 n_o m_o u^y u^x$	$\gamma^2 n_o m_o u^y u^y$	$\gamma^2 n_o m_o u^y u^x$
z	$\gamma^2 n_o m_o c u^z$	$\gamma^2 n_o m_o u^z u^x$	$\gamma^2 n_o m_o u^z u^y$	$\gamma^2 n_o m_o u^z u^z$

In a frame with 4-Velocity **U**

$$\begin{matrix} \gamma^2 n_o m_o c^2 & \gamma^2 n_o m_o c u^j \\ \gamma^2 n_o m_o c u^i & \gamma^2 n_o m_o u^i u^j \end{matrix}$$

$$\begin{matrix} \gamma^2 n_o m_o c^2 & \gamma^2 n_o m_o c\, dx^j/dt \\ \gamma^2 n_o m_o c\, dx^i/dt & \gamma^2 n_o m_o\, dx^i/dt\, dx^j/dt \end{matrix}$$

$$\begin{matrix} \gamma n_o E & \gamma n_o c p^j \\ \gamma n_o c p^i & \gamma n_o m u^i u^j \end{matrix}$$

$$\begin{matrix} \gamma^2 \rho_{mo} c^2 & \gamma^2 \rho_{mo} c u^j \\ \gamma^2 \rho_{mo} c u^i & \gamma^2 \rho_{mo} u^i u^j \end{matrix}$$

$$\begin{matrix} nE & ncp^j \\ ncp^i & nmu^i u^j \end{matrix}$$

$$\begin{matrix} \gamma^2 \rho_{eo} & \gamma^2 \rho_{eo} u^j/c \\ \gamma^2 \rho_{eo} u^i/c & \gamma^2 \rho_{eo} u^i u^j/c^2 \end{matrix}$$

$$\begin{matrix} \gamma^2 \rho_{eo} & \gamma^2 \rho_{eo} \beta^j \\ \gamma^2 \rho_{eo} \beta^i & \gamma^2 \rho_{eo} \beta^i \beta^j \end{matrix}$$

[Time·Space] Stress-EnergyDensity Tensor:
(temporal) EnergyDensity = MassDensity * c^2
(mixed) HeatFlux / c
(spatial) Pressure & Viscous Shear
all have the same dimensional measurement units:
[Pa = J/m³ = N/m² = kg/m·s²]

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor V^{μ} = **V** = (v^0,**v**)
SR 4-CoVector:OneForm
(0,1)-Tensor V_{μ} = (v_0,-**v**)

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Technically, all these are 4D Stress-EnergyDensity Tensors That they are usually called "Stress-Energy" Tensors is a lazy abbreviation which causes confusion

Equation of State
EoS[$T^{\mu\nu}$] = w = p_o/ρ_{eo}
4-Scalar

Trace[$T^{\mu\nu}$] = $\eta_{\mu\nu}T^{\mu\nu}$ = $T^{\mu}{}_{\mu}$ = T
V·V = $V^{\mu}\eta_{\mu\nu}V^{\nu}$ = [(v^0)² - **v·v**] = ($v^0{}_o$)²
= Lorentz Scalar Invariant

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SRQM Study: Physical 4-Tensors
SR Perfect Fluid 4-Tensor Special-Cases

4-Vector SRQM Interpretation of QM

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http://scirealm.org/SRQM.pdf

Perfect Fluid Stress-Energy

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} \rightarrow_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & p & 0 & 0 \\ y & 0 & 0 & p & 0 \\ z & 0 & 0 & 0 & p \end{array}$$

$\rho_e = \rho_m c^2$ $\quad 0^{0j}$

0^{i0} $\quad p\delta^{ij}$

4-Tensor
Symmetric,
Spatial Isotropic

2 [DoF]

[Pa]

$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$

$Tr[T^{\mu\nu}] = \rho_{eo} - 3p_o$

In a frame with 4-Velocity **U**

$$\begin{matrix} \gamma^2(\rho_{eo}+p_o)-p_o\eta^{00} & \gamma^2(\rho_{eo}+p_o)\beta^j-p_o\eta^{0j} \\ \gamma^2(\rho_{eo}+p_o)\beta^i-p_o\eta^{i0} & \gamma^2(\rho_{eo}+p_o)\beta^i\beta^j-p_o\eta^{ij} \end{matrix}$$

$$\begin{matrix} \gamma^2(\rho_{eo}+p_o)-p_o & \gamma^2(\rho_{eo}+p_o)\beta^j \\ \gamma^2(\rho_{eo}+p_o)\beta^i & \gamma^2(\rho_{eo}+p_o)\beta^i\beta^j-p_o\eta^{ij} \end{matrix}$$

$$\begin{matrix} \gamma^2(\rho_{eo}+p_o)-p_o & \gamma^2(\rho_{eo}+p_o)\beta^j \\ \gamma^2(\rho_{eo}+p_o)\beta^i & \gamma^2(\rho_{eo}+p_o)\beta^i\beta^j+p_o\delta^{ij} \end{matrix}$$

$$\begin{matrix} \gamma^2(w_o)-p_o & \gamma^2(w_o)\beta^j \\ \gamma^2(w_o)\beta^i & \gamma^2(w_o)\beta^i\beta^j+p_o\delta^{ij} \end{matrix}$$

$T_{perfluid}^{\mu\nu} = (\rho_{eo}+p_o)\overline{T}^{\mu}\overline{T}^{\nu} - (p_o)\eta^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu}$

[Time·Space] Stress-EnergyDensity Tensor:
(temporal) EnergyDensity = MassDensity * c^2
(mixed) HeatFlux / c
(spatial) Pressure = Momentum Flux Density & Viscous Shear
all have the same dimensional measurement units:
[Pa = J/m³ = N/m² = kg/m·s²]

Rest Enthalpy $w_o = \rho_{eo}+p_o$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Technically, all these are 4D Stress-EnergyDensity Tensors
That they are usually called "Stress-Energy" Tensors is a lazy abbreviation which causes confusion

Equation of State
$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$
4-Scalar

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: Physical 4-Tensors
SR Fluid:Dust:Vacuum 4-Tensors
Also known as GR "Solutions"

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

4-ForceDensity [N/m³ = Pa/m]
$F_{den} = F_{den}^{\nu} = -\partial_\mu T^{\mu\nu} = -\partial\cdot T^{\mu\nu}$
{= 0 = 0^ν if conserved}

Maxwell 4D EM Stress-Energy Tensor

$T^{\mu\nu} \rightarrow -(1/\mu_o)[F^{\mu\alpha}F^{\nu}{}_{\alpha} - (1/4)\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}]$

$\rightarrow$ {No Rest Frame,LightLike,Null}

$$\begin{matrix} \tfrac{1}{2}(\varepsilon_o e^2 + b^2/\mu_o) & s^{0j}/c \\ s^{i0}/c & -\sigma^{ij} \end{matrix}$$

[Pa] 6 [DoF]

4-Tensor { formula in (+,-,-,-) }
Symmetric

$Tr[T^{\mu\nu}]=0$

EM field

No Heat Flux
No Viscosity

$Q^\mu = 0^\mu$
$\Pi^{\mu\nu} = 0^{\mu\nu}$

Full Relativistic Fluid Stress-Energy(Density)

$T^{\mu\nu} \rightarrow$
$(\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} + (\bar{T}^\mu H^\nu{}_\sigma Q^\sigma + Q^\sigma H^\mu{}_\sigma \bar{T}^\nu)/c + \Pi^{\mu\nu}$

$\rightarrow$ {MCRF}

[J/m³] [N/m²] [Pa] 10 [DoF]

$$\begin{matrix} \rho_e = \rho_m c^2 & Q^{0j}/c \\ Q^{i0}/c & p\delta^{ij} + \Pi^{ij} \end{matrix}$$

$Q^\mu \bar{T}_\mu = 0$

$\Pi^{\mu\nu}\bar{T}_\mu = 0$

4-Tensor
Symmetric
10 independent components

$Tr[T^{\mu\nu}] = \rho_{eo} - 3p_o$
because ViscousShear is traceless

$(\rho_{eo}) = \Phi_o$
$(p_o) = \Phi_o/3$
$(Q^\mu) = \Phi_o \hat{N}^\mu$
$(\Pi^{\mu\nu}) = \Phi_o * Diag[0, -1/3, -1/3, +2/3]$

Perfect Fluid Stress-Energy

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} \rightarrow$ {MCRF}

$$\begin{matrix} & t & x & y & z \\ t & [\rho_e & 0 & 0 & 0] \\ x & [0 & p & 0 & 0] \\ y & [0 & 0 & p & 0] \\ z & [0 & 0 & 0 & p] \end{matrix}$$

$$\begin{matrix} \rho_e = \rho_m c^2 & 0^{0j} \\ 0^{i0} & p\delta^{ij} \end{matrix}$$

2 [DoF]

4-Tensor
Symmetric,
[Pa] Spatial Isotropic

$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$

$Tr[T^{\mu\nu}] = \rho_{eo} - 3p_o$

$(p_o) = 0$

$(p_o) = -\rho_{eo}$

$(p_o) = (\rho_{eo}) = 0$

$(p_o) = \rho_{eo}/3$

(Cold) Matter-Dust

$T^{\mu\nu} \rightarrow P^\mu N^\nu = m_o U^\mu n_o U^\nu = (\rho_{eo})V^{\mu\nu} \rightarrow$ {MCRF}

$$\begin{matrix} & t & x & y & z \\ t & [\rho_e & 0 & 0 & 0] \\ x & [0 & 0 & 0 & 0] \\ y & [0 & 0 & 0 & 0] \\ z & [0 & 0 & 0 & 0] \end{matrix}$$

$$\begin{matrix} \rho_e = \rho_m c^2 & 0^{0j} \\ 0^{i0} & 0^{ij} \end{matrix}$$

[Pa] 1 [DoF]

$EoS[T^{\mu\nu}] = w = 0$

$Tr[T^{\mu\nu}] = \rho_{eo}$

Stress-Energy 4-Tensor
Symmetric, Spatial Isotropic, Pressureless

Null-Dust : "Gravitational Wave" Incoherent EM Mixture

$T^{\mu\nu} \rightarrow \Phi_o N^\mu N^\nu = \Phi_o N^{\mu\nu} \rightarrow$ {MCRF_z-dir_no rest frame}

$$\begin{matrix} & t & x & y & z \\ t & [\Phi & 0 & 0 & \pm\Phi] \\ x & [0 & 0 & 0 & 0] \\ y & [0 & 0 & 0 & 0] \\ z & [\pm\Phi & 0 & 0 & \Phi] \end{matrix}$$

$$\begin{matrix} \Phi^{00} & \Phi^{0j} \\ \Phi^{i0} & \Phi^{ij} \end{matrix}$$

$EoS[T^{\mu\nu}] = w = 1/3$

$Tr[T^{\mu\nu}] = 0$

Stress-Energy 4-Tensor
[Pa] 3-4? [DoF] Symmetric, Null

Lambda Vacuum

$T^{\mu\nu} \rightarrow (\rho_{eo})\eta^{\mu\nu} = (\Lambda)\eta^{\mu\nu} \rightarrow$ {MCRF}

Dark Energy?

$$\begin{matrix} & t & x & y & z \\ t & [\rho_e & 0 & 0 & 0] \\ x & [0 & -\rho_e & 0 & 0] \\ y & [0 & 0 & -\rho_e & 0] \\ z & [0 & 0 & 0 & -\rho_e] \end{matrix}$$

$$\begin{matrix} \rho_e = \rho_m c^2 & 0^{0j} \\ 0^{i0} & -\rho_e\delta^{ij} \end{matrix}$$

1 [DoF]

$EoS[T^{\mu\nu}] = w = -1$

$Tr[T^{\mu\nu}] = 4\rho_{eo}$

Stress-Energy 4-Tensor
[Pa] Symmetric, Spatial Isotropic

PhotonGas=RadiationFluid

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-\rho_{eo}/3)H^{\mu\nu} \rightarrow$ {MCRF?}

$$\begin{matrix} & t & x & y & z \\ t & [\rho_e & 0 & 0 & 0] \\ x & [0 & \rho_e/3 & 0 & 0] \\ y & [0 & 0 & \rho_e/3 & 0] \\ z & [0 & 0 & 0 & \rho_e/3] \end{matrix}$$

$$\begin{matrix} \rho_e = \rho_m c^2 & 0^{0j} \\ 0^{i0} & (\rho_e/3)\delta^{ij} \end{matrix}$$

$EoS[T^{\mu\nu}] = w = 1/3$

$Tr[T^{\mu\nu}] = 0$

1 [DoF] **Stress-Energy 4-Tensor**
[Pa] Symmetric, Spatial Isotropic

Zero:Nothing Vacuum

$T^{\mu\nu} \rightarrow 0^{\mu\nu} \rightarrow$ {MCRF}

$$\begin{matrix} & t & x & y & z \\ t & [0 & 0 & 0 & 0] \\ x & [0 & 0 & 0 & 0] \\ y & [0 & 0 & 0 & 0] \\ z & [0 & 0 & 0 & 0] \end{matrix}$$

$$\begin{matrix} 0 & 0^{0j} \\ 0^{i0} & 0^{ij} \end{matrix}$$

$EoS[T^{\mu\nu}] = w$ =undefined

$Tr[T^{\mu\nu}] = 0$

0 [DoF] **Stress-Energy 4-Tensor**
[Pa] Symmetric, Isotropic

[Time·Space] Stress-EnergyDensity Tensor:
(temporal) EnergyDensity = MassDensity * c^2
(mixed) HeatFlux / c
(spatial) Pressure & Viscous Shear
all have the same dimensional measurement units:
[Pa = J/m³ = N/m² = kg/m·s²]

see on Wikipedia:
Fluid Solution
Perfect Fluid
Dust Solution
Null Dust Solution
Vacuum Solution
Electrovacuum Solution
Lambdavacuum Solution
Scalar Field Solution
etc.

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Technically, all these are 4D Stress-EnergyDensity Tensors
That they are usually called "Stress-Energy" Tensors is a
lazy abbreviation which causes confusion

Equation of State
$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$
4-Scalar

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4-Vector SRQM Interpretation of QM

SRQM Study: Physical 4-Tensors

Some SR 4-Tensors and Symbols

A Tensor Study of Physical 4-Vectors

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http://scirealm.org/SRQM.pdf

4-UnitTemporal
$\bar{T}=T^{\mu}=\gamma(1,\beta)$
$=\gamma(1,u/c)=U/c$

[Unit]
[Dimensionless]

4-ForceDensity [N/m³ = Pa/m]
$F_{den} = F_{den}{}^{\nu} = -\partial_{\mu}T^{\mu\nu} = -\partial\cdot T^{\mu\nu}$
{=0ᵛ if conserved}

energy density MCRF ρ_{eo}

negative pressure MCRF $-p_o$

Temporal "(V)ertical" Projection (2,0)-Tensor

$P^{\mu\nu} \rightarrow V^{\mu\nu} = \|^{\mu\nu} = \bar{T}^{\mu}\bar{T}^{\nu} = U^{\mu}U^{\nu}/c^2$
$\rightarrow \mathrm{Diag}[1,0^{ij}]_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array} \qquad \begin{matrix} 1 & 0^{0j} \\ 0^{i0} & 0^{ij} \end{matrix}$$

4-Tensor
Symmetric
Spatial Isotropic
[Unit Dimensionless]

$Tr[V^{\mu\nu}]=1$

SR:Minkowski Metric

$\partial[R] = \partial^{\mu}R^{\nu} = \eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu}$
$\rightarrow \mathrm{Diag}[1,-I_{(3)}]=\mathrm{Diag}[1,-\delta^{ij}]$
{Cartesian/rectangular basis}

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & -1 \end{array} \qquad \begin{matrix} 1 & 0^{0j} \\ 0^{i0} & -\delta^{ij} \end{matrix}$$

4-Tensor
Symmetric, Spatial Isotropic
[Unit Dimensionless]

$EoS[\eta^{\mu\nu}]=w=-1$

$Tr[\eta^{\mu\nu}]=4$

Spatial "(H)orizontal" Projection (2,0)-Tensor

$P^{\mu\nu} \rightarrow H^{\mu\nu} = \perp^{\mu\nu} = \eta^{\mu\nu} - \bar{T}^{\mu}\bar{T}^{\nu}$
$\rightarrow \mathrm{Diag}[0,-I_{(3)}]=\mathrm{Diag}[0,-\delta^{ij}]_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & -1 \end{array} \qquad \begin{matrix} 0 & 0^{0j} \\ 0^{i0} & -\delta^{ij} \end{matrix}$$

4-Tensor
Symmetric
Spatial Isotropic
[Unit Dimensionless]

$Tr[H^{\mu\nu}]=3$

Perfect Fluid Stress-Energy

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} \rightarrow_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & p & 0 & 0 \\ y & 0 & 0 & p & 0 \\ z & 0 & 0 & 0 & p \end{array} \qquad \begin{matrix} \rho_e=\rho_m c^2 & 0^{0j} \\ 0^{i0} & p\delta^{ij} \end{matrix}$$

[J/m³] [N/m²] [Pa] 4-Tensor
Symmetric,
Spatial Isotropic

$EoS[T^{\mu\nu}]=w=p_o/\rho_{eo}$

$Tr[T^{\mu\nu}]=\rho_{eo}-3p_o$

$(p_o) = 0$

$(p_o) = -\rho_{eo}$

$(p_o) = \rho_{eo}/3$

$(p_o) = (\rho_{eo}) = 0$

(Cold) Matter-Dust

$T^{\mu\nu} \rightarrow P^{\mu}N^{\nu}=m_oU^{\mu}n_oU^{\nu}=(\rho_{eo})V^{\mu\nu} \rightarrow_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array} \qquad \begin{matrix} \rho_e=\rho_m c^2 & 0^{0j} \\ 0^{i0} & 0^{ij} \end{matrix}$$

[Pa]

$EoS[T^{\mu\nu}]=w=0$

$Tr[T^{\mu\nu}]=\rho_{eo}$

Stress-Energy 4-Tensor
Symmetric, Spatial Isotropic, Pressureless

PhotonGas=RadiationFluid

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-\rho_{eo}/3)H^{\mu\nu} \rightarrow_{\{MCRF?\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & \rho_e/3 & 0 & 0 \\ y & 0 & 0 & \rho_e/3 & 0 \\ z & 0 & 0 & 0 & \rho_e/3 \end{array} \qquad \begin{matrix} \rho_e=\rho_m c^2 & 0^{0j} \\ 0^{i0} & (\rho_e/3)\delta^{ij} \end{matrix}$$

$EoS[T^{\mu\nu}]=w=1/3$

$Tr[T^{\mu\nu}]=0$

Stress-Energy 4-Tensor
[Pa] Symmetric, Spatial Isotropic

Lambda Vacuum

$T^{\mu\nu} \rightarrow (\rho_{eo})\eta^{\mu\nu} = (\Lambda)\eta^{\mu\nu} \rightarrow_{\{MCRF\}}$

Dark Energy?

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & -\rho_e & 0 & 0 \\ y & 0 & 0 & -\rho_e & 0 \\ z & 0 & 0 & 0 & -\rho_e \end{array} \qquad \begin{matrix} \rho_e=\rho_m c^2 & 0^{0j} \\ 0^{i0} & -\rho_e\delta^{ij} \end{matrix}$$

$EoS[T^{\mu\nu}]=w=-1$

$Tr[T^{\mu\nu}]=4\rho_{eo}$

Stress-Energy 4-Tens
[Pa] Symmetric, Spatial Isotropic

Faraday EM Tensor

$F^{\alpha\beta} = \partial^{\alpha}A^{\beta} - \partial^{\beta}A^{\alpha} = \partial \wedge A$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & -e^x/c & -e^y/c & -e^z/c \\ x & +e^x/c & 0 & -b^z & +b^y \\ y & +e^y/c & +b^z & 0 & -b^x \\ z & +e^z/c & -b^y & +b^x & 0 \end{array} \qquad \begin{matrix} 0 & -e^{0j}/c \\ +e^{i0}/c & -\varepsilon^{ij}{}_k b^k \end{matrix} \qquad \begin{matrix} 0 & -e/c \\ +e^T/c & -\nabla\wedge a \end{matrix}$$

[T]

4-Tensor
Anti-symmetric

$Tr[F^{\mu\nu}]=0$

Maxwell 4D EM Stress-Energy Tensor

$T^{\mu\nu} \rightarrow -(1/\mu_o)[F^{\mu\alpha}F^{\nu}{}_{\alpha}-(1/4)\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}] \rightarrow_{\{No\ Rest\ Frame,LightLike,Null\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \frac{1}{2}(\varepsilon_o e^2+b^2/\mu_o) & s^x/c & s^y/c & s^z/c \\ x & s^x/c & -\sigma^{xx} & -\sigma^{xy} & -\sigma^{xz} \\ y & s^y/c & -\sigma^{yx} & -\sigma^{yy} & -\sigma^{yz} \\ z & s^z/c & -\sigma^{zx} & -\sigma^{zy} & -\sigma^{zz} \end{array} \qquad \begin{matrix} \frac{1}{2}(\varepsilon_o e^2+b^2/\mu_o) & s^{0j}/c \\ s^{i0}/c & -\sigma^{ij} \end{matrix}$$

[Pa]

4-Tensor { formula in (+,-,-,-) }
Symmetric

$Tr[T^{\mu\nu}]=0$

Zero:Nothing Vacuum

$T^{\mu\nu} \rightarrow 0^{\mu\nu} \rightarrow_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array} \qquad \begin{matrix} 0 & 0^{0j} \\ 0^{i0} & 0^{ij} \end{matrix}$$

$EoS[T^{\mu\nu}]=w$
=undefined

$Tr[T^{\mu\nu}]=0$

Stress-Energy 4-Te
[Pa] Symmetric, Isotropic

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Note that the Projection Tensors & the Minkowski Metric are [unit dimensionless=1]. EnergyDensity (temporal) & Pressure (spatial) have the same dimensional measurement units. [Pa = J/m³ = N/m² = kg/m·s²]

Equation of State
$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$
4-Scalar

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4-Vector SRQM Interpretation of QM

SRQM Study: Physical 4-Tensors

Metric Sign-Convention:Signature

A Tensor Study of Physical 4-Vectors

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Ultimately, the Sign-Convention or Metric Signature is based on the definition of the 4-Gradient ∂^μ since it is used to create the Minkowski Metric $\eta^{\mu\nu} = \partial^\mu[R^\nu]$:
I prefer the (+,-,-,-) Metric Signature:SignConvention, and use it in this study
= {temporal,0th,+},Timelike+,ParticlePhysics,WestCoast convention,Mostly-minuses

[1/m] **4-Gradient**

$\partial = \partial_R = \partial_x = \partial^\mu = \partial/\partial R_\mu$

$=(-\partial_t/c,\nabla)$
$\rightarrow(-\partial_t/c,\partial_x,\partial_y,\partial_z)$
$=(-\partial/c\partial t,\partial/\partial x,\partial/\partial y,\partial/\partial z)$

for (-,+,+,+) Spacelike+,Relativity, EastCoast convention

preferred in this treatise
$=(\partial_t/c,-\nabla)$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

for (+,-,-,-) Timelike+,ParticlePhys, WestCoast convention

[m] **4-Position**

$R=R^\mu=(ct,r)\in$<Event>
$\rightarrow(ct,x,y,z)$

SR:Minkowski Metric

$\partial[R] = \partial^\mu[R^\nu] = \eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu}$

$\rightarrow$Diag[-1,+$I_{(3)}$]=Diag[-1,+δ^{ij}]
{Cartesian/rectangular basis}

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & -1 & 0 & 0 & 0 \\ x & 0 & +1 & 0 & 0 \\ y & 0 & 0 & +1 & 0 \\ z & 0 & 0 & 0 & +1 \end{array} \qquad \begin{matrix} -1 & 0^{0j} \\ 0^{i0} & +\delta^{ij} \end{matrix}$$

for (-,+,+,+) Spacelike+,Relativity, EastCoast convention

$\rightarrow$Diag[+1,-$I_{(3)}$]=Diag[+1,-δ^{ij}]
{Cartesian/rectangular basis}

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & +1 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & -1 \end{array} \qquad \begin{matrix} +1 & 0^{0j} \\ 0^{i0} & -\delta^{ij} \end{matrix}$$

for (+,-,-,-) Timelike+,ParticlePhys, WestCoast convention

4-Tensor [Unit Dimensionless]
Symmetric, Spatial Isotropic

Perfect Fluid 4D Stress-Energy Tensor (technically an EnergyDensity)

$T^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu}$
$T^{\mu\nu} = (\rho_{mo}+p_o/c^2)U^\mu U^\nu + \{\{sc\}\}\ p_o\eta^{\mu\nu}$

$T^{\mu\nu} = (\rho_{mo}+p_o/c^2)U^\mu U^\nu + p_o\eta^{\mu\nu}$ for (-,+,+,+) Spacelike+,Relativity,EastCoast convention

$T^{\mu\nu} = (\rho_{mo}+p_o/c^2)U^\mu U^\nu - p_o\eta^{\mu\nu}$ for (+,-,-,-) Timelike+,ParticlePhys,WestCoast convention

$Tr[T^{\mu\nu}] = (\rho_e) - (p) - (p) - (p) =$
$\eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = \rho_{eo} - 3p_o$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & p & 0 & 0 \\ y & 0 & 0 & p & 0 \\ z & 0 & 0 & 0 & p \end{array} \qquad \begin{matrix} \rho_e=\rho_m c^2 & 0^{0j} \\ 0^{i0} & p\delta^{ij} \end{matrix}$$

[J/m³] [N/m²] [Pa] **4-Tensor**
Symmetric, Spatial Isotropic

Maxwell 4D EM Stress-Energy Tensor (technically an EnergyDensity)

$T^{\mu\nu} = \{\{sc\}\}\ (1/\mu_o)[F^{\mu\alpha}F^\nu{}_\alpha-(1/4)\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}]$

$T^{\mu\nu} = +(1/\mu_o)[F^{\mu\alpha}F^\nu{}_\alpha-(1/4)\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}]$ for (-,+,+,+) Spacelike+,Relativity,EastCoast conv.

$T^{\mu\nu} = -(1/\mu_o)[F^{\mu\alpha}F^\nu{}_\alpha-(1/4)\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}]$ for (+,-,-,-) Timelike+,ParticlePhys,WestCoast convention

$Tr[T^{\mu\nu}] = 0$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \frac{1}{2}(\varepsilon_o e^2+b^2/\mu_o) & s^x/c & s^y/c & s^z/c \\ x & s^x/c & -\sigma^{xx} & -\sigma^{xy} & -\sigma^{xz} \\ y & s^y/c & -\sigma^{yx} & -\sigma^{yy} & -\sigma^{yz} \\ z & s^z/c & -\sigma^{zx} & -\sigma^{zy} & -\sigma^{zz} \end{array} \qquad \begin{matrix} \frac{1}{2}(\varepsilon_o e^2+b^2/\mu_o) & s^{0j}/c \\ s^{i0}/c & -\sigma^{ij} \end{matrix}$$

[J/m³] [N/m²] [Pa] **4-Tensor**
Symmetric

with $\sigma^{ij} = \varepsilon_o e^i e^j + b^i b^j/\mu_o - \frac{1}{2}(\varepsilon_o e^2+b^2/\mu_o)\delta^{ij}$
= the 3D Maxwell Stress Tensor
& $\mathbf{s} = (1/\mu_o)\mathbf{e}\times\mathbf{b}$ = PoyntingVector

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Stress-Energy Tensors are technically EnergyDensities, not Energies: EnergyDensity (temporal) & Pressure:Stress (spatial) have the same dimensional measurement units. [Pa = J/m³ = N/m² = kg/m·s²]

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Special Relativity → Quantum Mechanics
RoadMap of SR→QM

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

4-Gradient=***Alteration*** of SR <Events>
SR SpaceTime "Flat" Minkowski 4D Metric
SR SpaceTime Dimension=4
SR Lorentz Transforms
SR Action → 4-Momentum
SR Phase → 4-WaveVector
SR ProperTime Derivative
SR & QM Invariant Waves

START HERE: 4-Position=***Location*** of SR <Events> in SpaceTime
follow the arrows

$\partial^\mu[R^\nu]=\eta^{\mu\nu}$ Minkowski Metric

$\partial_\nu[R^{\mu'}]=\Lambda^{\mu'}{}_\nu$ Lorentz Transform

$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4$

$Tr[\eta^{\mu\nu}]=4$

SpaceTime $\partial\cdot R=\partial_\mu R^\mu=4$ Dimension

4-Position R^μ
$R=(ct,r)\in$<Event>

$U\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$
ProperTime Derivative

4-Velocity=***Motion*** of SR <Events> in SpaceTime as both particles & waves

$R\cdot R=(ct)^2-r\cdot r$
$=(c\tau)^2=(i|r_o|)^2$

$R\cdot U=c^2\tau$

$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$
$=-(m_oc/\hbar)^2=-(\omega_o/c)^2$
$=(\partial_\tau/c)^2$

4-Gradient ∂^μ
$\partial=\partial/\partial R_\mu$
$=(\partial_t/c,-\nabla)=-iK$

ProperTime
$U\cdot\partial=d/d\tau=\gamma d/dt$
Derivative

4-Velocity U^μ
$U=\gamma(c,u)=dR/d\tau$

Matter Wave
$v_{group}*v_{phase}=c^2$
Rest Angular Frequency ω_o

$-K\cdot R=\Phi_{phase,free}$
SR Phase

$-P\cdot R=S_{action,free}$
SR Action

Phase & Action Lorentz Scalars

$U\cdot U=\gamma^2(c^2-u\cdot u)$
$=(c)^2$

$K\cdot U=\omega_o$

ω_o/c^2

SR d'Alembertian & Klein-Gordon Relativistic Quantum Wave Relation
Schrödinger QWE is {|v|<<c} limit of KG QWE
[SR → QM]

$-\partial[\Phi_{phase,free}]=K$

$-\partial[S_{action,free}]=P$

4-WaveVector Complex Plane-Waves
$K_T=-\partial[\Phi]$
$K=i\partial$

Hamilton-Jacobi
$P_T=-\partial[S]$

(-i)

Einstein
$E=mc^2=\gamma m_oc^2=\gamma E_o$
Rest Mass m_o : Rest Energy E_o

$P\cdot U=E_o$

m_o
E_o/c^2

4-WaveVector=***Substantiation*** of SR Wave <Events>
oscillations proportional to mass:energy & 3-momentum

4-WaveVector K^μ
$K=(\omega/c,k)=(\omega/c,\omega\hat{n}/v_{phase})$
$=(1/c\bar{\lambda},\hat{n}/\bar{\lambda})=(\omega_o/c^2)U=P/\hbar$

4-Momentum P^μ
$P=(mc,p)=(E/c,p)=m_oU$

4-Momentum=***Substantiation*** of SR Particle <Events>
mass:energy & 3-momentum

Einstein, de Broglie Relation
Dirac:Planck Constant $\hbar=h/2\pi$
$P=\hbar K$

$P\cdot K=m_o\omega_o$

$K\cdot K=(\omega/c)^2-k\cdot k$
$=(m_oc/\hbar)^2=(\omega_o/c)^2=(1/c\bar{\lambda}_o)^2$

$P\cdot P=(E/c)^2-p\cdot p$
$=(m_oc)^2=(E_o/c)^2$

ω_o/E_o
($1/\hbar$)

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
(QM Principles)

$Trace[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Chart:
Special Relativity → Quantum Mechanics
SR→QM Interpretation Simplified

4-Vector SRQM Interpretation of QM

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SRQM: The [SR→QM] Interpretation of Quantum Mechanics

Special Relativity (SR) Axioms: Invariant Interval + LightSpeed (c) as Universal Physical Constant lead to SR, although technically SR is itself the Minkowski-SpaceTime low-curvature:"flat" limiting-case of GR.

{ $c,\tau,m_o,\hbar,i$ } = {c: SpeedOfLight, τ: ProperTime, m_o: RestMass, $\hbar$: Dirac/PlanckReducedConstant($\hbar=h/2\pi$), i: ImaginaryNumber}: are all Empirically-Measured SR Lorentz-Invariant Physical and/or Mathematical Constants $i = +\sqrt{[-1]} = (0,1)_{complex\#}$

Standard SR 4-Vectors:		*Related by these SR Lorentz Invariants:*	
4-Position	$\mathbf{R} = (ct,\mathbf{r})$	$\in$ <Event> $\in$ <Time·Space>	$(\mathbf{R}\cdot\mathbf{R}) = (c\tau)^2 = (i\lvert\mathbf{r_o}\rvert)^2$
4-Velocity	$\mathbf{U} = \gamma(c,\mathbf{u})$	$= (\mathbf{U}\cdot\partial)\mathbf{R} = (d/d\tau)\mathbf{R} = d\mathbf{R}/d\tau$	$(\mathbf{U}\cdot\mathbf{U}) = (c)^2$
4-Momentum	$\mathbf{P} = (E/c,\mathbf{p})$	$= m_o\mathbf{U}$	$(\mathbf{P}\cdot\mathbf{P}) = (m_oc)^2$
4-WaveVector	$\mathbf{K} = (\omega/c,\mathbf{k})$	$= \mathbf{P}/\hbar$	$(\mathbf{K}\cdot\mathbf{K}) = (m_oc/\hbar)^2$
4-Gradient	$\partial = (\partial_t/c,-\nabla)$	$= -i\mathbf{K}$	$(\partial\cdot\partial) = (-im_oc/\hbar)^2 = -(m_oc/\hbar)^2$ = QM Relation → RQM → QM

KG Equation: $|v|<<c$

SR + Empirically Measured Physical Constants lead to RQM via the (KG) Klein-Gordon Quantum Eqn, and thence to QM via the low-velocity limit { $|\mathbf{v}| << c$ }, giving the Schrödinger Eqn. This fundamental KG Relation also leads to the other

Quantum Wave Equations:	RQM (massless, no rest-frame, Lorentzian) { $\lvert\mathbf{v}\rvert = c : m_o = 0$ }	RQM (with non-zero mass, Lorentzian) { $0 <= \lvert\mathbf{v}\rvert < c : m_o > 0$ }	QM (limit-case from RQM, Galilean) { $0 <= \lvert\mathbf{v}\rvert << c : m_o > 0$ }
spin=0 boson field = 4-Scalar:	Free Scalar Wave (Higgs)	Klein-Gordon (KG)	Schrödinger (regular QM)
spin=1/2 fermion field = 4-Spinor:	Weyl	Dirac (RQM w/ EM charge)	Pauli (QM w/ EM charge)
spin=1 boson field = 4-Vector:	Maxwell (EM photonic)	Proca	

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SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM 4-Vector Topic Index

SR & QM via 4-Vector Diagrams

4-Vector SRQM Interpretation of QM

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Mostly SR Stuff

4-Vector Basics, SR 4-Vectors = Physical 4D (1,0)-Tensors, Coordinate-Independent Objects
Paradigm Assumptions: Right & Wrong
Minkowski:SR SpaceTime, <Events> ∈ <Time·Space>, WorldLines, 4D Minkowski Metric
SR {4-Scalars, 4-Vectors, 4-Tensors} & Tensor Invariants, Cayley-Hamilton Theorem
SR Lorentz Transforms, CPT Symmetry, Trace Identification, Antimatter, Feynman-Stueckelberg
Fundamental Physical Constants: Lorentz Scalar Invariants = SR 4-Scalars = 4D (0,0)-Tensors
Projection Tensors: Temporal "(V)ertical" & Spatial "(H)orizontal": (V),(H) refer to Light-Cone
Stress-Energy Tensors: Relativistic Fluids→(PerfectFluid, Dust, Radiation, EM, DarkEnergy, etc)
Invariant Intervals, Measurement, Metrics, Metric Signature
SpaceTime Kinematics & Dynamics, ProperTime Derivative
Einstein's $E = mc^2 = \gamma m_o c^2 = \gamma E_o$, Rest Mass ($m_o$):Rest Energy ($E_o$), Scalar Invariants
SpaceTime Orthogonality: Time-like 4-Velocity, Space-like 4-Acceleration
Relativity of Simultaneity:Stationarity ; **Invariance/Absolutes** of Causality:Topology
Relativity: Time Dilation (←| clock moving |→), Length Contraction (|→ruler moving←|)
Invariants: Proper Time (| clock at rest |) , Proper Length (| ruler at rest |)
Temporal Ordering: (Time-like) Causality is **Absolute**; (Space-like) Simultaneity is *Relative*
Spatial Ordering: (Time-like) Stationarity is *Relative* ; (Space-like) Topology is **Absolute**
SR Motion * Lorentz Scalar = Interesting Physical 4-Vector
SR Conservation Laws & Local Continuity Equations, Symmetries
SR Wave-Particle Relation, Invariant d'Alembertian Wave Eqn, SR Waves, 4-WaveVector
Relativistic Doppler Effect, Relativistic Aberration Effect
SpaceTime is 4D = (1+3)D: $\partial\cdot\mathbf{R}=\partial_\mu R^\mu=4$, $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4$, $Tr[\eta^{\mu\nu}]=4$, $\mathbf{A} = A^\mu = (a^\mu) = (a^0,a^1,a^2,a^3)$ = 4 comps
Minimal Coupling = Interaction with a (Vector)Potential, usually the EM 4-VectorPotential
Conservation of 4-TotalMomentum (TotalEnergy=Hamiltonian & 3-total-momentum)
SR Hamiltonian:Lagrangian Connection
Lagrangian, Lagrangian Density
Hamilton-Jacobi Equation (differential ∂), Relativistic Action (integral $\int$)
4-Position:4-Velocity Relation, Euler-Lagrange Equations
Noether's Theorem, Continuous Symmetries, Conservation Laws, Continuity Equations
Relativistic Equations of Motion, Lorentz Force Equation
c^2 Invariant Relations, The Speed-of-Light (c)
Thermodynamic 4-Vectors, Unruh-Hawking Temperature, Particle Distributions

Mostly QM & SRQM Stuff

Advanced SRQM 4-Vectors
Where is Quantum Gravity?
Relativistic Quantum Wave Equations (RWE : QWE)
Klein-Gordon Equation (KG) / Fundamental Quantum Relation $(\partial\cdot\partial) = -(m_o c/\hbar)^2$
RoadMap from SR to QM: SR→QM, SRQM 4-Vector Connections
QM Schrödinger Relation
QM Axioms? - No, (QM Principles derived from SR) = SRQM
Relativistic Wave Equations: based on mass & spin & relative velocity:energy
RWE's: Klein-Gordon, Dirac, Proca, Maxwell, Weyl, Pauli, Schrödinger, etc.
Classical Limits: SR's { $|\mathbf{v}| << c$ } ; QM's { $\hbar|\nabla\cdot\mathbf{p}| << (\mathbf{p}\cdot\mathbf{p})$ or $|\nabla\cdot\mathbf{k}| << (\mathbf{k}\cdot\mathbf{k})$}
Photon Polarization
Linear PDE's→{Principle of Superposition, Hilbert Space, <Bra|:|Ket> Notation}
Canonical QM Commutation Relations ← derived from SR
Heisenberg Uncertainty Principle (due to non-zero commutation)
Pauli Exclusion Principle (Fermion spin n/2), Bose Aggregation Principle (Boson spin n)
Complex 4-Vectors, Quantum Probability, Imaginary values
CPT Theorem, Lorentz Invariance, Poincaré Invariance, Isometry
Hermitian Generators, Unitarity:Anti-Unitarity
QM → Classical Correspondence Principle, similar to SR → Classical Low Velocity
The Compton Effect = Photon:Electron Interaction (neglecting Spin Effects)
Photon Diffraction, Crystal-Electron Diffraction, The Kapitza-Dirac Effect
The ($\hbar$) Relation, Einstein-de Broglie, Planck:Dirac, Wave-Particle
The Aharonov-Bohm Effect (integral $\int$), The Josephson Junction Effect (differential ∂)
Dimensionless Quantities
SRQM Symmetries:
Hamilton-Jacobi Equation (differential ∂), Relativistic Action (integral $\int$)
Differential (∂) on (4-Vector) vs. Integral(I $\int$) on (4-Scalar)
Schrödinger Relations vs. Cyclic Imaginary Time ↔ Inverse Temperature
4-Velocity:4-Position vs. Euler-Lagrange Equations
Matter-AntiMatter: Trace Identification of Lorentz Transforms, CPT
Quantum Relativity: GR is *NOT* wrong, *Never bet against Einstein* :)
Quantum Mechanics is Derivable from Special Relativity, SR→QM: SRQM

SRQM = The [SR→QM] Interpretation of Quantum Mechanics
= Special Relativity → Quantum Mechanics

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)

Special Relativity → Quantum Mechanics

Paradigm Background Assumptions (part 1)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

There are some paradigm assumptions that need to be cleared up:

The real, physical world *IS NOT* Euclidean 3-dimensional (3D) with absolute background time.
Classical and quantum 3D physics is a great approximation; but only for Galilean, slow-moving objects |**v**|<<c.
3D physics uses {3-vectors = 3D (1,0)-tensors}, has 3D Euclidean invariants like lengths (Pythagorean theorem),
has 1D Euclidean scalar invariants like absolute time, but it does not contain or predict many of
the physical properties and relationships that we now know to be true from SR & RQM.
Also, these 1D & 3D Euclidean invariants have been empirically-proven to *NOT* be invariant in the real world.
This is based on a century+ of physics experiments and observations confirming the fact of 4D *Relativity*.
These 1D & 3D scalar invariants are actually just relativistic components of 4-Vectors in 4D.
The fact that 4D Scalar Invariants don't display similar relativistic variance is a good indication that our universe is empirically 4D.

The real, physical world *IS* a locally Minkowskian 4-Dimensional SpaceTime (4D),
with *relativistically-interconnected* (1 time + 3 space) covariant dimensions.
Time and space are interconnected in a very specific Lorentzian way, via SpaceTime 4D *Relativistic* Metrics,
which give a great many special relationships and invariances that 3D physics misses entirely.
These properties are easily explained using SR:Minkowskian Physical {4-Vectors = 4D (1,0)-Tensors}.

3D physics can be obtained as a limiting-case approximation from 4D Physics by using relative speed |**v**|<<c.
Classical Mechanics (CM) is just a low-speed limiting-case of Special Relativity (SR)
Quantum Mechanics (QM) is just the low-speed limiting-case of Relativistic Quantum Mechanics (RQM)
This is related to (1+3) Time·Space Splitting

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

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(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

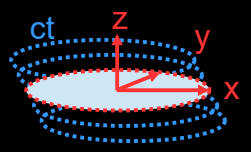
Special Relativity → Quantum Mechanics
Paradigm Background Assumptions (part 2)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

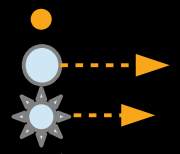
There are some paradigm assumptions that need to be cleared up:

Minkowskian:SR 4D Physical 4-Vectors *ARE NOT* generalizations of Classical/Quantum 3D physical 3-vectors.
While a "mathematical" Euclidean (n+1)D-vector is the generalization of a Euclidean (n)D-vector,
the "physical/physics" analogy ends there.

Minkowskian:SR 4D Physical 4-Vectors *ARE* the primitive elements of 4D Minkowski:SR SpaceTime.
Classical/Quantum physical 3-vectors are just the **spatial** components of SR Physical {4-Vectors = 4D (1,0)-Tensors}.
There is also a fundamentally-related classical/quantum physical scalar related to each and every type of 3-vector,
which is just the temporal component scalar of a given SR Physical **SpaceTime** 4-Vector.



4-Position $\mathbf{R} = R^\mu = (r^\mu) = (r^0,r^i) = (ct,\mathbf{r}) \rightarrow (ct=r^t,r^x,r^y,r^z) = (ct,x,y,z)$
4-Momentum $\mathbf{P} = P^\mu = (p^\mu) = (p^0,p^i) = (E/c=mc,\mathbf{p}=m\mathbf{u}) \rightarrow (E/c=p^t,p^x,p^y,p^z)$
4-CurrentDensity $\mathbf{J} = J^\mu = (j^\mu) = (j^0,j^i) = (\rho c,\mathbf{j}=\rho\mathbf{u}) \rightarrow (\rho c=j^t, j^x, j^y, j^z)$



These Classical/Quantum {scalar}+{**3-vector**} are the dual {temporal}+{**spatial**} components
of a single SR Time·Space 4-Vector = (temporal scalar * $c^{\pm1}$, **spatial 3-vector**)
with SR LightSpeed factor ($c^{\pm1}$) to give correct overall dimensional measurement units.

While different observers may see different *relative* "values" of the Classical/Quantum components (v^0,v^1,v^2,v^3)
from their point-of-view:frame-of-reference in SpaceTime, each will see the same actual SR 4-Vector $\mathbf{V} = V^\mu$
and its magnitude^2 = $\mathbf{V}\cdot\mathbf{V} = V^\mu V_\mu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0_o)^2$ at a given <Event> in SpaceTime.
Magnitudes^2 can be {+ = temporal : 0 = null : - = **spatial**} in (+,-,-,-) Special Relativity,
due to the { metric signature = $(1,3,0)^+$ } Lorentzian metric ⊂ pseudo-Riemannian metric (non-positive-definite)

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Special Relativity → Quantum Mechanics

Paradigm Background Assumptions (part 3)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

There are some paradigm assumptions that need to be cleared up:

Relativistic Physics **IS NOT** the generalization of Classical or Quantum Physics.
Classical & Quantum Physics **ARE** the low-relative-speed { |**v**| << c } limiting-case approximation of Relativistic Physics.

This includes (Newtonian) Classical Mechanics and Classical QM (NRQM: meaning the Non-Relativistic Schrödinger QM Equation – it is not fundamental).
The rules of standard QM are just the low-relative-speed approx. { |**v**|<<c } of RQM rules. Classical EM is for the most part already compatible with Special Relativity.
However, Classical EM doesn't include or take into account intrinsic spin, even though spin is a result of SR Poincaré Invariance, not QM.

So far, in all of my research, if there was a way to get a result classically,
then there was usually a much simpler way to get the result using tensorial 4-Vectors and SRQM *relativistic* thinking.
Likewise, a lot of QM results make much more sense when approached from SRQM (ex: Temporal vs. **Spatial** relations).
4-Vector formulations are all extremely easy to derive in SRQM and are all relativistically covariant and give **invariant** results.

Einstein Energy:Mass Eqn: $\mathbf{P} = m_o\mathbf{U} \rightarrow \{ E = mc^2 = \gamma m_o c^2 = \gamma E_o : \mathbf{p} = m\mathbf{u} = \gamma m_o \mathbf{u} \}$

Hamiltonian: $H = \gamma(\mathbf{P_T}\cdot\mathbf{U})$ {Relativistic} $\rightarrow (T + V) = (E_{kinetic} + E_{potential})$ {Classical-limit only, |**u**| << c}
Lagrangian: $L = -(\mathbf{P_T}\cdot\mathbf{U})/\gamma$ {Relativistic} $\rightarrow (T - V) = (E_{kinetic} - E_{potential})$ {Classical-limit only, |**u**| << c}

{differential 4-Vector formats}
SR/QM Wave Eqn {inv of Phase Eqn}: $\mathbf{K_T} = -\partial[\Phi_{phase}] = \mathbf{P_T}/\hbar \rightarrow \{ \omega_T = -\partial_t[\Phi] : \mathbf{k_T} = \nabla[\Phi] \}$
Hamilton-Jacobi Eqn {inv of Action Eqn}: $\mathbf{P_T} = -\partial[S_{action}] = \hbar\mathbf{K_T} \rightarrow \{ E_T = -\partial_t[S] : \mathbf{p_T} = \nabla[S] \}$

{integral 4-Scalar formats}
SR Action Eqn {inv of H-J Eqn}: $\Delta S_{action} = -\int_{path}\mathbf{P_T}\cdot d\mathbf{X} = -\int_{path}(\mathbf{P_T}\cdot\mathbf{U})d\tau = \int_{path} L\, dt$
SR/QM Phase Eqn {inv of Wave Eqn}: $\Delta\Phi_{phase} = -\int_{path}\mathbf{K_T}\cdot d\mathbf{X} = -\int_{path}(\mathbf{K_T}\cdot\mathbf{U})d\tau = \Delta S_{action}/\hbar$

{advanced mechanics}
Euler-Lagrange Equation: $(\mathbf{U} = (d/d\tau)\mathbf{R}) \rightarrow (\partial_R = (d/d\tau)\partial_U)$ {the easy derivation}
Hamilton's Equations: $(d/d\tau)[\mathbf{X}] = (\partial/\partial\mathbf{P_T})[H_o]$ & $(d/d\tau)[\mathbf{P_T}] = (\partial/\partial\mathbf{X})[H_o]$

{SR wave mechanics – requires a 4-WaveVector **K** as solution}
d'Alembertian Wave Equation: $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla$, with solutions $\sim \Sigma_n (A_n)\, e^{\pm(\mathbf{K}_n\cdot\mathbf{X})}$

Einstein-de Broglie Relation: $\mathbf{P} = \hbar\mathbf{K} \rightarrow \{ E = \hbar\omega : \mathbf{p} = \hbar\mathbf{k} \}$
Complex Plane-Wave Relation: $\mathbf{K} = i\partial \rightarrow \{ \omega = i\partial_t : \mathbf{k} = -i\nabla \}$
Schrödinger Relations: $\mathbf{P} = i\hbar\partial \rightarrow \{ E = i\hbar\partial_t : \mathbf{p} = -i\hbar\nabla \}$

Canonical QM Commutation Relations inc. QM Time,Energy:
$[P^\mu,X^\nu] = i\hbar\eta^{\mu\nu} \rightarrow \{ [x^0,p^0] = [ct,E/c] = [t,E] = -i\hbar : [x^j,p^k] = i\hbar\delta^{jk} \}$
$[\partial^\mu,X^\nu] = \eta^{\mu\nu} \rightarrow \{ [x^0,\partial^0] = [ct,\partial_t/c] = [t,\partial_t] = -1 : [x^j,\partial^k] = +\delta^{jk} \}$

Total Momentum: $\mathbf{P_T} = \mathbf{P} + q\mathbf{A} \rightarrow \{ E_T = E + q\varphi : \mathbf{p_T} = \mathbf{p} + q\mathbf{a} \}$
Minimal Coupling: $\mathbf{P} = \mathbf{P_T} - q\mathbf{A} \rightarrow \{ E = E_T - q\varphi : \mathbf{p} = \mathbf{p_T} - q\mathbf{a} \}$

{Physical Inverse Effects}
Josephson-Junction {differential 4-Vector format}: $\mathbf{A} = -(\hbar/q)\partial[\Delta\Phi_{pot}]$
Aharonov-Bohm {integral 4-Scalar format}: $\Delta\Phi_{pot} = -(q/\hbar)\int_{path}\mathbf{A}\cdot d\mathbf{X}$

Compton Scattering: $\Delta\lambda\!\!\!^{-} = (\lambda\!\!\!^{-}' - \lambda\!\!\!^{-}) = (\hbar/m_o c)(1 - \cos[\varnothing])$
Klein-Gordon Relativistic Quantum Wave Eqn: $\partial\cdot\partial = -(m_o c/\hbar)^2$

Special Relativity → Quantum Mechanics
Paradigm Background Assumptions (part 4)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

There are some paradigm assumptions that need to be cleared up:

We will **NOT** be employing the commonly-(mis)used Newtonian classical limits {c→∞} and {ħ→0}.
Neither of these is a valid physical assumption, for the following reasons:

[1]

Both (c) and (ħ=h/2π) are unchanging Universal Physical Constants and Lorentz Scalar **Invariants**.
Taking a limit where these change is non-physical. They are CONSTANT. Tensor math shows them Invariant.
Many, many experiments verify that these physical constants have not changed over the lifetime of the universe.
This is one reason for the 2019 Redefinition of SI Base Units on Fundamental Constants $\{c,h,e,k_B,N_A,K_{CD},\Delta\nu_{Cs}\}$.

[2]

Photons/waves have energy (E) via momentum (E=|**p**|c) & frequency (E=ħω) : (ω = 2πν)$_{\{\text{ angular [rad/s], circular[cycle/s] , 2π rad = 1 cycle }\}}$
Let E = |**p**|c. If c→∞, then E→∞. Then Classical EM light rays/waves have infinite energy.
Let E = ħω = hν. If ħ→0, then E→0. Then Classical EM light rays/waves have zero energy.

Obviously neither of these energy results is true in the Newtonian/Classical limit.
In Classical EM and Classical Mechanics, LightSpeed (c) remains a large but finite constant.
Likewise, Dirac's (Planck-reduced) Constant (ħ=h/2π) remains very small but never becomes zero.

The **correct way** to take the limits is via:
The low-speed non-relativistic limit { |**v**| << c }, which is a physically-occurring situation.
The Hamilton-Jacobi non-quantum limit { ħ|∇·**p**| << (**p**·**p**) } or { |∇·**k**| << (**k**·**k**)}, which is a physically-occurring situation.

Special Relativity → Quantum Mechanics
Paradigm Background Assumptions (part 5)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

There are some paradigm assumptions that need to be cleared up:

While we are discussing units, note that the following are *ALL* fundamental Relativistic Invariants, meaning also that they are *ALL* Lorentz Scalar Invariants = 4D (0,0)-Tensors:

Universal Constants (True for all Time·Space)

c: LightSpeed in Vacuum Constant [m/s] (maximum speed of causality) $E \sim pc$

G: Gravitational Constant [$m^3/kg\cdot s^2 = N\cdot m^2/kg^2$] (GR curvature) $E \sim -GMm/r$

h,ħ=h/2π: Planck's Constant,Dirac's Constant(Planck Reduced) [J·s] (QM action, waves) $E \sim \hbar\omega = h\nu$

k_B: Boltzmann's Constant [J/°K] (Stat Mech, temperature, entropy) $E \sim k_B T$

ε_o: Electric Constant [$F/m = C^2\cdot s^2/kg\cdot m^3$]

μ_o: Magnetic Constant [$H/m = kg\cdot m/C^2$] $E/V \sim \frac{1}{2}(\varepsilon_o e^2 + b^2/\mu_o)$, with $\varepsilon_o\mu_o=1/c^2$

Particle-Dependent Constants {interestingly, also the "No Hair" scalar parameters of a BlackHole}

m_o: Rest Mass [kg] $E \sim m_o c^2$

q: EM Charge [C] $E \sim -(1/4\pi\varepsilon_o)Qq/r$ or $E \sim q\Phi$: ex. E ~ eV

s_o: Intrinsic Spin [J·s] $E \sim (g/2)\boldsymbol{\mu}\cdot\mathbf{b}$, with $\boldsymbol{\mu}=q\mathbf{s}/2m$

Fluid-Dependent Scalars

ρ_{eo}: MRCF Fluid EnergyDensity (temporal) [$Pa = J/m^3 = N/m^2 = kg/m\cdot s^2$]

p_o: MRCF Fluid Pressure (spatial) [$Pa = J/m^3 = N/m^2 = kg/m\cdot s^2$]

$T_{perfectfluid}^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu}$

These scalars join with Unit-Projection 4-Tensors to give property descriptions to particles and fluids:

4-UnitTemporal $\overline{\mathbf{T}} = \gamma(1,\boldsymbol{\beta})$ 4-Velocity $\mathbf{U} = (c)\overline{\mathbf{T}}$ 4-Momentum $\mathbf{P} = (cm_o)\overline{\mathbf{T}}$

4-UnitSpatial $\overline{\mathbf{S}} = \gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{\mathbf{n}},\hat{\mathbf{n}})$ 4-Spin $\mathbf{S} = (s_o)\overline{\mathbf{S}}$ along with the 4-Position $\mathbf{R} = (ct,\mathbf{r})$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Special Relativity → Quantum Mechanics

Paradigm Background Assumptions (part 6)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

There are some paradigm assumptions that need to be cleared up:

We will *NOT* be implementing the common {→lazy and extremely misguided} convention of setting physical constants to the value of (dimensionless) unity, often called "Natural Units", to hide them from equations; nor using mass (m) in place of RestMass (m_o). Likewise for other components vs Lorentz Scalars with rest-value-naughts ($_o$), like Energy (E) vs RestEnergy (E_o).

One sees this very often in the literature (ex. LightSpeed c→1) . The usual excuse cited is "For the sake of brevity" or "For the sake of simplicity". Well, the "sake of brevity" forsakes "clarity". There is nothing physically "natural" about "natural units".

The *ONLY* situations in which setting constants to unity (1) is practical or advisable is in numerical simulation or mathematical analysis. When teaching physics, or trying to understand physics: it helps when equations are dimensionally, unit-wise, correct. In other words, the physics technique of "dimensional analysis" is a powerful tool that should not be disdained. i.e. Brevity only aids speed of computation, Clarity aids understanding.

The situation of using "naught = $_o$" for rest-values, such as (m_o) for RestMass and (E_o) for RestEnergy: is intrinsic to SR, is a very good idea, absolutely adds clarity, identifies Lorentz Scalar Invariants, and will be explained in more detail later. Essentially, *relativistic* gamma (γ) pairs with **invariant** (Lorentz scalar:rest value $_o$) to make a *relativistic* component: { $m = \gamma m_o$; $E = \gamma E_o$; $\rho = \gamma \rho_o$; $\omega = \gamma \omega_o$ } Note the multiple equivalent ways that one can write 4-Vectors of SpaceTime (Time·Space) using these rules: {$t = \gamma t_o$; $L = L_o/\gamma$; $V = V_o/\gamma$; etc.}

$$\text{4-Momentum } \mathbf{P} = P^\mu = (p^\mu) = (p^0,p^i) = (mc=E/c,\mathbf{p}=m\mathbf{u}) = (\gamma m_o c=\gamma E_o/c,\mathbf{p}=\gamma m_o\mathbf{u}) = -\partial_R[\, S_{action,free} \,] = -\partial_U[\, L_{o,RestLagrangian,free} \,]$$
$$= m_o\mathbf{U} = m_o\gamma(c,\mathbf{u}) = \gamma m_o(c,\mathbf{u}) = m(c,\mathbf{u}) = (mc,m\mathbf{u}) = (mc,\mathbf{p}) = mc(1,\boldsymbol{\beta}) = m_o c\gamma(1,\boldsymbol{\beta}) = (m_o c)\overline{\mathbf{T}}$$
$$= (E_o/c^2)\mathbf{U} = (E_o/c^2)\gamma(c,\mathbf{u}) = \gamma(E_o/c^2)(c,\mathbf{u}) = (E/c^2)(c,\mathbf{u}) = (E/c,E\mathbf{u}/c^2) = (E/c,\mathbf{p}) = (E/c)(1,\boldsymbol{\beta}) = (E_o/c)\gamma(1,\boldsymbol{\beta}) = (E_o/c)\overline{\mathbf{T}}$$

This notation makes clear what is { *relativistically-varying=(frame-dependent)* vs. **Invariant=(frame-independent)** } and { Temporal vs. **Spatial** } BTW, I prefer the "Particle Physics" Metric-Signature-Convention (+,-,-,-)={temporal:0th:+}. {Makes rest values positive, fewer minus signs to deal with} Show the physical constants and rest naughts ($_o$) in the work. They deserve the respect and you will benefit. You can always set constants to unity later, when you are doing your numerical simulations.

SR 4-Tensor
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(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Special Relativity → Quantum Mechanics

Paradigm Background Assumptions (part 7)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

There are some paradigm assumptions that need to be cleared up:

Some physics books on ElectroMagnetism (EM) say that:
Electric field **E** and the Magnetic field **B** are the "real" physical objects, and that
EM scalar-potential φ and the EM 3-vector-potential "**A**" are just "calculational/mathematical" artifacts.

Neither statement is relativistically correct: See Jefimenko's equations & Liénard-Wiechert Potential.

All of these physical EM properties: {**E**,**B**,φ,"**A**"} are actually just the components of SR tensors, and as such, their values will *relativistically* vary in different observers' reference-frames.

Given this SR knowledge, and to match our 4-Vector notation, we demote the physical property symbols, (the tensor "components") to their lower-case equivalents {**e**,**b**,φ,**a**} to match other 4-Vector comps. see Wolfgang Rindler's works as example

The truly SR **invariant** physical objects are:
The 4-Gradient ∂, the 4-VectorPotential **A**, their combination via the exterior (wedge=^) product into the Faraday EM 4-Tensor $F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha = (\partial \wedge \mathbf{A})$, and their combination via the inner (dot=·) product into the Lorenz Gauge 4-Scalar $(\partial \cdot \mathbf{A}) = 0$. Yes, Lorenz, not Lorentz.

Temporal-spatial components of 4-Tensor $F^{\alpha\beta}$: electric 3-vector field $\mathbf{e} = e^i = e^{i0}$
Spatial-spatial components of 4-Tensor $F^{\alpha\beta}$: magnetic 3-vector field $\mathbf{b} = b^k = (½)\, \varepsilon_{ij}{}^{k} F^{ij}$
Temporal component of 4-Vector $\mathbf{A}=A^\mu$: EM scalar-potential φ
Spatial components of 4-Vector $\mathbf{A}=A^\mu$: EM 3-vector-potential **a**

Note that the Speed-of-Light (c) plays a prominent role in the component definitions.
Also, QM requires the 4-VectorPotential **A** as explanation of the Aharonov-Bohm Effect.
The physical measurability of the AB Effect proves the reality of the 4-VectorPotential **A**.
Again, all the lower and higher-rank SR tensors can be built from fundamental 4-Vectors.

4-Gradient
$\partial=\partial^\mu=\partial/\partial R_\mu=(\partial_t/c,-\nabla)$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

Lorenz Gauge: Conservation of EM (Vector)Potential
$\partial\cdot\mathbf{A}=(\partial_t/c)(\varphi/c)-(-\nabla)\cdot(\mathbf{a})$
$=(\partial_t\varphi/c^2)+\nabla\cdot\mathbf{a}=0$

4-(EM)VectorPotential
$\mathbf{A}=A^\mu=(\varphi/c,\mathbf{a})$
$\mathbf{A}_{EM}=A_{EM}{}^\mu=(\varphi_{EM}/c,\mathbf{a}_{EM})$

Faraday EM Tensor

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha = \partial \wedge \mathbf{A} \rightarrow$$

$$\begin{bmatrix} F^{tt} & F^{tx} & F^{ty} & F^{tz} \\ F^{xt} & F^{xx} & F^{xy} & F^{xz} \\ F^{yt} & F^{yx} & F^{yy} & F^{yz} \\ F^{zt} & F^{zx} & F^{zy} & F^{zz} \end{bmatrix} = \begin{bmatrix} 0 & -e^x/c & -e^y/c & -e^z/c \\ +e^x/c & 0 & -b^z & +b^y \\ +e^y/c & +b^z & 0 & -b^x \\ +e^z/c & -b^y & +b^x & 0 \end{bmatrix} = \begin{bmatrix} 0 & , -e^{0j}/c \\ +e^{i0}/c, & -\varepsilon^{ij}{}_k b^k \end{bmatrix} = \begin{bmatrix} 0 & , -\mathbf{e}/c \\ +\mathbf{e}^T/c, & -\nabla\wedge\mathbf{a} \end{bmatrix}$$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Special Relativity → Quantum Mechanics

Paradigm Background Assumptions (part 8)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

There are some paradigm assumptions that need to be cleared up:

A number of QM philosophies make the assertion that particle "properties" do not "exist" until measured.
The assertion is based on the QM Heisenberg Uncertainty Principle, and more specifically on quantum non-zero commutation, in which a measurement on one property of a particle alters a different non-commuting property of the same particle.

That is an incorrect analysis. Properties define particles: what they do & how they interact with other particles. Particles and their properties "exist" as <events> independently of human intervention or observation. The correct way to analyze this is to understand what a measurement is: an arrangement of some number of particles in a particular manner as to allow an observer to get information:knowledge about one or more of the "subject particle's" properties. Typically this involves "counting" spacetime <events> and using SR **invariant** intervals as a basis-of-measurement.

Some properties are indeed non-commuting. This simply means that it is not possible to arrange a set of particles (the measuring device) in such a way as to measure (ie. obtain "complete" information about) both of the "subject particle's" non-commuting properties at the same spacetime <event>.
The measurement arrangement <events> can be done at best sequentially, and the temporal order of these <events> makes a difference in observed results.
EPR-Bell, however, allows one to "infer" (due to conservation:continuity laws) properties on a "distant" subject particle by making a measurement on a different "local" {space-like-separated but entangled} particle. This does *not* imply FTL signaling nor non-locality.
The (psi-epistemic) measurement just updates local partial-information one already has about particles that interacted/entangled then separated.

So, a better way to think about it is this: The "Measurement→InformationUpdate" of a property does not "exist" until a physical setup <event> is arranged. Non-commuting properties require different physical arrangements in order for the properties to be measured, and the temporally-first measurement alters that particle's properties in a minimum sort of way, which affects the latter measurement. All observers agree on Causality, the time-order of temporally-separated spacetime <events>. However, individual observers may have different sets of partial information about the same particle(s).

This objective, realist view makes way more sense than the subjective belief that a particle's actual properties don't exist until it is "observed", which is about as unscientific and laughable a statement as I can imagine.

****Relativity is the System-of-Measurement that QM has been looking for: (Ψ) Psi-Epistemic****

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Special Relativity → Quantum Mechanics

Paradigm Background Assumptions (part 9)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

There are some paradigm assumptions that need to be cleared up:

****A well-formulated and correctly-used notation is critical for understanding physics****

Unfortunately, there are a number of "sloppy" notations seen in relativistic and quantum physics.

Incorrect: Using T^{ii} as a Trace of 3D tensor T^{ij}, or $T^{\mu\mu}$ as a Trace of 4D Tensor $T^{\mu\nu}$
T^{ii} is actually just the diagonal part of 3-tensor T^{ij}, the components: $T^{ii} = \text{Diag}[T^{11},T^{22},T^{33}]$
The Trace operation requires a paired upper-lower index combination (Einstein Sum), which then gets summed over.
$T_i^{\,i}$ is the Trace of 3-tensor T^{ij}: $T_i^{\,i} = T_1^{\,1}+T_2^{\,2}+T_3^{\,3} = \text{3-trace}[T^{ij}] = \delta_{ij}T^{ij} = +T^{11}+T^{22}+T^{33}$ in the Euclidean Metric $E^{ij} = \delta^{ij} = \text{Diag}[+1,+1,+1]$

$T^{\mu\mu}$ is actually just the diagonal part of 4-Tensor $T^{\mu\nu}$, the components: $T^{\mu\mu} = \text{Diag}[T^{00},T^{11},T^{22},T^{33}]$
The Trace operation requires a paired upper-lower index combination (Einstein Sum), which then gets summed over.
$T_\mu^{\,\mu}$ is the Trace of 4-Tensor $T^{\mu\nu}$: $T_\mu^{\,\mu} = T_0^{\,0}+T_1^{\,1}+T_2^{\,2}+T_3^{\,3} = \text{4-Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = +T^{00}-T^{11}-T^{22}-T^{33}$ in the Minkowskian Metric $\eta^{\mu\nu} = \text{Diag}[+1,-\delta^{ij}]$

Incorrect: Hiding factors of LightSpeed (c) in relativistic equations, ex. E = m
The use of "natural units" leads to a lot of ambiguity, and one loses the ability to do proper dimensional analysis.
Wrong: E=m: Energy [J = kg·m²/s²] is *not* identical to mass [kg], not in dimensional units nor in reality.
Correct: $E=mc^2$: Energy is related to mass via the Speed-of-Light (c) [m/s], ie. mass is a type of concentrated energy.

Incorrect: Using m instead of m_o for rest mass; Using E instead of E_o for rest energy
Correct: $E = mc^2 = \gamma m_o c^2 = \gamma E_o$
E & m, and **p** are *relativistic* internal components of 4-Momentum $\mathbf{P}=(E/c,\mathbf{p})=(mc,\mathbf{p})$ which vary in different reference-frames.
E_o & m_o are Lorentz Scalar **Invariants**, the SR Rest Values, which are the same, even in different reference-frames: $\mathbf{P}=m_o\mathbf{U}=(E_o/c^2)\mathbf{U}$

Special Relativity → Quantum Mechanics

Paradigm Background Assumptions (part 10)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

There are some paradigm assumptions that need to be cleared up:

Wrong: $[x^i,p^j] = i\hbar\delta^{ij}$
Right: $[x^j,p^k] = i\hbar\delta^{jk}$
Better: $[P^\mu,X^\nu] = i\hbar\eta^{\mu\nu}$
because: $[\partial^\mu,X^\nu] = \eta^{\mu\nu}$

Incorrect: Using the same symbol for a tensor-index and a component
The biggest offender in many books for this one is quantum commutation.
Unclear because (i) means two different things in the same equation.
Correct way: ($i = \sqrt{[-1]}$) is the imaginary unit ; { j,k } are tensor-indicies
In general, any equation which uses complex-number math should reserve (i) for the imaginary, not as a tensor-index.

Incorrect: Using the 4-Gradient:Gradient One-Form notation incorrectly
The 4-Gradient is a 4-Vector, a 4D (1,0)-Tensor, uses an upper index, and has a negative spatial component ($-\nabla$) in (+,-,-,-) SR.
The Gradient (4D) One-Form, its more natural tensor form, a 4D (0,1)-Tensor, uses a lower index in SR.
4-Gradient: $\partial=\partial^\mu=(\partial_t/c,-\nabla)=(\partial_t/c,-\nabla)$ Gradient (4D) One-Form: $\partial=\partial_\mu=(\partial_t/c,+\nabla)=(\partial_t/c,\nabla)$

Incorrect: Mixing styles in 4-Vector naming conventions
There is pretty much universal agreement on the 4-Momentum $\mathbf{P}=P^\mu=(p^\mu)=(p^0,p^i)=(E/c,\mathbf{p})=(mc,\mathbf{p})=(E/c,\mathbf{p})=(mc,\mathbf{p})$
Do not in the same document use 4-Potential $\mathbf{A}=(\varphi,\mathbf{A})$: This is wrong on many levels, inc. dimensional units.
The correct form is 4-VectorPotential $\mathbf{A}=A^\mu=(a^\mu)=(a^0,a^i)=(\varphi/c,\mathbf{a})=(\varphi/c,\mathbf{a})$, with ($\varphi$)=the scalar-potential & ($\mathbf{a}$)=the 3-vector-potential

For all SR 4-Vectors, one should use a **consistent** notation:
The UPPER-CASE SpaceTime (Time·Space) 4-Vector Names match the lower-case **spatial** 3-vector names
There is a LightSpeed (c) factor in the temporal component to give overall matching dimensional units for the entire 4-Vector
4-Vector components are typically lower-case with a few exceptions, mainly energy (E) vs. energy-density { (e),(ρ_e),(ρ_E) }

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Old Wrong Idea

Old Paradigm: QM (as I was taught...)

SR and QM as separate theories

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John B. Wilson
SciRealm@aol.com
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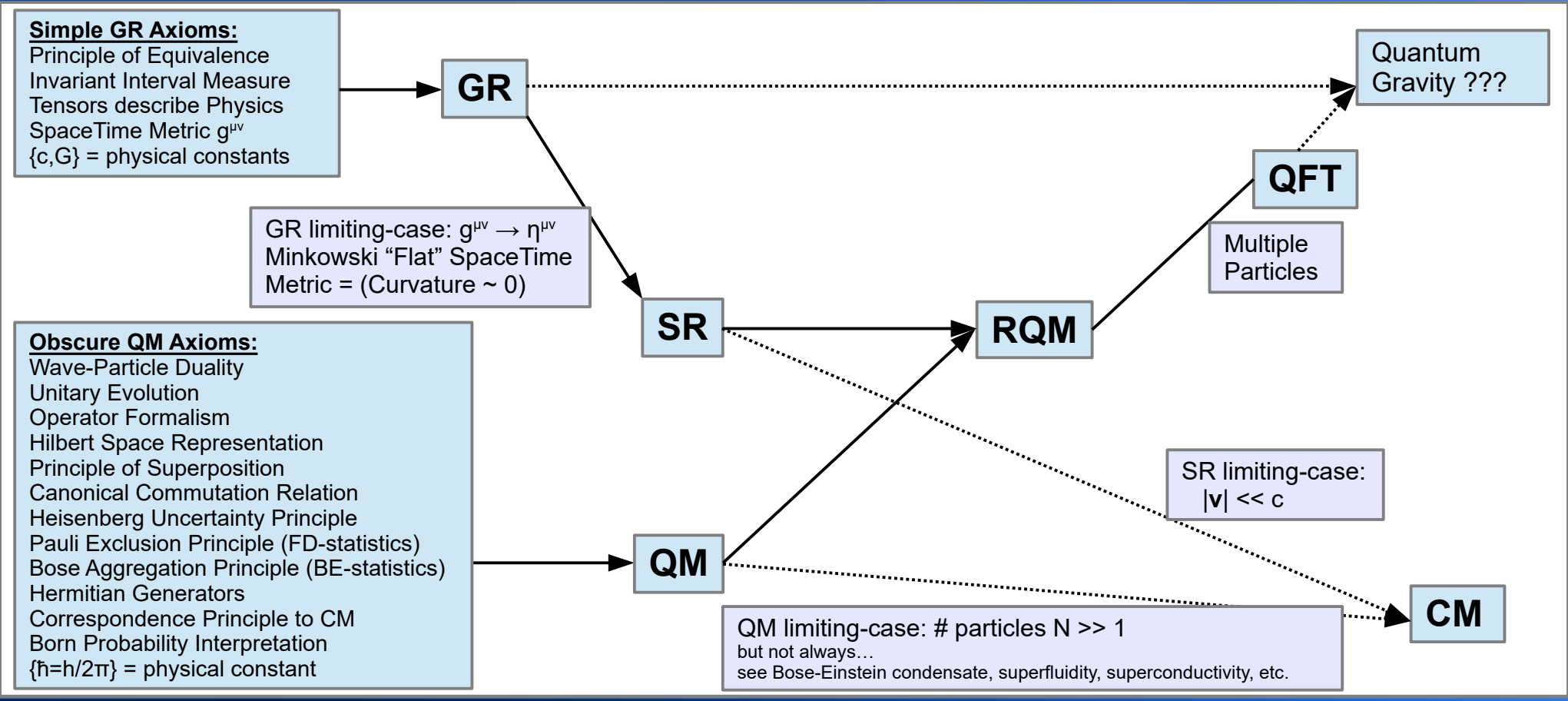

This was the QM paradigm that I was taught while in Grad School: everyone trying for Quantum Gravity

Old Paradigm: QM (years later...)

SR and QM still as separate theories

QM limiting-case better defined, still no QG

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Simple GR Axioms:
Principle of Equivalence
Invariant Interval Measure
Tensors describe Physics
SpaceTime Metric $g^{\mu\nu}$
{c,G} = physical constants

GR

Quantum Gravity ???

Yet another "would be" fortuitous merging???

50+ years searching for QG with no success...

QFT

Multiple Particles

GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$
Minkowski "Flat" SpaceTime
Metric = (Curvature ~ 0)

SR

Another fortuitous merging??

RQM

Obscure QM Axioms:
Wave-Particle Duality
Unitary Evolution
Operator Formalism
Hilbert Space Representation
Principle of Superposition
Canonical Commutation Relation
Heisenberg Uncertainty Principle
Pauli Exclusion Principle (FD-statistics)
Bose Aggregation Principle (BE-statistics)
Hermitian Generators
Correspondence Principle to CM
Born Probability Interpretation
{ħ=h/2π} = physical constant

QM

SR limiting-case:
$|\mathbf{v}| << c$

A fortuitous merging?

CM

QM limiting-case:
{ $\hbar|\nabla\cdot\mathbf{p}| << (\mathbf{p}\cdot\mathbf{p})$ } or
{ $|\nabla\cdot\mathbf{k}| << (\mathbf{k}\cdot\mathbf{k})$ } or { $\psi \rightarrow \mathrm{Re}[\psi]$ }

It is known that QM + SR "join nicely" together to form RQM, but problems with RQM + GR...

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study:
Physical Theories as Venn Diagram
Which regions are empirically real?

4-Vector SRQM Interpretation of QM

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SciRealm@aol.com
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Reality

GR: General Relativity

QM: Quantum Mechanics

Quantum Gravity?

SR: Special Relativity

GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$ Minkowski "Flat" SpaceTime = (Curvature ~ 0)

QM physicists think these areas, anything outside of QM, doesn't exist…

Hence the attempt to Quantize Gravity: Unsuccessful for 50+ years…

A new approach is needed: SR→QM (SRQM) fits the facts...

CM: Classical Mechanics

SR limiting-case: $|\mathbf{v}| << c$

QM limiting-case: $\hbar|\nabla\cdot\mathbf{p}| << (\mathbf{p}\cdot\mathbf{p})$

RQM: Relativistic QM

Many-Worlds Interpretations
Non-local interactions
Instantaneous QM entangled connections
Instantaneous Physical Wavefunction Collapse
Spacetime Dimensions >4
Hidden:Alternate Dimensions
Super-Symmetry
String Theory
Alternate Gravity Theories
Slews of hypothetical new particles
etc.

Quantum Mysticism

Old Wrong Ideas

Basically, lots of stuff for which there's no empirical evidence… & loads of hype...

Many QM physicists believe that the regions outside of QM don't exist…
SRQM Interpretation would say that the regions outside of GR probably don't exist...

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

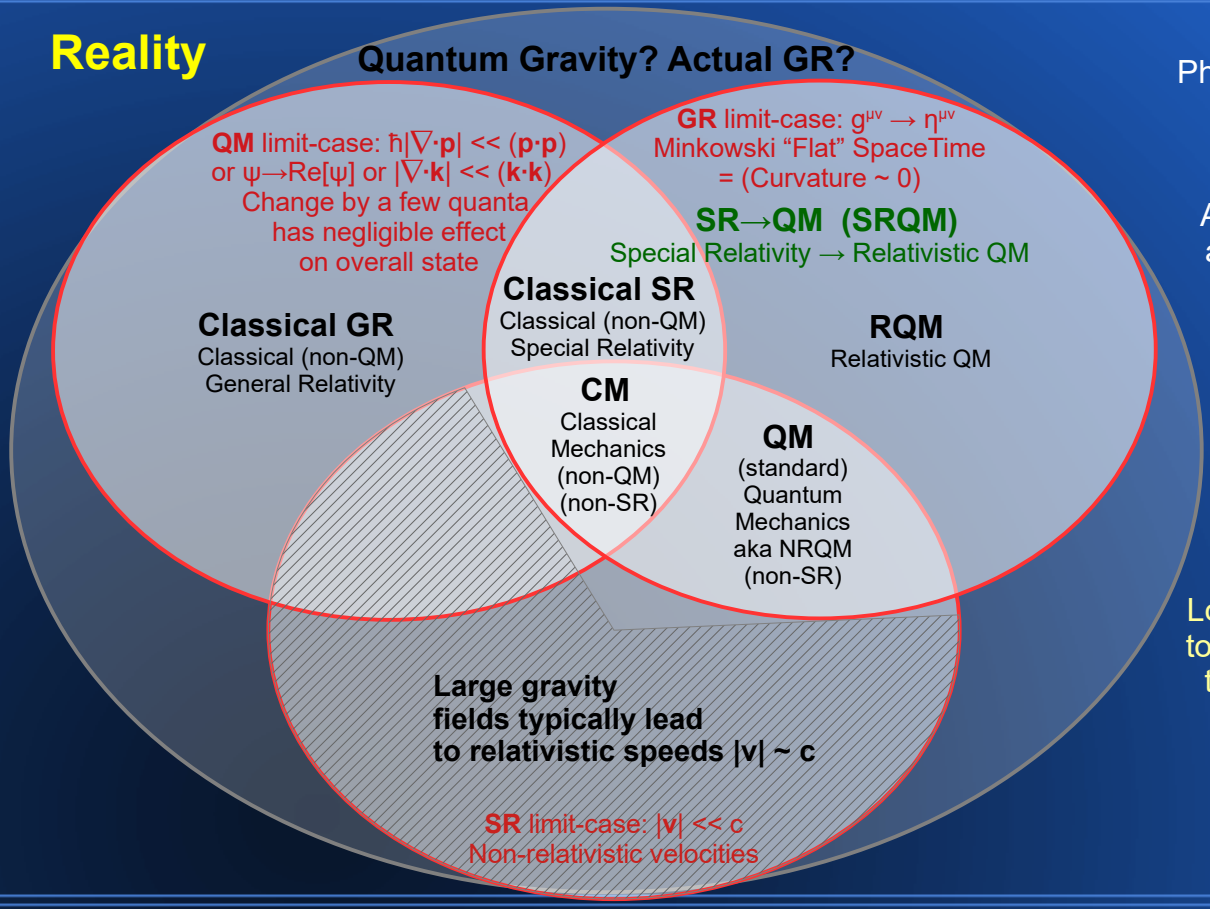
SRQM Study: Regimes of Physics

Physical Limit-Cases as Venn Diagram

Which limit-regions use which physics?

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf



Instead of taking the Physical Theories as set, examine Physical Reality and then apply various limiting-conditions.

What do we then call the various regions?

As we move inwards from any region on the diagram, we are adding more stringent conditions which give physical limiting-cases of "larger, more encompassing" theories.

If one is in Classical GR, one can get Classical SR by moving toward the Minkowski "Flat" SpaceTime limit.

If one is in RQM, one can get Classical SR by moving toward the Hamilton-Jacobi non-QM limit, or to standard QM by moving toward the SR low-velocity limit.

Looking at it this way, I can define SRQM to be equivalent to Minkowski SpaceTime, which contains RQM, and leads to Classical SR, or QM, or CM by taking additional limits.

My assertion:
There is no "Quantized Gravity"
Actual GR contains SRQM and Classical GR.
Perhaps "Gravitizing QM"...

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)

Special Relativity → Quantum Mechanics

Background: Proven Physics

A Tensor Study of Physical 4-Vectors

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Both General Relativity (GR) and Special Relativity (SR) have passed very stringent tests of multiple varieties.
Likewise, Relativistic Quantum Mechanics (RQM) and standard Quantum Mechanics (QM) have passed all tests within their realms of validity:
{ generally micro-scale systems: ex. Single particles, ions, atoms, molecules, electric circuits, atomic-force microscopes, etc.,
but also a few special macro-scale systems: ex. Bose-Einstein condensates, super-currents, super-fluids, long-distance entanglement, etc.}.

To-date, however, there is no observational/experimental indication that quantum effects "alter" the fundamentals of either SR or GR.
Likewise, there are no known violations, QM or otherwise, of Local Lorentz Invariance (LLI) nor of Local Position/Poincaré Invariance (LPI).
In fact, in all known experiments where both SR/GR and QM are present, QM respects the principles of SR/GR, whereas SR/GR modify the results of QM.
All tested quantum-level particles, atoms, isotopes, super-positions, spin-states, etc. obey GR's Universality of Free-Fall & Equivalence Principle and SR's
{ $E = mc^2$ } and speed-of-light (c) communication/signaling limit. Meanwhile, quantum-level atomic clocks are used to measure gravitational red:blue-shift effects.
i.e. GR gravitational frequency-shift (gravitational time-dilation) alters atomic=quantum-level timing. ***Think about that for a moment...***

Some might argue that QM modifies the results of SR, such as via non-commuting measurements. However, that is an alteration of CM expectations, not SR expectations. In fact, there is a basic non-zero commutation relation fully within SR:($[\partial^\mu, X^\nu] = \eta^{\mu\nu}$) which will be derived from purely SR Principles in this treatise. The actual commutation part (Commutator [a,b]) is not about (ħ) or (i), which are just invariant Lorentz Scalar multipliers.

On the other hand, GR Gravity *does* induce changes in quantum interference patterns and hence modifies QM:
See the COW gravity-induced neutron QM interference experiments, the LIGO & VIRGO & KAGRA gravitational-wave detections via QM interferometry, and now also QM atomic matter-wave gravimeters via QM interferometry, ie. gravitational potential modifies atomic-level (quantum) timing.
Likewise, SR induces fine-structure splitting of spectral lines of atoms, “quantum” spin, spin magnetic moments, spin-statistics (fermions & bosons), antimatter, QED, Lamb shift, relativistic heavy-atom effects (liquid mercury, yellowish color of gold, lead batteries having higher voltage than classically predicted, heavy noble-gas interactions, relativistic chemistry...), etc. - essentially requiring QM to be RQM to be valid. QM is instead seen to be the limiting-case of RQM for { $|\mathbf{v}| << c$ }.

Some QM scientists say that quantum entanglement is "non-local", but you still can't send any real messages/signals/information/particles faster than SR's speed-of-light (c). The only “non-local” aspect is the alteration of probability-distributions based on knowledge-changes obtained via measurement.
A local measurement can only alter the “partial information” already-known about the probability-distribution of a distant (entangled) system.
There is no FTL-communication-with nor alteration-of the distant particle. Getting a Stern-Gerlach “up” here doesn’t cause the distant entangled particle to suddenly start moving “down” there. One only knows “now” that it “would” go down “if” the distant experimenter actually performs the measurement.

QM respects the principles of SR/GR, whereas SR/GR modify the results of QM

Special Relativity → Quantum Mechanics
Background: GR Principles
Known Physics ↔ Empirically Tested

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Principles/Axioms and Mathematical Consequences of General Relativity (GR):

Equivalence Principle: Inertial Motion = Geodesic Motion, Universality of Free-Fall, Mass Equivalency ($Mass_{inertial} = Mass_{gravitational}$)

Relativity Principle: SpaceTime (M) has a Lorentzian ⊂ pseudo-Riemannian Metric ($g^{\mu\nu}$) & SR:Minkowski Space rules apply locally ($g^{\mu\nu} \rightarrow \eta^{\mu\nu}$)
(Minkowksi)

General Covariance Principle: Tensors describe Physics, General Laws of Physics are independent of arbitrarily chosen Coordinate-Systems

Invariance Principle: Invariant Interval Measure comes from Tensor Invariance Properties, 4D SpaceTime (Time·Space) from Invariant Trace[$g^{\mu\nu}$] = 4

Causality Principle: Minkowski Diagram/Light-Cone gives { Time-Like (+), Light-Like:Null (0), Space-Like (-) } Measures and Causality Conditions

Einstein:Riemann's Ideas about Matter & Curvature:
Riemann(g) has 20 independent components → too many
Ricci(g) has 10 independent components = enough to describe/specify a gravitational field = # of Poincaré Invariances (10)

{c,G} are Fundamental Physical Constants; so are {$\hbar = h/2\pi$}, but less well-known that this actually comes from SR

SR:Minkowski Space is the GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$ Minkowski "Flat" SpaceTime Metric = (Curvature ~ 0)

To-date, there are no known violations of any of these GR Principles.
GR has passed EVERY observational test to-date, in both weak and strong field regimes.

It is vitally important to keep the mathematics grounded in known physics.
There are too many instances of trying to apply top-down, theoretical-only mathematics to physics.
(ex. String Theory, SuperSymmetry: no physical evidence to-date; SuperGravity: physically disproven)
Progress in science doesn't work that way: Nature itself is the arbiter of what math works with physics. Tensor mathematics applies well to known physics {SR and GR}, which have been empirically extremely well-tested in a huge variety of physical situations. Tensors describe physics.

All known experiments to date comply with all of these Principles, including QM and RQM

Old Wrong Idea

A Tensor Study of Physical 4-Vectors

Old Paradigm: QM Axioms (for comparison)

SR and QM still as separate theories

QM limiting-case better defined, still no QG

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Simple GR Axioms:
Principle of Equivalence
Invariant Interval Measure
Tensors describe Physics
SpaceTime Metric $g^{\mu\nu}$
{c,G} = physical constants

GR

Quantum Gravity ???

Yet another "would be" fortuitous merging???

50+ years searching for QG with no success...

QFT

Multiple Particles

GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$
Minkowski "Flat" SpaceTime
Metric = (Curvature ~ 0)

Another fortuitous merging??

SR

RQM

Obscure QM Axioms:
Wave-Particle Duality
Unitary Evolution
Operator Formalism
Hilbert Space Representation
Principle of Superposition
Canonical Commutation Relation
Heisenberg Uncertainty Principle
Pauli Exclusion Principle (FD-statistics)
Bose Aggregation Principle (BE-statistics)
Hermitian Generators
Correspondence Principle to CM
Born Probability Interpretation
{ħ=h/2π} = physical constant

SR limiting-case:
$|\mathbf{v}| << c$

A fortuitous merging?

QM

CM

QM limiting-case:
$\{ \hbar|\nabla\cdot\mathbf{p}| << (\mathbf{p}\cdot\mathbf{p}) \}$ or
$\{ |\nabla\cdot\mathbf{k}| << (\mathbf{k}\cdot\mathbf{k}) \}$ or $\{ \psi \rightarrow \mathrm{Re}[\psi] \}$

It is known that QM + SR "join nicely" together to form RQM, but problems with RQM + GR...

New Paradigm: SRQM or [SR→QM]

QM derived from SR + a few empirical facts
Simple and fits the data

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John B. Wilson
SciRealm@aol.com
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Simple GR Axioms:
Principle of Equivalence
Invariant Interval Measure
Tensors describe Physics
SpaceTime Metric $g^{\mu\nu}$
{c,G} = physical constants

GR

GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$
Minkowski "Flat" SpaceTime
Metric = (Curvature ~ 0)

(properties)
SR 4-vector:
$\mathbf{R}=(ct,\mathbf{r})$ 4-Position
$\mathbf{U}=\gamma(c,\mathbf{u})$ 4-Velocity
$\mathbf{P}=(E/c,\mathbf{p})$ 4-Momentum
$\mathbf{K}=(\omega/c,\mathbf{k})$ 4-WaveVector
$\partial=(\partial_t/c,-\nabla)$ 4-Gradient

(relations)
SR 4-vector:
$\mathbf{R}\in$<Event>
$\mathbf{U}=d\mathbf{R}/d\tau$
$\mathbf{P}=(m_o)\mathbf{U}$
$\mathbf{K}=(1/\hbar)\mathbf{P}$
$\partial=(-i)\mathbf{K}$

~~Quantum Gravity ???~~

QFT

Multiple Particles

SR → **SRQM** → **RQM**

SR limiting-case:
$|\mathbf{v}| << c$

QM

CM

Derived RQM **Principles:**
Wave-Particle Duality
Unitary Evolution
Operator Formalism
Hilbert Space Representation
Principle of Superposition
Canonical Commutation Relation
Heisenberg Uncertainty Principle
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Hermitian Generators
{$\hbar=h/2\pi$} = physical constant

Derived QM **Principles:**
Correspondence Principle to CM
Born Probability Interpretation

QM limiting-case:
{ $\hbar|\nabla\cdot\mathbf{p}| << (\mathbf{p}\cdot\mathbf{p})$ } or { $\psi\rightarrow\mathrm{Re}[\psi]$ } or
{ $|\nabla\cdot\mathbf{k}| << (\mathbf{k}\cdot\mathbf{k})$ }
Change by a few quanta has negligible effect on overall state

This new paradigm explains why RQM "miraculously fits" SR, but not necessarily GR

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

New Paradigm: SRQM w/ EM

QM, EM, CM derived from SR + a few empirical facts

4-Vector SRQM Interpretation of QM

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John B. Wilson
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Simple GR Axioms:
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GR

GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$
Minkowski "Flat" SpaceTime
Metric = (Curvature ~ 0)

SR

(properties)
SR 4-vector & EM tensor:
$\mathbf{R}=(ct,\mathbf{r})$ $\quad \mathbf{A}=(\varphi/c,\mathbf{a})$
$\mathbf{U}=\gamma(c,\mathbf{u})$ $\quad \mathbf{J}=(c\rho,\mathbf{j})$
$\mathbf{P}=(E/c,\mathbf{p})$
$\mathbf{K}=(\omega/c,\mathbf{k})$ $\quad F^{\alpha\beta}=[\ 0\ ,-e^{0j}/c\ ]$
$\partial=(\partial_t/c,-\nabla)$ $\quad [+e^{i0}/c,-\varepsilon^{ij}{}_k b^k]$
$\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f})$
$\mathbf{N}=(nc,\mathbf{n})$

(relations)
SR 4-vector & EM tensor:
$\mathbf{R}\in$<Event> $\mathbf{A}=(\varphi_o/c^2)\mathbf{U}$
$\mathbf{U}=d\mathbf{R}/d\tau$ $\quad \mathbf{J}=(\rho_o)\mathbf{U}=(q)\mathbf{N}$
$\mathbf{P}=(m_o)\mathbf{U}$ $\quad \mathbf{K}=-\partial[\Phi_{phase}]$
$\mathbf{K}=(1/\hbar)\mathbf{P}$ $\quad F^{\alpha\beta}=\partial^\alpha A^\beta - \partial^\beta A^\alpha$
$\partial=(-i)\mathbf{K}$ $\quad \mathbf{U}\cdot F^{\alpha\beta}=(1/q)\mathbf{F}$
$\mathbf{F}=d\mathbf{P}/d\tau$ $\quad \partial\cdot F^{\alpha\beta}=(\mu_o)\mathbf{J}$
$\mathbf{N}=(n_o)\mathbf{U}$ $\quad \partial\cdot\mathbf{J}=0$

~~Quantum Gravity ???~~

SRQM

RQM QED
q=0
q≠0
A≠0

QFT

Multiple Particles

SR limiting-case:
$|\mathbf{v}| << c$

QM EM w/ spin
q=0
q≠0
A≠0

CM EM
q=0
q≠0
A≠0

Derived RQM **Principles:**
Wave-Particle Duality
Unitary Evolution
Operator Formalism
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This new paradigm explains why RQM "miraculously fits" SR, but not necessarily GR

(+,-,-,-) SR → QM Physics

Good But Limited Idea

A Tensor Study of Physical 4-Vectors

Classical SR w/ EM Paradigm (for comparison)

CM & EM derived from SR + a few empirical facts

4-Vector SRQM Interpretation of QM

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John B. Wilson
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Simple GR Axioms:
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Tensors describe Physics
SpaceTime Metric $g^{\mu\nu}$
{c,G} = physical constants

GR

GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$
Minkowski "Flat" SpaceTime
Metric = (Curvature ~ 0)

SR

(properties)
SR 4-vector & EM tensor:
$\mathbf{R}=(ct,\mathbf{r})$ $\mathbf{A}=(\varphi/c,\mathbf{a})$
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$\mathbf{K}=(\omega/c,\mathbf{k})$ $F^{\alpha\beta}=[\ 0\ ,-e^{0j}/c\ ]$
$\partial=(\partial_t/c,-\nabla)$ $[+e^{i0}/c,-\varepsilon^{ij}{}_k b^k]$
$\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f})$
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(relations)
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$\mathbf{P}=(m_o)\mathbf{U}$ $\mathbf{K}=-\partial[\Phi_{phase}]$
[redacted] $F^{\alpha\beta}=\partial^\alpha A^\beta - \partial^\beta A^\alpha$
$\partial=(-i)\mathbf{K}$ $\mathbf{U}\cdot F^{\alpha\beta}=(1/q)\mathbf{F}$
$\mathbf{F}=d\mathbf{P}/d\tau$ $\partial\cdot F^{\alpha\beta}=(\mu_o)\mathbf{J}$
$\mathbf{N}=(n_o)\mathbf{U}$ $\partial\cdot\mathbf{J}=0$

SR EM
q=0
q≠0
A≠0

SR limiting-case:
$|\mathbf{v}| << c$

CM EM
q=0
q≠0
A≠0

The entire classical SR→{EM,CM} structure is based on the limiting-case of quantum effects being negligible.

Notice that only the SR 4-Vector relation: **K=(1/ħ)P** is missing from the Classical Interpretation…

All of the SR 4-Vectors, including (**K** & ∂), are still present in the Classical setting.

K is used in the Relativistic Doppler Effect and EM waves.
∂ is used in the SR Conservation/Continuity Equations, Maxwell Equations, Hamilton-Jacobi, Lorenz Gauge, etc.
∂=(-i)**K** may be somewhat controversial, but it is the equation for complex plane-waves, which are still used in classical EM.

Background Inherent Assumption

QM limiting-case:
$\{\ \hbar|\nabla\cdot\mathbf{p}| << (\mathbf{p}\cdot\mathbf{p})\ \}$ or $\{\ \psi\rightarrow \mathrm{Re}[\psi]\ \}$ or
$\{\ |\nabla\cdot\mathbf{k}| << (\mathbf{k}\cdot\mathbf{k})\ \}_{(\text{doesn't depend on } \hbar)}$
Hamilton-Jacobi non-quantum limit
Change by a few quanta has negligible effect on overall state

This (Classical=non-QM) SR→{EM,CM} approximation-paradigm has been working successfully for decades...

SRQM = New Paradigm:
SRQM View as Venn Diagram Ranges of Validity

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

The SRQM view:
Each level (range of validity) is a subset of the larger level.

GR
General Relativity

SRQM
Special Relativity → Relativistic QM
GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$ Minkowski "Flat" SpaceTime = (Curvature ~ 0)

QM
Non-relativistic Quantum Mechanics
SRQM limiting-case: $|\mathbf{v}| << c$

CM
Classical Mechanics
QM limiting-case: $\hbar|\nabla\cdot\mathbf{p}| << (\mathbf{p}\cdot\mathbf{p})$
or $\psi \rightarrow \mathrm{Re}[\psi]$ or $|\nabla\cdot\mathbf{k}| << (\mathbf{k}\cdot\mathbf{k})$
Change by a few quanta has negligible effect on overall state

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
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(0,0)-Tensor S or S_o
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John B. Wilson
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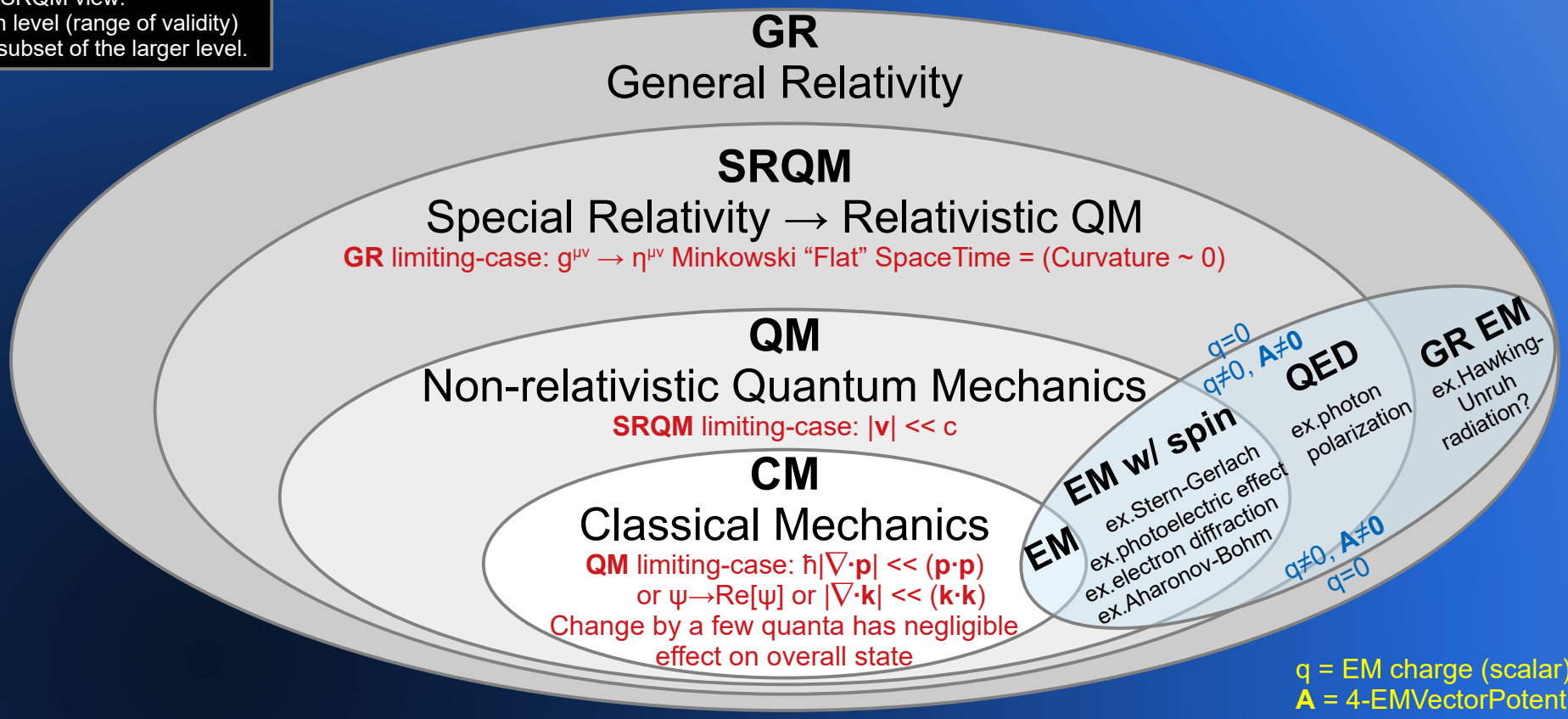

q = EM charge (scalar)
A = 4-EMVectorPotential

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM:
SR language beautifully expressed with Physical 4-Vectors

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
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Newton's laws of classical physics are greatly simplified by the use of physical 3-vector notation, which converts 3 separate space components, which may be different (*relative*) in various coordinate systems, into a single **invariant** object: a 3D vector, with an **invariant** 3D magnitude (but not 4D **invariant**). The basis-values of these components can differ in certain {*relativistic*} ways, via Galilean transforms, yet still refer to the same overall 3-vector object.

3-vector = 3D (1,0)-tensor
$\mathbf{a} = a^i = (a^i) = (\mathbf{a}) = (a^1,a^2,a^3)$

→ (a^x,a^y,a^z) Cartesian/Rectangular 3D basis
→ (a^r,a^θ,a^z) Polar/Cylindrical 3D basis
→ (a^r,a^θ,a^Φ) Spherical 3D basis

$\mathbf{a}\cdot\mathbf{a}=a^j\,\delta_{jk}\,a^k=(a^1)^2+(a^2)^2+(a^3)^2=|\mathbf{a}|^2$

The scalar products of either type: {3D,4D} are basis-independent. However, unlike the 3D magnitude² (only +)=Riemannian=positive-definite, the 4D magnitude² can be (+/0/-)=pseudo-Riemannian→CausalConditions

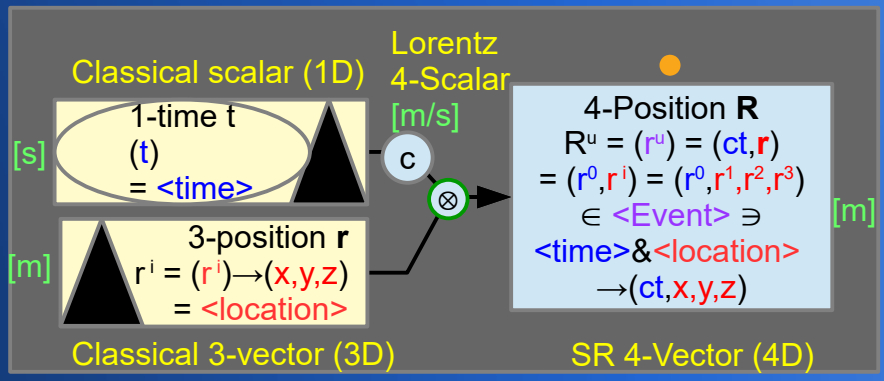
$\mathbf{A}\cdot\mathbf{A}=A^\mu\eta_{\mu\nu}A^\nu=(a^0)^2-\mathbf{a}\cdot\mathbf{a}=(a^0{}_o)^2$

4-Vector = 4D (1,0)-Tensor
$\mathbf{A} = A^\mu = (a^\mu) = (a^0,a^i) = (a^0,\mathbf{a}) = (a^0,a^1,a^2,a^3)$

→ (a^t,a^x,a^y,a^z) Cartesian/Rectangular 4D basis
→ (a^t,a^r,a^θ,a^z) Polar/Cylindrical 4D basis
→ $(a^t,a^r,a^\theta,a^\varphi)$ Spherical 4D basis

I style classical 3D objects this way (by a triangle/wedge Δ) to emphasize that they are actually just the separated components of SR 4-Vectors.

The triangle/wedge Δ (3 sides) represents splitting the components into a scalar and 3-vector.



SR is able to expand the concept of mathematical vectors into the Physical 4-Vector, which combines both (time) and (space) components into a single (Time·Space) object: These 4-Vectors are elements of Minkowski 4D SR SpaceTime. They have Lorentzian (*relative*) components but **invariant** 4D Magnitudes. There is a Speed-of-Light factor (c) in the temporal component to make the dimensional units match.
ex. $\mathbf{R} = (ct,\mathbf{r})$: overall dimensional units of [length] = SI Unit [m]
This also allows the 4-Vector name to match up with the 3-vector name.

In this presentation:
{Temporal,0th,+}=(+,-,-,-) metric signature, giving $\mathbf{A}\cdot\mathbf{A} = A^\mu\eta_{\mu\nu}A^\nu = [(a^0)^2 - \mathbf{a}\cdot\mathbf{a}] = (a^0{}_o)^2$
4-Vectors will use Upper-Case Letters, ex. **A**; 3-vectors will use lower-case letters, ex. **a**; I always put the (c) dimensional factor in the temporal component. Vectors of both types will be in **bold** font; components and scalars in normal font and usually lower-case. 4-Vector name will match with 3-vector name. Tensor form will usually be normal font with tensor indicies: { Greek Time·Space index (0,1..3): ex. $\mathbf{A} = A^\mu$ } or { Latin SpaceOnly index (1..3): ex. $\mathbf{a} = a^k$ }

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Classical (scalar 3-vector)
3D Galilean Invariant
not Lorentz Invariant

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SR 4-Vectors are primitive elements of Minkowski SpaceTime 4D←(1+3)D

SciRealm.org
John B. Wilson
SciRealm@aol.com
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We want to be clear, however, that SR 4-Vectors are **NOT** generalizations of Classical or Quantum 3-vectors.

SR 4-Vectors are the primitive elements of Minkowski SpaceTime (Time·Space) = 4D←(1+3)D, which incorporate both: a {temporal scalar element} and a {**spatial 3-vector element**} as components. Temporals and **Spatials** are metrically distinct, but can mix in SR.
4-Vector **A** = $A^\mu = (a^\mu) = (a^0,a^1,a^2,a^3) = (a^0,a^i = \mathbf{a}) \rightarrow (a^t,a^x,a^y,a^z)$ with component scalar $(a^0) \rightarrow (a^t)$ & component 3-vector $(a^i = \mathbf{a}) \rightarrow (a^x,a^y,a^z)$

It is the {Classical (Newtonian) or Quantum} 3-vector (**a**) which is a limiting-case approximation of the spatial part of SR 4-Vector (**A**) for { |**v**| << c }.

i.e. The energy (E) and 3-momentum (**p**) as "separate" entities occurs only in the low-velocity limit { |**v**| << c } of the Lorentz Boost Transform. They are actually part of a single 4D entity: the 4-Momentum **P** = (E/c,**p**); with the components: temporal energy (E), **spatial 3-momentum** (**p**), dependent on a frame-of-reference, while the overall 4-Vector **P** is invariant. Likewise with time (t), **space 3-position** (**r**) in the 4-Position **R**=(ct,**r**).

SR is 4D Minkowskian; obeys Lorentz⊂Poincaré Invariance.

CM is 3D Euclidean; obeys Galilean Invariance.

(E) can intermix with (**p**) via a Lorentz Boost Transformation $\Lambda^{\mu'}{}_\nu \rightarrow B^{\mu'}{}_\nu$

Spatial components can intermix via a Lorentz Rotation Transform $\Lambda^{\mu'}{}_\nu \rightarrow R^{\mu'}{}_\nu$

(t) can intermix with (**r**) via a Lorentz Boost Transformation $\Lambda^{\mu'}{}_\nu \rightarrow B^{\mu'}{}_\nu$

[kg·m/s] 4-Momentum **P**=(E/c,**p**) —Classical limiting-case |**v**| << c→ 4-Momentum$_{CM}$ $\mathbf{P}_{CM}$=(E/c, **p**)

Minkowski (1+3)D → 4D [TimeSpace] — Lorentz Invariant

Euclidean 1D [Time] + 3D [Space] — Galilean Invariant

[m] 4-Position **R**=(ct,**r**) —Classical limiting-case |**v**| << c→ 4-Position$_{CM}$ $\mathbf{R}_{CM}$=(ct, **r**)

c → (E) is totally independent of (**p**) only classically → energy E [kg·m²/s²]; 3-momentum **p** = $p^i \rightarrow (p^x,p^y,p^z)$ [kg·m/s]

1/c → (t) is totally independent of (**r**) only classically → time t [s]; 3-position **r** = $r^i \rightarrow (x,y,z)$ [m]

Spatial components can intermix via a Galilean (space-only) Rotation Transform $R^{i'}{}_k$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Classical (scalar 3-vector)
3D Galilean Invariant
not Lorentz Invariant

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM:
SR 4-Vectors & Lorentz Scalars
Rest Values ("naughts"=$_o$) are Lorentz Scalars

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

$\mathbf{A}\cdot\mathbf{A} = (a^0a^0 - \mathbf{a}\cdot\mathbf{a}) = (a^0{}_o)^2$, where $(a^0{}_o)$ is the rest-value, the value of the temporal coordinate when the spatial coordinate is zero ($\mathbf{a}\to\mathbf{0}$).
The "rest-values" of several physical properties are all invariant Lorentz scalars.

$\mathbf{P} = (mc,\mathbf{p})$ $\qquad$ $\mathbf{K} = (\omega/c,\mathbf{k})$
$\mathbf{P}\cdot\mathbf{P} = (mc)^2 - \mathbf{p}\cdot\mathbf{p}$ $\qquad$ $\mathbf{K}\cdot\mathbf{K} = (\omega/c)^2 - \mathbf{k}\cdot\mathbf{k}$
($\mathbf{P}\cdot\mathbf{P}$) and ($\mathbf{K}\cdot\mathbf{K}$) are Lorentz Scalars. We can choose a frame that may simplify the expressions.

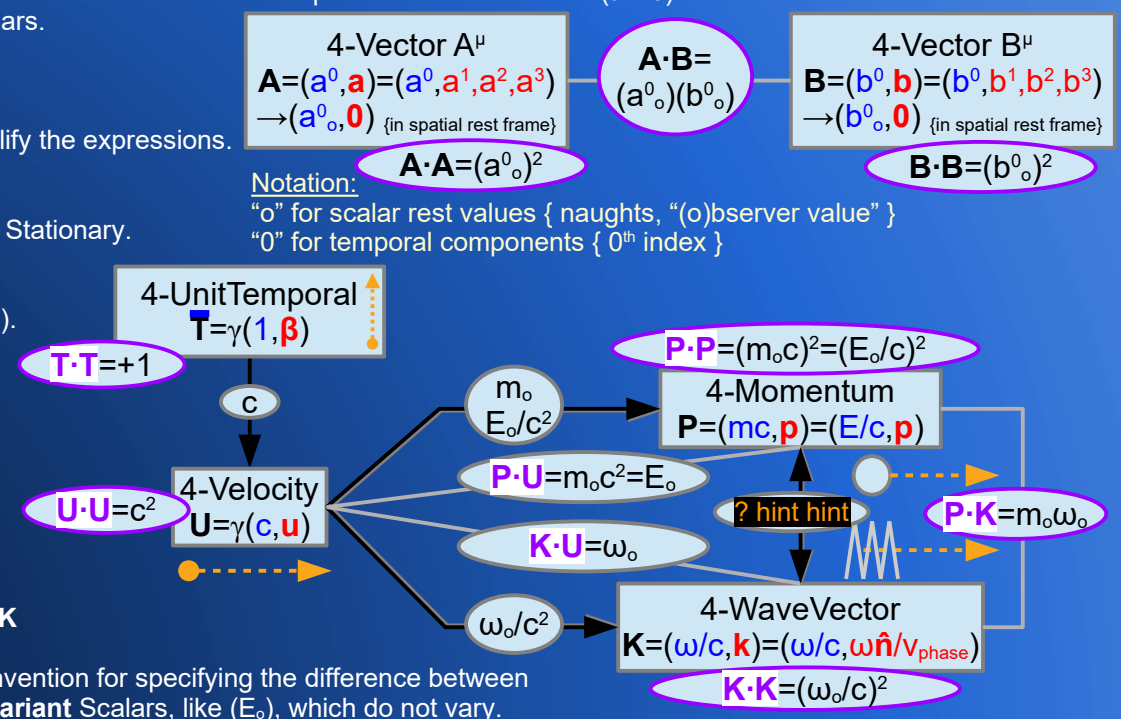
Choose a frame in which the spatial component is zero.
This is known as the "rest-frame" of the 4-Vector. It is not moving spatially = Stationary.

$\mathbf{P}\cdot\mathbf{P} = (mc)^2 - \mathbf{p}\cdot\mathbf{p} = (m_oc)^2$ $\qquad$ $\mathbf{K}\cdot\mathbf{K} = (\omega/c)^2 - \mathbf{k}\cdot\mathbf{k} = (\omega_o/c)^2$
The resulting simpler expressions then give the "rest values", indicated by ($_o$).
RestMass (m_o) and RestAngularFrequency (ω_o)
They are Invariant Lorentz Scalars by construction.

This leads to simple relations between 4-Vectors.
$\mathbf{P} = (m_o)\mathbf{U} = (E_o/c^2)\mathbf{U}$ $\qquad$ $\mathbf{K} = (\omega_o/c^2)\mathbf{U}$

And gives nice Scalar Product relations between 4-Vectors as well.
$\mathbf{P}\cdot\mathbf{U} = (m_o)\mathbf{U}\cdot\mathbf{U} = (m_o)c^2 = (E_o)$ $\qquad$ $\mathbf{K}\cdot\mathbf{U} = (\omega_o/c^2)\mathbf{U}\cdot\mathbf{U} = (\omega_o/c^2)c^2 = (\omega_o)$

$$\mathbf{P}\cdot\mathbf{K} = (m_o\omega_o) \rightarrow \mathbf{P} = (m_oc^2/\omega_o)\mathbf{K} = (E_o/\omega_o)\mathbf{K} \rightarrow \mathbf{P} = (\text{scalar invariant})\mathbf{K}$$



This property of SR equations is a very good reason to use the "naught" convention for specifying the difference between *relativistic* component values which can vary, like (E), versus RestValue **Invariant** Scalars, like (E_o), which do not vary.
They are usually related via a Lorentz Factor: { $m = \gamma m_o$; $E = \gamma E_o$; $\omega = \gamma\omega_o$ }, as seen in the relations of **P** , **K** , **U** , and **T**.

$$\mathbf{P} = (mc,\mathbf{p}) = (m_o)\mathbf{U} = (m_o)\gamma(c,\mathbf{u}) = (\gamma m_oc,\gamma m_o\mathbf{u}) = (mc,m\mathbf{u}) = (mc,\mathbf{p}) = (m_oc)\mathbf{T} = (m_oc)\gamma(1,\boldsymbol{\beta}) = (mc)(1,\boldsymbol{\beta})$$
$$\mathbf{P} = (E/c,\mathbf{p}) = (E_o/c^2)\mathbf{U} = (E_o/c^2)\gamma(c,\mathbf{u}) = (\gamma E_o/c,\gamma E_o\mathbf{u}/c^2) = (E/c,E\mathbf{u}/c^2) = (E/c,\mathbf{p}) = (E_o/c)\mathbf{T} = (E_o/c)\gamma(1,\boldsymbol{\beta}) = (E/c)(1,\boldsymbol{\beta})$$

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(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
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(0,0)-Tensor S or S_o
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$\text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study:
Manifest Invariance
Invariant SR 4-Vector Relations

4-Vector SRQM Interpretation of QM

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SciRealm@aol.com
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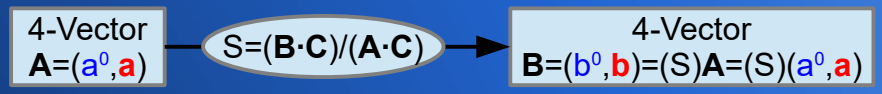
Relations among just tensors, ex. 4-Vectors and Lorentz 4-Scalars, are Manifestly Invariant, meaning that they are true in all inertial reference frames.

Consider a particle at a SpaceTime (Time·Space) <Event> that has properties described by 4-Vectors **A** and **B**:

One possible relationship is that the two 4-Vectors are related by a Lorentz 4-Scalar (S): ex. **B** = (S) **A**.
How can one determine this? Answer: Make an experiment that empirically measures the tensor invariant [**B·C** / **A·C**].
If **B** = (S) **A** then **B·C** = (S) **A·C**, giving (S) = [**B·C** / **A·C**]

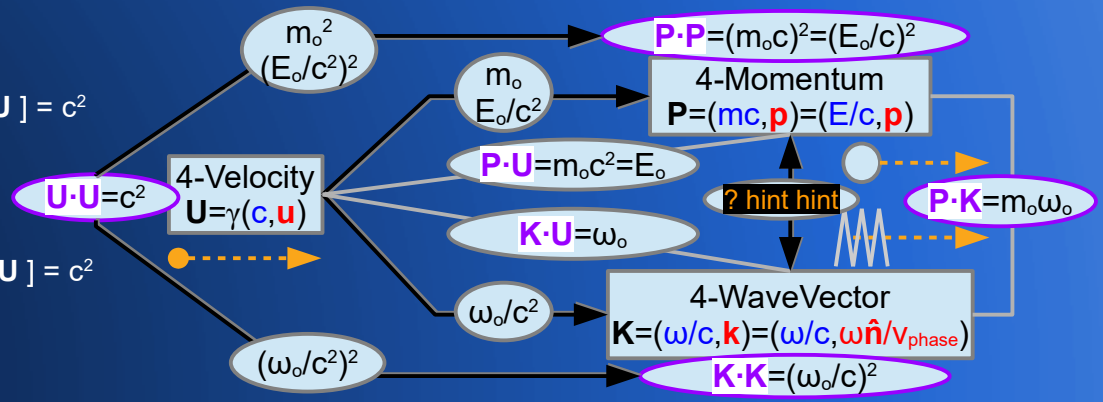
if **C=A**, then (S) = [**B·A** / **A·A**] This basically a standard vector projection.
if **C=other**, Invariant result mediated by another 4-Vector **C**, always possible.



Run the experiment many times. If you always get the same result for (S), then it is likely that the relationship is true, and thus invariant.

Example: Measure (S_P) = [**P·U** / **U·U**] for a given particle type.
Repeated measurement always give (S_P) = m_o
This makes sense because we know [**P·U**] = $\gamma(E - \mathbf{p}\cdot\mathbf{u}) = E_o$ and [**U·U**] = c^2
Thus, 4-Momentum **P** = (E_o/c^2)**U** = (m_o)**U** = (m_o)*4-Velocity **U**

Example: Measure (S_K) = [**K·U** / **U·U**] for a given particle type.
Repeated measurement always give (S_K) = (ω_o/c^2)
This makes sense because we know [**K·U**] = $\gamma(\omega - \mathbf{k}\cdot\mathbf{u}) = \omega_o$ and [**U·U**] = c^2
Thus, 4-WaveVector **K** = (ω_o/c^2)**U** = (ω_o/c^2)*4-Velocity **U**



Since **P** and **K** are both related to **U**, this would also mean that the
4-Momentum **P** is related to the 4-WaveVector **K** in a particular Lorentz Invariant manner for each given particle type… a major hint for later...

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

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(1,0)-Tensor V^{μ} = **V** = $(v^0,\mathbf{v})$
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(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SR 4-Vectors & Lorentz Scalars
Frame-Invariant Equations
SRQM Diagramming Method

4-Vector SRQM Interpretation of QM

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4-Vectors are 4D (1,0)-Tensors, Lorentz 4-Scalars are 4D (0,0)-Tensors, 4-CoVectors are 4D (0,1)-Tensors, (m,n)-Tensors have (m) # upper-indices and (n) # lower-indices V^μ, S, C_μ, $T^{\alpha\beta\gamma..\{m\ indices\}}{}_{\mu\nu..\{n\ indices\}}$

Any equation which employs only Tensors, such as those with only 4-Vectors and Lorentz 4-Scalars, (ex. $\mathbf{P} = P^\mu = m_o\mathbf{U} = m_oU^\mu$) is automatically Frame-Invariant, or coordinate-frame-independent. One's *frame-of-reference* plays no role in the form of the overall equations. This is also known as being "Manifestly-Invariant", not showing inner components. This is exactly what Einstein meant by his postulate: "The laws of physics should have the same form for all inertial observers". Use of the RestFrame-naught ($_o$) helps show this. It is seen when the **spatial** part ($\mathbf{v}$) of a magnitude can be set to zero (= at-rest). The temporal part (v^0) would then equal the rest value ($v^0{}_o$).

4-Vector = 4D (1,0)-Tensor

$$\mathbf{V} = \overline{\mathbf{V}} = V^\mu = (v^\mu) = (v^0,v^i) = (v^0,\mathbf{v}) = (v^0,v^1,v^2,v^3) \rightarrow (v^t,v^x,v^y,v^z)_{\{rectangular\ basis\}}$$
$$\rightarrow (v^0{}_o,\mathbf{0})_{\{Temporal\ Interval,\ Spatially\ At\text{-}Rest,\ Stationary\}}$$
$$\rightarrow (0,\mathbf{v}_o)_{\{Spatial\ interval,\ Temporally\ At\text{-}Rest,\ Simultaneous\}}$$

$$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = (v^0)^2 - \mathbf{v}\cdot\mathbf{v}$$
$$= (v^0{}_o)^2 \text{ Temporal Interval, Spatially At-Rest, Stationary}$$
$$= -(|\mathbf{v}_o|)^2 = (i|\mathbf{v}_o|)^2 \text{ Spatial Interval, Temporally At-Rest, Simultaneous}$$

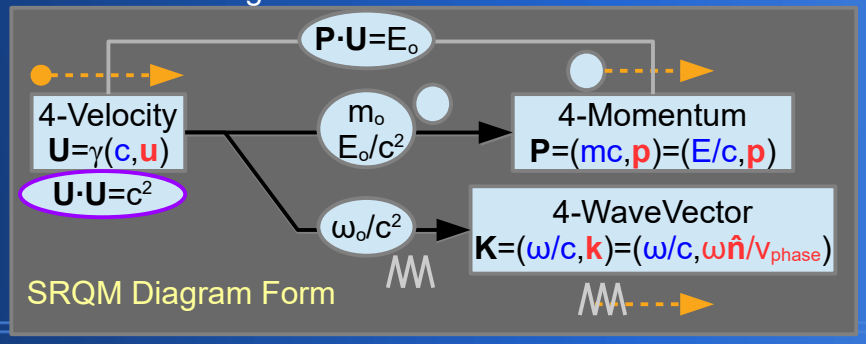
The components (v^0,v^1,v^2,v^3) of the 4-Vector $\mathbf{V}$ can *relativistically* vary depending on the observer and their choice of coordinate system, but the 4-Vector $\mathbf{V} = V^\mu$ itself is **invariant**. Equations using only 4-Tensors, 4-Vectors, and Lorentz 4-Scalars are true for all inertial observers. The SRQM Diagramming Method makes this easy to see in a visual format, and will be used throughout this treatise. The following examples are SR Time·Space frame-invariant equations:

$$\mathbf{U}\cdot\mathbf{U} = (c)^2$$
$$\mathbf{U} = \gamma(c,\mathbf{u})$$
$$\mathbf{P} = (mc,\mathbf{p}) = (E/c,\mathbf{p}) = m_o\mathbf{U} = (E_o/c^2)\mathbf{U}$$
$$\mathbf{K} = (\omega/c,\mathbf{k}) = (\omega/c,\omega\hat{\mathbf{n}}/v_{phase}) = (\omega_o/c^2)\mathbf{U}$$
$$\mathbf{P}\cdot\mathbf{U} = E_o$$

Equation Form

The SRQM Diagram Form, on the right, has all of the info of the Equation Form, but shows overall relationships and symmetries among the 4-Vectors much more clearly.

Blue: Temporal components
Red: Spatial components
Purple: Mixed Time·Space components



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Some SR Mathematical Tools

Definitions, Approximations, Misc.

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$\boldsymbol{\beta} = \mathbf{v}/c$; $\beta = |\boldsymbol{\beta}|$: dimensionless Velocity Beta Factor { β=(0..1); rest at (β=0); speed-of-light (c) at (β=1) }

$\gamma = 1/\sqrt{[1-\beta^2]} = 1/\sqrt{[1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: dimensionless Lorentz Relativistic Gamma Factor { γ=(1..∞); rest at (γ=1); speed-of-light (c) at (γ=∞) }

$(1+x)^n \sim (1 + nx + O[x^2])$ for { $|x| << 1$ } Approximation used for SR→Classical limiting-cases, typically used on the Relativistic Gamma γ

Lorentz Transformation $\Lambda^{\mu'}{}_{\nu} = \partial X^{\mu'}/\partial X^{\nu} = \partial_{\nu}[X^{\mu'}]$: a relativistic frame-shift, such as a (spatial) Rotation or (spacetime) Velocity-Boost.
It transforms a 4-Vector in the following way: $X^{\mu'} = \Lambda^{\mu'}{}_{\nu} X^{\nu}$: with Einstein summation over the paired indices, and the (') indicating an alternate frame.
A typical Lorentz Boost Transformation $\Lambda^{\mu'}{}_{\nu} \rightarrow B^{\mu'}{}_{\nu}$ for a linear-velocity frame-shift (x,t)-Boost in the x̂-direction:

Lorentz x-Boost Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow B^{\mu'}{}_{\nu}$ =

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \gamma & -\beta\gamma & 0 & 0 \\ x & -\beta\gamma & \gamma & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array} = \begin{array}{c|cccc} & t & x & y & z \\ \hline t & \cosh[w] & -\sinh[w] & 0 & 0 \\ x & -\sinh[w] & \cosh[w] & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array} \leftarrow \begin{array}{cc} \gamma & -\gamma\beta_j \\ -\gamma\beta^i & (\gamma-1)\beta^i\beta_j/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\delta^i_j \end{array}$$

General Time-Space Boost

Symmetric Mixed 4-Tensor

SR:Minkowski Metric

$\partial[\mathbf{R}] = \partial^{\mu}[R^{\nu}] = \eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu} \rightarrow$

$\mathrm{Diag}[+1,-1,-1,-1] = \mathrm{Diag}[1,-I_{(3)}] = \mathrm{Diag}[1,-\delta^{jk}]$

{in Cartesian form} "Particle Physics" Convention

$\{\eta_{\mu\mu}\} = 1/\{\eta^{\mu\mu}\} : \eta_{\mu}{}^{\nu} = \delta_{\mu}{}^{\nu}$ $\mathrm{Tr}[\eta^{\mu\nu}]=4$

SR:Lorentz Transform

$\partial_{\nu}[R^{\mu'}] = \partial R^{\mu'}/\partial R^{\nu} = \Lambda^{\mu'}{}_{\nu}$

$\Lambda^{\mu}{}_{\nu} = (\Lambda^{-1})_{\nu}{}^{\mu} : \Lambda^{\mu}{}_{\alpha}(\Lambda^{-1})^{\alpha}{}_{\nu} = \eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$

$\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$

$\mathrm{Det}[\Lambda^{\mu}{}_{\nu}]=\pm1$ $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$

$\mathrm{Tr}[\Lambda^{\mu}{}_{\nu}]=\{-\infty..+\infty\}$

=Lorentz Transform Type

Original $A^{\nu} = (a^t, a^x, a^y, a^z)$ {for x̂-boost Lorentz Transform}

Boosted $A^{\mu'} = (a^t, a^x, a^y, a^z)' = \Lambda^{\mu'}{}_{\nu}A^{\nu} \rightarrow B^{\mu'}{}_{\nu}A^{\nu} = (\gamma a^t - \gamma\beta a^x, -\gamma\beta a^t + \gamma a^x, a^y, a^z)$

$\mathbf{A'\cdot B'} = (\Lambda^{\mu'}{}_{\nu}A^{\nu})\cdot(\Lambda^{\rho'}{}_{\sigma}B^{\sigma}) = \mathbf{A\cdot B} = A^{\mu}\eta_{\mu\nu}B^{\nu} = (a^0b^0 - a^1b^1 - a^2b^2 - a^3b^3) = (a^0b^0 - \mathbf{a\cdot b}) = (a^0{}_o b^0{}_o)$

$= A^{\mu}B_{\mu} = \Sigma_{u=0..3}[a^u b_u] = (a^0b_0 + a^1b_1 + a^2b_2 + a^3b_3)$

$= A_{\nu}B^{\nu} = \Sigma_{v=0..3}[a_v b^v] = (a_0b^0 + a_1b^1 + a_2b^2 + a_3b^3)$

using the Einstein Summation Convention where upper:lower paired-indices are summed over.

SpaceTime

$\partial\cdot\mathbf{R} = \partial_{\mu}R^{\mu} = 4$

Dimension

$\partial[\mathbf{X}] = \partial^{\mu}[X^{\nu}] = (\partial_t/c, -\nabla)(ct, \mathbf{x}) = \mathrm{Diag}[\partial_t/c[ct], -\nabla[\mathbf{x}]] = \mathrm{Diag}[1, -\mathbf{I}_{(3)}] = \mathrm{Diag}[1,-1,-1,-1] = \eta^{\mu\nu}$ Minkowski "Flat" SpaceTime Metric

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= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: Ordering of Time·Space <Events>

Temporal Causality vs. Spatial Topology

Simultaneity vs. Stationarity

Venn Diagram

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
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Properties of Minkowski:SR SpaceTime <Events>

Time-Like Ordering of…

Time-Like Separated <Events>

Time-Like Invariant Interval
$\mathbf{\Delta R}\cdot\mathbf{\Delta R}=(c\Delta t)^2 - \mathbf{\Delta r}\cdot\mathbf{\Delta r} \rightarrow +(c\Delta\tau)^2$

Causal: **Invariant** = Absolute Temporal Order (A→B→C)
{ ProperTime ($t_o = \tau$) for | clock at-rest | }
{ Time Dilation ($t = \gamma t_o = \gamma\tau$) for ...←| moving clock |→ }
All observers agree on temporal order of time-separated events, although temporal event separation may be ←|Time-Dilated|→.

Light-Like (Null) Separated <Events>

$\mathbf{U}\cdot\mathbf{U}=c^2$

Light-Like Invariant Interval
$\mathbf{\Delta R}\cdot\mathbf{\Delta R}=(c\Delta t)^2 - \mathbf{\Delta r}\cdot\mathbf{\Delta r} \rightarrow 0$

Causal: **Invariant** = Absolute Temporal Order (A→B→C)
All observers agree on temporal order of light-separated events, and on the invariant TimeSpace <Event> interval measurement.
All observers measure invariant LightSpeed (c) in their own frames.

Space-Like Separated <Events>

Space-Like Invariant Interval
$\mathbf{\Delta R}\cdot\mathbf{\Delta R}=(c\Delta t)^2 - \mathbf{\Delta r}\cdot\mathbf{\Delta r} \rightarrow -(|\mathbf{\Delta r_o}|)^2$

Non-Causal: *Relative* → Relativity of Simultaneity (A←?→B)
Simultaneity: (only if in reference-frame with Same-Time occurrence)
("no wait" for simultaneous events, "wait" in all other reference frames)
Any 2 space-separated <events> may occur in any temporal order = frame-dependent

Temporal (+)

Null (0)

Spatial (−)

Space-Like Ordering of…

Time-Like Separated <Events>

Non-Topological: *Relative* → Relativity of Stationarity (A←?→B)
Stationarity: (only if in reference-frame with Same-Place occurrence)
("no motion" for stationary particle/worldline, "motion" in all other frames)
Any 2 time-separated <events> may occur in any spatial order = frame-dependent

Light-Like (Null) Separated <Events>

$\mathbf{U}\cdot\mathbf{U}=c^2$

Topological: **Invariant** = Absolute Spatial Order (A→B→C)
All observers agree on spatial order/topology of light-separated events, and on the invariant TimeSpace <Event> interval measurement.
All observers measure invariant LightSpeed (c) in their own frames.

Space-Like Separated <Events> (co-linear)

Topological: **Invariant** = Absolute Spatial Order (A→B→C) or (C→B→A) by rotation|reflection
{ ProperLength (L_o) for | ruler at-rest | }
{ Length Contraction ($L = L_o/\gamma$) for ...|→ moving ruler ←| }
All observers agree on spatial order/topology of space-separated events, although spatial event separation may be |→Length-Contracted←|.

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(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

4-Displacement (between <events>)
$\mathbf{\Delta R}=\Delta R^{\mu}=(c\Delta t,\mathbf{\Delta r})=\mathbf{R_2}-\mathbf{R_1}$ {finite}
$\mathbf{dR}=dR^{\mu}=(cdt,\mathbf{dr})$ {infintesimal}

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
Special Relativity via 4-Vectors

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Focus on a few of the main SR Physical 4-Vectors:

4-Position
$\mathbf{R}=R^\mu=(r^\mu)=(r^0,r^i)=(ct,\mathbf{r})\in$ <Event>
$=(r^0,r^1,r^2,r^3)\rightarrow(ct,x,y,z)$

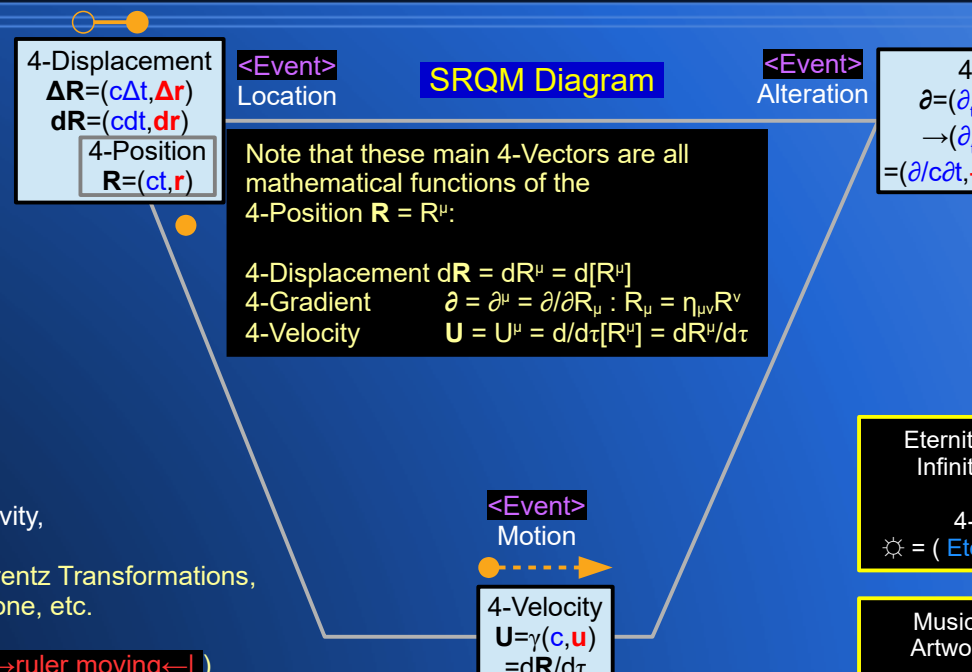
<Event> Location

4-Velocity
$\mathbf{U}=U^\mu=dR^\mu/d\tau=(u^\mu)=(u^0,u^i)=\gamma(c,\mathbf{u})$
$=(u^0,u^1,u^2,u^3)\rightarrow\gamma(c,u^x,u^y,u^z)$

<Event> Motion

4-Gradient
$\partial=\partial_R=\partial^\mu=\partial/\partial R_\mu=(\partial^\mu)=(\partial^0,\partial^i)=(\partial_t/c,-\nabla)$
$=(\partial^0,\partial^1,\partial^2,\partial^3)\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

<Event> Alteration



Note that these main 4-Vectors are all mathematical functions of the 4-Position $\mathbf{R} = R^\mu$:

4-Displacement $d\mathbf{R} = dR^\mu = d[R^\mu]$
4-Gradient $\partial = \partial^\mu = \partial/\partial R_\mu : R_\mu = \eta_{\mu\nu}R^\nu$
4-Velocity $\mathbf{U} = U^\mu = d/d\tau[R^\mu] = dR^\mu/d\tau$

These 4-Vectors give some of the main classical results of Special Relativity, including 4D SR:Minkowski Space concepts like:
The 'flat' Minkowski Metric, SpaceTime (Time·Space) Dimension = 4, Lorentz Transformations, <Events>, Invariant Interval Measure, Minkowski Diagrams, The Light-Cone, etc.

Relativity: Time Dilation (←| clock moving |→), Length Contraction (|→ruler moving←|)
Invariants: Proper Time (| clock at rest |) , Proper Length (| ruler at rest |)

Temporal 1D Ordering of:{ (Time-like <event> separations)=Causality is **Absolute** , (Space-like <event> separations)→Simultaneity is *Relative* }
Spatial 3D Ordering of:{ (Time-like <event> separations)→Stationarity is *Relative* , (Space-like <event> separations)=Topology is **Absolute** }

Use of the Lorentz Scalar Product to make Lorentz Invariants, Continuity Equations, etc.
The Invariant Speed-of-Light (c), Invariant Proper:Rest Measurement (Time & Space), Relative Components of 4-Vectors=Lorentz Covariance.
Invariant SR Wave Equations, via the d'Alembertian (Lorentz Scalar Product of 4-Gradient with itself), leads to a 4-WaveVector **K** solution.

Eternity is to time as Infinity is to space

4-Universe
☼ = (Eternity , **Infinity**)

Music is to time as Artwork is to space

4-Creativity
☼ = (Music , **Artwork**)

SR is a theory about the relations between 4D Time·Space <Events>, ie. how their intervals are "measured"

SR 4-Tensor
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The Basis of most all Classical SR Physics is in the SR Minkowski Metric of "Flat" SpaceTime $\eta^{\mu\nu} = \partial^\mu[R^\nu] = \partial[\mathbf{R}]$, which is generated from the 4-Gradient $\partial = \partial^\mu$ and 4-Position $\mathbf{R} = R^\nu$ and and determines the invariant measurement interval $\mathbf{R}\cdot\mathbf{R} = R^\mu\eta_{\mu\nu}R^\nu$ between <Events>.

This Minkowski Metric $\eta^{\mu\nu}$ provides the relations between the 4-Vectors of SR: 4-Position $\mathbf{R} = R^\mu$, 4-Gradient $\partial = \partial^\mu$, 4-Velocity $\mathbf{U} = U^\mu$

The Tensor **Invariants** of these 4-Vectors give the:
Invariant Interval Measures → Causality:Topology, from $\mathbf{R}\cdot\mathbf{R}$
Invariant |Magnitude| LightSpeed (c), from $\mathbf{U}\cdot\mathbf{U}$
Invariant d'Alembertian Wave Equation & 4-WaveVector $\mathbf{K}$, from $\partial\cdot\partial$

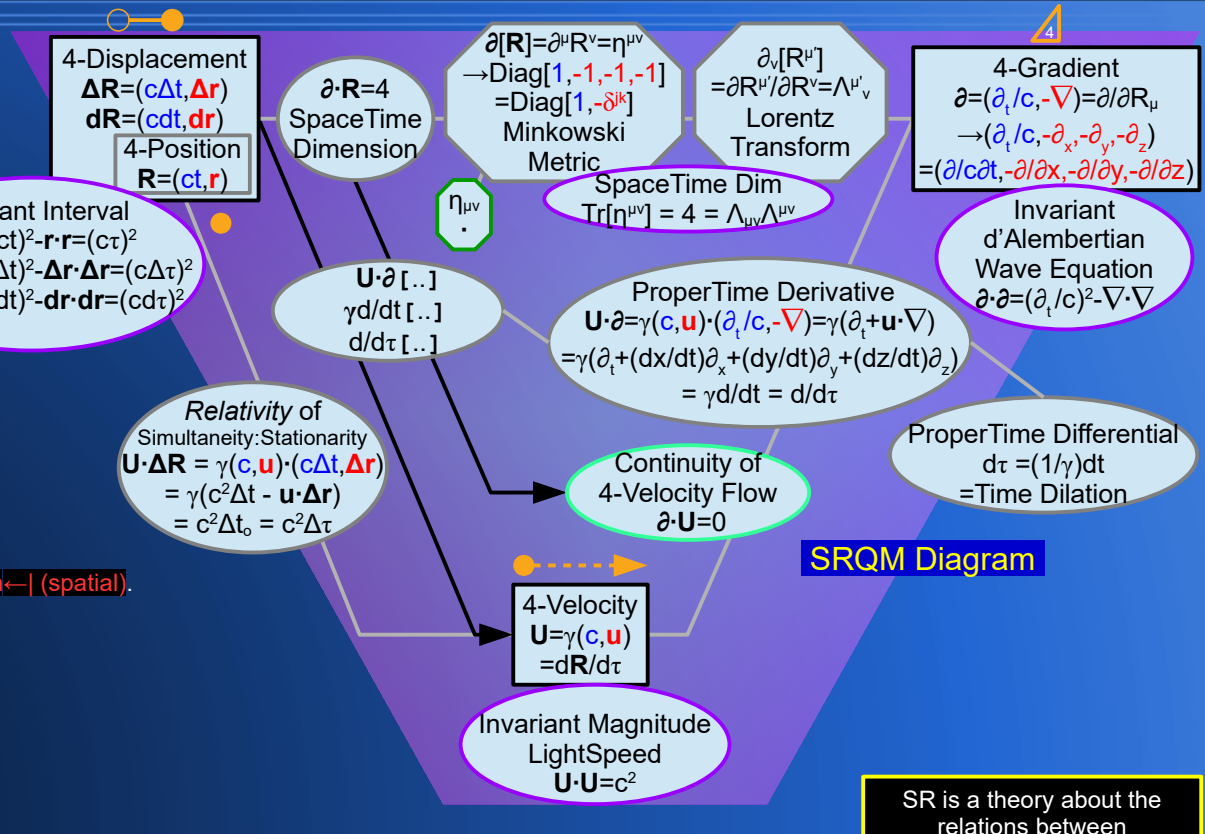
The relation between 4-Gradient ∂ and 4-Position $\mathbf{R}$ gives the Dimension of SpaceTime = (4), the Minkowski Metric ($\eta^{\mu\nu}$), and the Lorentz Transformations ($\Lambda^{\mu'}{}_\nu$).

The relation between 4-Gradient ∂ and 4-Velocity $\mathbf{U}$ gives the **invariant** ProperTime Derivative ($d/d\tau$). Rearranging gives the **invariant** ProperTime Differential ($d\tau$), which gives *relativistic* ←|Time Dilation|→ (temporal) & |→Length Contraction←| (spatial).

The ProperTime Derivative $d/d\tau$:
acting on 4-Position $\mathbf{R}$ gives 4-Velocity $\mathbf{U}$
acting on the SpaceTime Dimension Lorentz Scalar gives the Continuity of 4-Velocity Flow.

The relation between 4-Displacement $\mathbf{\Delta R}$ and 4-Velocity $\mathbf{U}$ gives *Relativity* of Simultaneity:Stationarity.

One of the most important properties is the Tensor Invariant Lorentz Scalar Product (dot = ·), provided by the lowered- index form of the Minkowski Metric $\eta_{\mu\nu}$.



From here, each object will be examined in turn...

SR is a theory about the relations between 4D Time·Space <Events>, ie. how their intervals are "measured"

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(+,-,-,-) SR → QM Physics

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SRQM Diagram:
The Basis of Classical SR Physics
4-Position, 4-Displacement, 4-Differential

4-Vector SRQM Interpretation of QM

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4-Displacement $\Delta R^\mu = \mathbf{\Delta R} = (c\Delta t, \mathbf{\Delta r}) = \mathbf{U}\Delta\tau = \mathbf{R_2} - \mathbf{R_1} = (ct_2 - ct_1, \mathbf{r_2} - \mathbf{r_1})$: {finite}
4-Differential $dR^\mu = \mathbf{dR} = (cdt, \mathbf{dr}) = \mathbf{U}d\tau$: {infintesimal}
4-Position $R^\mu = \mathbf{R} = (ct, \mathbf{r}) = (c{*}\text{when}, \mathbf{where}) = (r^\mu) \in$ <Event> $\rightarrow (ct, x, y, z)$

The 4-Position **R** = (ct,**r**) {alt. notation = **X**} is essentially one of the most fundamental 4-Vectors of SR.
It is the SpaceTime location of an <Event>,
the basic element (∈) of Minkowski 4D SpaceTime:
a time (t) & a place (**r**) → (c*when,**where**) = (ct,**r**) = (r^μ) = **R**.
Technically, the 4-Position is just one of the possible properties of an <Event>, which may also have a 4-Velocity, 4-Momentum, 4-Spin, etc.
I emphasize the 4-Position **R** as "∈" of an <Event> since that is its most basic property.

The 4-Position **R** = (ct,**r**) relates time to **space** via the fundamental invariant physical constant (c): the Speed-of-Light = "(c)elerity ; (c)eleritas", which is used to give consistent dimensional units across all SR 4-Vector components.

The 4-Position is a specific type of 4-Displacement, for which one of the endpoints is the <Origin> = 4-Zero **Z** = 4-Origin **O** = (c*now,**here**).

$\mathbf{R_2} \rightarrow \mathbf{R}$, $\mathbf{R_1} \rightarrow \mathbf{O}$
$\mathbf{\Delta R} = \mathbf{R_2} - \mathbf{R_1} \rightarrow \mathbf{R} - \mathbf{O} = \mathbf{R}$

4-Zero **Z**, 4-Origin **O**
$0^\mu = (0, 0^j) = (0, \mathbf{0}) = (0,0,0,0) = (c{*}\text{now}, \mathbf{here}) = (0^\mu) \in$ <Origin>

As such, any "defined" 4-Position, like the 4-Zero, is Lorentz Invariant (point rotations and boosts), but not Poincaré Invariant (Lorentz + time & space translations), since the translations can move it.

The more general 4-Displacement and 4-Differential(Displacement) are invariant under both Lorentz and Poincaré transformations, since neither of their endpoints are "pinned" this way.

The 4-Differential(Displacement) is just the infinitesimal version of the finite 4-Displacement, and is used in the calculus of SR. $\mathbf{U} = d\mathbf{R}/d\tau$: $d\mathbf{R} = \mathbf{U}d\tau$

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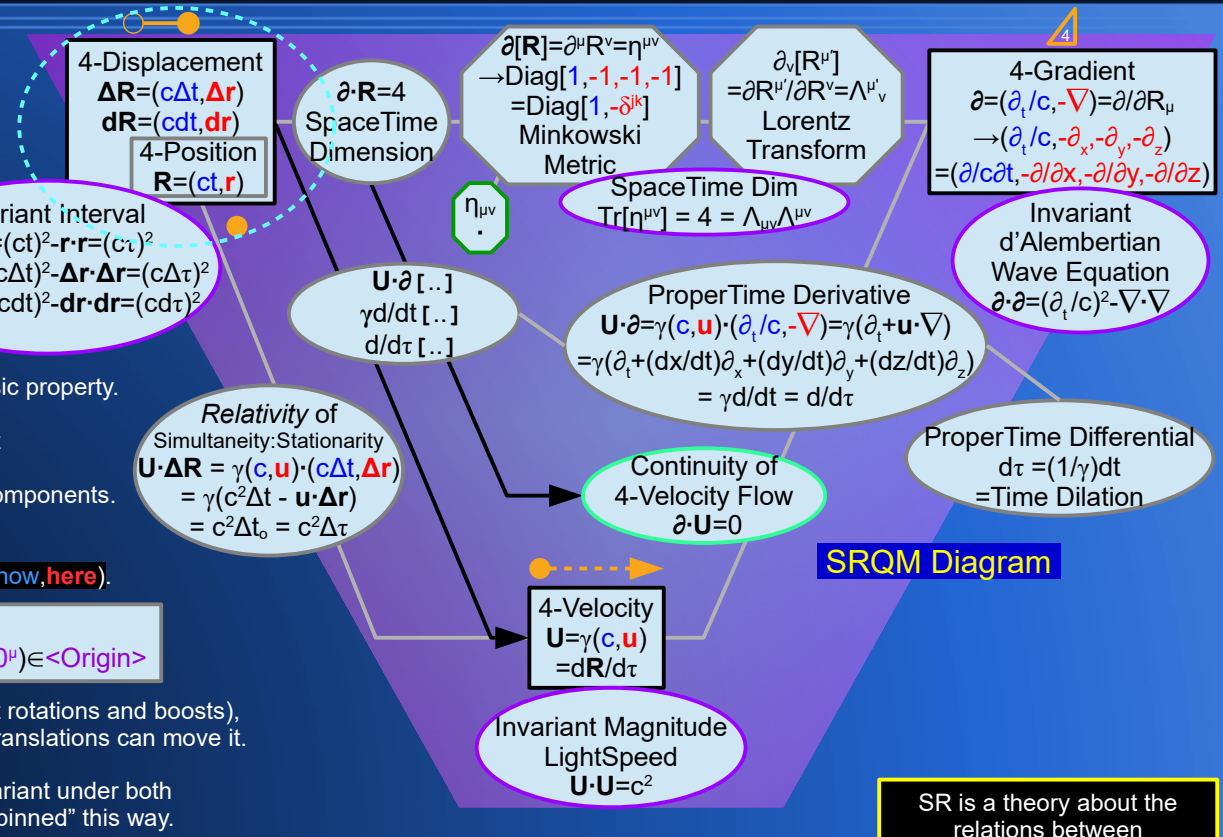
Lorentz Scalar

4-Position $\mathbf{R} = R^\mu = (ct, \mathbf{r}) = (r^\mu) \in$ <Event>
$\mathbf{R} = \int d\mathbf{R} = \int \mathbf{U}d\tau = \int \gamma(c,\mathbf{u})d\tau = \int (c,\mathbf{u})\gamma d\tau = \int (c,\mathbf{u})dt = (ct,\mathbf{r})$
$\mathbf{R} = \Sigma\Delta\mathbf{R} = \Sigma\mathbf{U}\Delta\tau = \Sigma\gamma(c,\mathbf{u})\Delta\tau = \Sigma(c,\mathbf{u})\gamma\Delta\tau = \Sigma(c,\mathbf{u})\Delta t = (ct,\mathbf{r})$



SR is a theory about the relations between 4D Time·Space <Events>, ie. how their intervals are "measured"

$\text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
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Invariant Intervals, Time·Space
Causality (time), LightSpeed, Topology (space)

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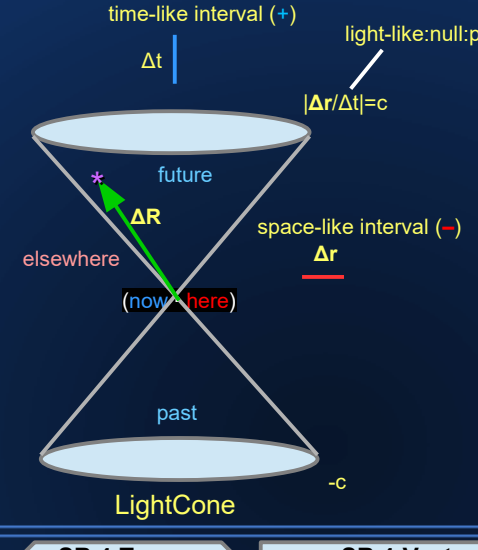
4-Displacement $\Delta R^\mu = \mathbf{\Delta R} = (c\Delta t, \mathbf{\Delta r}) = \mathbf{U}\Delta\tau = \mathbf{R_2}-\mathbf{R_1} = (ct_2-ct_1, r_2-r_1)$: {finite}

4-Differential $dR^\mu = \mathbf{dR} = (cdt, \mathbf{dr}) = \mathbf{U}d\tau$: {infintesimal}

4-Position $R^\mu = \mathbf{R} = (ct, \mathbf{r}) = (c*\text{when}, \text{where}) = (r^\mu) \in \langle\text{Event}\rangle \to (ct, x, y, z)$

The Invariant Interval is the Lorentz Scalar Product of the {4-Position, 4-Displacement, 4-Differential} with itself, giving a magnitude-squared, which may be (+/0/–).

$$\mathbf{R\cdot R} = (ct)^2 - \mathbf{r\cdot r} = (ct_o)^2 = (c\tau)^2 = -(r_o)^2 = (ir_o)^2$$
$$\mathbf{\Delta R\cdot\Delta R} = (c\Delta t)^2 - \mathbf{\Delta r\cdot\Delta r} = (c\Delta t_o)^2 = (c\Delta\tau)^2 = -(\Delta r_o)^2 = (i\Delta r_o)^2$$
$$\mathbf{dR\cdot dR} = (cdt)^2 - \mathbf{dr\cdot dr} = (cdt_o)^2 = (cd\tau)^2 = -(dr_o)^2 = (idr_o)^2$$



The 4D SpaceTime Intervals are Invariant:
meaning that all observers must agree on their magnitudes, regardless of differing reference frames. This leads to the idea of ProperTime ($\Delta\tau = \Delta t_o$), which is the time-displacement measured by a clock at-rest, and ProperLength ($L_o = |\Delta x_o|$), which is the space-displacement measured by a ruler at-rest.
This also leads to the various Causality Conditions of SR, and the concept of the (Minkowski Diagram) Light-Cone. The differential form **dR·dR** is apparently also still true in the curved spacetime of GR.

4-Displacement
$\mathbf{\Delta R} = (c\Delta t, \mathbf{\Delta r})$
$\mathbf{dR} = (cdt, \mathbf{dr})$

4-Position
$\mathbf{R} = (ct, \mathbf{r})$

Invariant Interval
$\mathbf{R\cdot R} = (ct)^2 - \mathbf{r\cdot r} = (c\tau)^2$
$\mathbf{\Delta R\cdot\Delta R} = (c\Delta t)^2 - \mathbf{\Delta r\cdot\Delta r} = (c\Delta\tau)^2$
$\mathbf{dR\cdot dR} = (cdt)^2 - \mathbf{dr\cdot dr} = (cd\tau)^2$

$\partial\cdot\mathbf{R} = 4$ SpaceTime Dimension

$\partial[\mathbf{R}] = \partial^\mu R^\nu = \eta^{\mu\nu} \to \mathrm{Diag}[1,-1,-1,-1] = \mathrm{Diag}[1,-\delta^{jk}]$ Minkowski Metric

$\partial_\nu[R^\mu'] = \partial R^{\mu'}/\partial R^\nu = \Lambda^{\mu'}{}_\nu$ Lorentz Transform

$\eta_{\mu\nu}$

SpaceTime Dim $\mathrm{Tr}[\eta^{\mu\nu}] = 4 = \Lambda_{\mu\nu}\Lambda^{\mu\nu}$

4-Gradient
$\partial = (\partial_t/c, -\nabla) = \partial/\partial R_\mu \to (\partial_t/c, -\partial_x, -\partial_y, -\partial_z) = (\partial/c\partial t, -\partial/\partial x, -\partial/\partial y, -\partial/\partial z)$

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

ProperTime Derivative
$\mathbf{U}\cdot\partial = \gamma(c,\mathbf{u})\cdot(\partial_t/c, -\nabla) = \gamma(\partial_t + \mathbf{u}\cdot\nabla) = \gamma(\partial_t + (dx/dt)\partial_x + (dy/dt)\partial_y + (dz/dt)\partial_z) = \gamma d/dt = d/d\tau$

Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\mathbf{\Delta R} = \gamma(c,\mathbf{u})\cdot(c\Delta t, \mathbf{\Delta r}) = \gamma(c^2\Delta t - \mathbf{u}\cdot\mathbf{\Delta r}) = c^2\Delta t_o = c^2\Delta\tau$

Continuity of 4-Velocity Flow
$\partial\cdot\mathbf{U} = 0$

ProperTime Differential
$d\tau = (1/\gamma)dt$
=Time Dilation

4-Velocity
$\mathbf{U} = \gamma(c,\mathbf{u}) = d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed
$\mathbf{U}\cdot\mathbf{U} = c^2$

SRQM Diagram

Absolute/Invariant:
Causality is to Time-like event separation as Topology is to Space-like event separation

Relativistic/Frame-Dependent:
Simultaneity is to Space-like event separation as Stationary is to Time-like event separation

$$\mathbf{\Delta R\cdot\Delta R} = [(c\Delta t)^2 - \mathbf{\Delta r\cdot\Delta r}] = (0)$$

$(c\Delta\tau)^2$	Time-like:Temporal	(+) {causal = 1D temporally-ordered, *spatially relative*}
(0)	Light-like:Null:Photonic	(0) {causal & topological, maximum signal speed (|ΔR/Δt|=c)}
$-(\Delta r_o)^2$	Space-like:Spatial	(–) {*temporally relative*, topological = 3D spatially-ordered}

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Absolute:Invariant (Ordering of Events)
Causality is temporal Topology : Topology is spatial Causality

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V\cdot V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v\cdot v}] = (v^0{}_o)^2$
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SpaceTime Dimension = 4D = (1+3)D

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4-Gradient ∂^μ
$\partial = \partial/\partial R_\mu = (\partial_t/c, -\nabla) = (\partial^\mu)$

SpaceTime
$\partial\cdot\mathbf{R} = \partial^\mu\eta_{\mu\nu}R^\nu = \partial_\nu R^\nu = 4$
Dimension

4-Position R^μ
$R = (ct, \mathbf{r}) = (r^\mu) \in$ <Event>

4-Displacement
$\Delta\mathbf{R} = (c\Delta t, \Delta\mathbf{r})$
$d\mathbf{R} = (cdt, d\mathbf{r})$

4-Position
$\mathbf{R} = (ct, \mathbf{r})$

$\partial\cdot\mathbf{R} = 4$
SpaceTime Dimension

$\partial[\mathbf{R}] = \partial^\mu R^\nu = \eta^{\mu\nu}$
$\rightarrow \mathrm{Diag}[1,-1,-1,-1]$
$= \mathrm{Diag}[1,-\delta^{jk}]$
Minkowski Metric

$\partial_\nu[R^{\mu'}]$
$= \partial R^{\mu'}/\partial R^\nu = \Lambda^{\mu'}{}_\nu$
Lorentz Transform

4-Gradient
$\partial = (\partial_t/c, -\nabla) = \partial/\partial R_\mu$
$\rightarrow (\partial_t/c, -\partial_x, -\partial_y, -\partial_z)$
$= (\partial/c\partial t, -\partial/\partial x, -\partial/\partial y, -\partial/\partial z)$

$\eta_{\mu\nu}$

SpaceTime Dim
$\mathrm{Tr}[\eta^{\mu\nu}] = 4 = \Lambda_{\mu\nu}\Lambda^{\mu\nu}$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R} = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (c\tau)^2$
$\Delta\mathbf{R}\cdot\Delta\mathbf{R} = (c\Delta t)^2 - \Delta\mathbf{r}\cdot\Delta\mathbf{r} = (c\Delta\tau)^2$
$d\mathbf{R}\cdot d\mathbf{R} = (cdt)^2 - d\mathbf{r}\cdot d\mathbf{r} = (cd\tau)^2$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

ProperTime Derivative
$\mathbf{U}\cdot\partial = \gamma(c,\mathbf{u})\cdot(\partial_t/c, -\nabla) = \gamma(\partial_t + \mathbf{u}\cdot\nabla)$
$= \gamma(\partial_t + (dx/dt)\partial_x + (dy/dt)\partial_y + (dz/dt)\partial_z)$
$= \gamma d/dt = d/d\tau$

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla$

Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\Delta\mathbf{R} = \gamma(c,\mathbf{u})\cdot(c\Delta t, \Delta\mathbf{r})$
$= \gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{r})$
$= c^2\Delta t_o = c^2\Delta\tau$

Continuity of 4-Velocity Flow
$\partial\cdot\mathbf{U} = 0$

ProperTime Differential
$d\tau = (1/\gamma)dt$
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SRQM Diagram

4-Velocity
$\mathbf{U} = \gamma(c,\mathbf{u})$
$= d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed
$\mathbf{U}\cdot\mathbf{U} = c^2$

$\partial\cdot\mathbf{R} = 4$: The 4-Divergence SpaceTime Dimension Relation
$= (\partial_t/c, -\nabla)\cdot(ct, \mathbf{r})$
$= [(\partial_t/c)*(ct) - (-\nabla)\cdot(\mathbf{r})]$
$= (\partial_t[t] + \nabla\cdot\mathbf{r})$
$= (\partial_t[t] + \partial_x[x] + \partial_y[y] + \partial_z[z])$
$= (\partial[t]/\partial t + \partial[x]/\partial x + \partial[y]/\partial y + \partial[z]/\partial z)$
$= (1+1+1+1)$
$= 4$

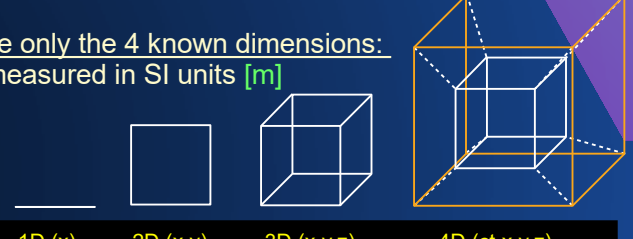
Alternate Tensorial Derivation:
$(\partial\cdot\mathbf{R}) = (\partial^\alpha\cdot R^\beta) = (\partial^\alpha\eta_{\alpha\beta}R^\beta) = \eta_{\alpha\beta}(\partial^\alpha R^\beta) = \eta_{\alpha\beta}(\eta^{\alpha\beta}) = \eta_\beta{}^\beta = \eta_\alpha{}^\alpha = \delta_\alpha{}^\alpha$
$= (\delta_0{}^0 + \delta_1{}^1 + \delta_2{}^2 + \delta_3{}^3) = (1+1+1+1) = 4$

This Tensor Invariant Lorentz Scalar relation gives the dimension of SpaceTime. The only way there can more dimensions is if there is another SpaceTime direction available. 4-Divergence ($\partial\cdot[..]$) is also used in SR Conservation Laws, ex. $(\partial\cdot\mathbf{J}) = 0$

All empirical evidence to-date indicates that there are only the 4 known dimensions:
1 temporal (t): measured in SI units = [s], with (ct): measured in SI units [m]
3 spatial (x, y, z) : measured in SI units = [m]

These are the 4 components that appear in:

4-Position
$\mathbf{R} = (ct, \mathbf{r}) \rightarrow (ct, x, y, z)$: measured in SI units [m]



0D () point	1D (x) line	2D (x,y) square	3D (x,y,z) cube	4D (ct,x,y,z) tesseract

The Tesseract, a 4D "cube", symbolizes 4D SpaceTime

SR : Minkowski
Time·Space is 4D
(1+3)D = 4D

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(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\delta^{\mu\nu} = \delta^\mu{}_\nu = \delta_\mu{}^\nu = \delta_{\mu\nu} = I_{(4)} = \{1 \text{ if } \mu=\nu, \text{ else } 0\} = \mathrm{Diag}[1,1,1,1]$
4D Kronecker Delta = 4D Identity

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
The Minkowski Metric ($\eta^{\mu\nu}$), Measurement

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

4-Gradient ∂^μ
$\partial=\partial/\partial R_\mu=(\partial_t/c,-\nabla)=(\partial^\mu)$

4-Position R^μ
$R=(ct,\mathbf{r})=(r^\mu)\in$<Event>

SR:Minkowski Metric
$\partial[\mathbf{R}] = \partial^\mu R^\nu = \eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu} \rightarrow$
$\mathrm{Diag}[1,-1,-1,-1] = \mathrm{Diag}[1,-I_{(3)}] = \mathrm{Diag}[1,-\delta^{jk}]$
{in Cartesian form} "Particle Physics" Convention
$\{\eta_{\mu\mu}\} = 1/\{\eta^{\mu\mu}\} : \eta_\mu{}^\nu = \delta_\mu{}^\nu$ $\mathrm{Tr}[\eta^{\mu\nu}]=4$

SR:Temporal Projection
"Vertical" $V^{\mu\nu} = \bar{T}^\mu\bar{T}^\nu \rightarrow$
$\mathrm{Diag}[1,0,0,0] = \mathrm{Diag}[1,0^{jk}]$

+

SR:Spatial Projection
"Horizontal" $H^{\mu\nu} = \eta^{\mu\nu}-\bar{T}^\mu\bar{T}^\nu \rightarrow$
$\mathrm{Diag}[0,-1,-1,-1] = \mathrm{Diag}[0,-\delta^{jk}]$

⊗

4-UnitTemporal
$\bar{T}=T^\mu=\gamma(1,\boldsymbol{\beta})=\mathbf{U}/c$

Derivation:
$\partial[\mathbf{R}] = \partial^\mu R^\nu$
$=(\partial_t/c,-\nabla)[(ct,\mathbf{r})]$

$=[\partial_t/c*ct, -\nabla ct]$
$[\partial_t/c*\mathbf{r}, -\nabla\mathbf{r}]$

$=[\partial_t t, \mathbf{0}]$
$[\mathbf{0}, -\nabla\mathbf{r}]$

$= \mathrm{Diag}[+1,-\delta^{jk}] = \eta^{\mu\nu}$

The component representation of 4-Vectors and the Minkowski Metric $\eta^{\mu\nu}$ will differ with the chosen basis,

$\mathbf{A}=A^\mu\rightarrow(a^t,a^x,a^y,a^z)$ $\eta^{\mu\nu}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$: Cartesian/Rectangular basis
$\mathbf{A}=A^\mu\rightarrow(a^t,a^r,a^\theta,a^z)$ $\eta^{\mu\nu}\rightarrow\mathrm{Diag}[1,-1,-1/r^2,-1]$: Polar/Cylindrical basis
$\mathbf{A}=A^\mu\rightarrow(a^t,a^r,a^\theta,a^\varphi)$ $\eta^{\mu\nu}\rightarrow\mathrm{Diag}[1,-1,-1/r^2,-1/(r\sin[\theta])^2]$: Spherical basis

Generally, diagonal components $[\eta^{\mu\mu}] = 1/[\eta_{\mu\mu}]$, non-diagonal = 0; and $\eta_\mu{}^\nu = \delta_\mu{}^\nu$

Alternate Tensorial Derivation: $\partial^\mu X^\nu = \eta^{\mu\sigma}\partial_\sigma X^\nu = \eta^{\mu\sigma}(\partial/\partial X^\sigma)X^\nu = \eta^{\mu\sigma}(\partial X^\nu/\partial X^\sigma) = \eta^{\mu\sigma}(\delta_\sigma{}^\nu) = \eta^{\mu\nu}$

4-Displacement
$\mathbf{\Delta R}=(c\Delta t,\mathbf{\Delta r})$
$\mathbf{dR}=(cdt,\mathbf{dr})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$
$\mathbf{\Delta R}\cdot\mathbf{\Delta R}=(c\Delta t)^2-\mathbf{\Delta r}\cdot\mathbf{\Delta r}=(c\Delta\tau)^2$
$\mathbf{dR}\cdot\mathbf{dR}=(cdt)^2-\mathbf{dr}\cdot\mathbf{dr}=(cd\tau)^2$

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\partial^\mu R^\nu=\eta^{\mu\nu}$
$\rightarrow\mathrm{Diag}[1,-1,-1,-1]$
$=\mathrm{Diag}[1,-\delta^{jk}]$
Minkowski Metric

$\partial_\nu[R^\mu]$
$=\partial R^\mu/\partial R^\nu=\Lambda^{\mu'}{}_{\nu'}$
Lorentz Transform

$\eta_{\mu\nu}$

SpaceTime Dim
$\mathrm{Tr}[\eta^{\mu\nu}] = 4 = \Lambda_{\mu\nu}\Lambda^{\mu\nu}$

4-Gradient
$\partial=(\partial_t/c,-\nabla)=\partial/\partial R_\mu$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$=\gamma(\partial_t+(dx/dt)\partial_x+(dy/dt)\partial_y+(dz/dt)\partial_z)$
$=\gamma d/dt = d/d\tau$

Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\mathbf{\Delta R} = \gamma(c,\mathbf{u})\cdot(c\Delta t,\mathbf{\Delta r})$
$= \gamma(c^2\Delta t - \mathbf{u}\cdot\mathbf{\Delta r})$
$= c^2\Delta t_o = c^2\Delta\tau$

Continuity of 4-Velocity Flow
$\partial\cdot\mathbf{U}=0$

ProperTime Differential
$d\tau=(1/\gamma)dt$
=Time Dilation

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed
$\mathbf{U}\cdot\mathbf{U}=c^2$

SRQM Diagram

"(V)ertical"
$V^{\mu\nu}$ Temporal

$H^{\mu\nu}$ Spatial
"(H)orizontal"
Light-Cone

The SR:Minkowski Metric $\eta^{\mu\nu}$ is the fundamental SR 4D (2,0)-Tensor, which shows how intervals are "measured" in SR Time·Space.
It is itself the low-mass = (Curvature ~ 0) limiting-case of the more general GR Metric $g^{\mu\nu}$. It can be split into temporal and spatial parts.
The Minkowski Metric can be used to raise/lower indices on other SR tensors, inc. 4-Vectors. The GR Metric $g^{\mu\nu}$ is used in strong gravity.

The SR : Minkowski Metric $\eta^{\mu\nu}$ is the "Flat SpaceTime" low-curvature limiting-case of the more general GR Metric $g^{\mu\nu}$.

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(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
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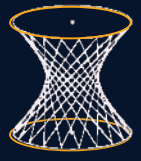

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$\partial=\partial/\partial R_\mu=(\partial_t/c,-\nabla)=(\partial^\mu)$

4-Position R^μ
$R=(ct,\mathbf{r})=(r^\mu)\in$<Event>

Tensorial Lorentz Transform $\Lambda^{\mu'}{}_\nu$
{ acting on 4-Vector [$A^{\mu'}=\Lambda^{\mu'}{}_\nu A^\nu$] }
$\partial_\nu[R^{\mu'}]=(\partial/\partial R^\nu)[R^{\mu'}]=(\partial/\partial R^\nu)[\Lambda^{\mu'}{}_\alpha R^\alpha]$
$=\Lambda^{\mu'}{}_\alpha(\partial/\partial R^\nu)[R^\alpha]=\Lambda^{\mu'}{}_\alpha\eta^\alpha{}_\nu=\Lambda^{\mu'}{}_\nu$

$\mathrm{Det}[\Lambda^\mu{}_\nu]=\pm1$

$\mathrm{Tr}[\Lambda^\mu{}_\nu]=\{-\infty..+\infty\}$
=Lorentz Transform Type

$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda_\mu{}^\nu\Lambda^\mu{}_\nu$

4-Displacement
$\mathbf{\Delta R}=(c\Delta t,\mathbf{\Delta r})$
$\mathbf{dR}=(cdt,\mathbf{dr})$

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$\partial\cdot\mathbf{R}=4$
SpaceTime Dimension

$\partial[\mathbf{R}]=\partial^\mu R^\nu=\eta^{\mu\nu}$
$\rightarrow\mathrm{Diag}[1,-1,-1,-1]$
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Minkowski Metric

$\partial_\nu[R^{\mu'}]$
$=\partial R^{\mu'}/\partial R^\nu=\Lambda^{\mu'}{}_\nu$
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$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

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$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
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$=c^2\Delta t_o=c^2\Delta\tau$

Continuity of 4-Velocity Flow
$\partial\cdot\mathbf{U}=0$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed
$\mathbf{U}\cdot\mathbf{U}=c^2$

General Lorentz Boost Transform (symmetric,continuous):

for a linear-velocity time-space-mixing frame-shift (Boost) in the $\mathbf{v}/c=\boldsymbol{\beta}=(\beta^1,\beta^2,\beta^3)$-direction: $\Lambda^{\mu'}{}_\nu\rightarrow B^{\mu'}{}_\nu=$

$$\begin{bmatrix}\gamma & -\gamma\beta_j\\ -\gamma\beta^i & (\gamma-1)\beta^i\beta_j/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\delta^i_j\end{bmatrix}$$

$\mathrm{Tr}[R^{\mu'}{}_\nu]$
$2[1+\cos(\theta)]=\{0..4\}$
Lorentz Rotation

$\mathrm{Tr}[\eta^{\mu'}{}_\nu]$
$=\{4\}$
Identity

$\mathrm{Tr}[B^{\mu'}{}_\nu]$
$2[1+\gamma]=\{4..+\infty\}$
Lorentz Boost

General Lorentz Rotation Transform (non-symmetric,orthogonal,continuous):

for an angular-displacement spatial-only frame-shift (Rotation) angle θ about the $\hat{\mathbf{n}}=(n^1,n^2,n^3)$-direction: $\Lambda^{\mu'}{}_\nu\rightarrow R^{\mu'}{}_\nu=$

$$\begin{bmatrix}1 & 0_j\\ 0^i & (\delta^i_j-n^in_j)\cos(\theta)-(\varepsilon^i{}_{jk}n^k)\sin(\theta)+n^in_j\end{bmatrix}$$

General Lorentz Discrete Transforms (symmetric,discrete):

Identity $I_{(4)}$	Time-Reverse	Parity	ComboPT
$\Lambda^{\mu'}{}_\nu\rightarrow\eta^{\mu'}{}_\nu=\delta^{\mu'}{}_\nu$	$\Lambda^{\mu'}{}_\nu\rightarrow T^{\mu'}{}_\nu$	$\Lambda^{\mu'}{}_\nu\rightarrow P^{\mu'}{}_\nu$	$\Lambda^{\mu'}{}_\nu\rightarrow(PT)^{\mu'}{}_\nu$
$=\mathrm{Diag}[1,\delta^i_j]$	$=\mathrm{Diag}[-1,\delta^i_j]$	$=\mathrm{Diag}[1,-\delta^i_j]$	$=\mathrm{Diag}[-1,-\delta^i_j]$
$\begin{bmatrix}1 & 0\\ 0 & \delta^i_j\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\ 0 & \delta^i_j\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & -\delta^i_j\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\ 0 & -\delta^i_j\end{bmatrix}$

Lorentz-Transform Properties:

$\Lambda^{\mu'}{}_\nu=\partial X^{\mu'}/\partial X^\nu$
$\Lambda^\mu{}_\nu=(\Lambda^{-1})^\nu{}_\mu$: $B^T=B$: $R^T=R^{-1}$
$\Lambda^{-1}(\boldsymbol{\beta})=\Lambda(-\boldsymbol{\beta})$: $\Lambda^{-1}(\theta)=\Lambda(-\theta)$
$\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta=\eta_{\alpha\beta}$
$\Lambda^\mu{}_\alpha\Lambda^{\alpha\nu}=\eta^{\mu\nu}$: $\Lambda^\mu{}_\alpha\Lambda^\alpha{}_\nu=\Lambda^{\mu\alpha}\Lambda_{\alpha\nu}=\eta^\mu{}_\nu=\delta^\mu{}_\nu$
$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=\Lambda^\mu{}_\nu\Lambda_\mu{}^\nu=4$: SpaceTime Dimension
$\mathrm{Det}[\Lambda^\mu{}_\nu]=\pm1$: (+)=Linearity; (-)=Anti-Linearity
$|\Lambda|=1$: $\Lambda^{-1}(\mathbf{0})=\Lambda(\mathbf{0})=I_{(4)}$
The Trace Invariant of the various Lorentz-Transforms leads to very interesting results: CPT Symmetry and Antimatter

SRQM Diagram

$\Lambda^\mu{}_\alpha(\Lambda^{-1})^\alpha{}_\nu=\Lambda^{\mu\alpha}(\Lambda^{-1})_{\alpha\nu}=\eta^\mu{}_\nu=\delta^\mu{}_\nu$

SR:Lorentz Transform
$\partial_\nu[R^{\mu'}]=\partial R^{\mu'}/\partial R^\nu=\Lambda^{\mu'}{}_\nu$
$\Lambda^\mu{}_\nu=(\Lambda^{-1})_\nu{}^\mu$
$\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta=\eta_{\alpha\beta}$
$\mathrm{Det}[\Lambda^\mu{}_\nu]=\pm1$ $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^\mu{}_\nu\Lambda_\mu{}^\nu$
$\mathrm{Tr}[\Lambda^\mu{}_\nu]=\{-\infty..+\infty\}$
=Lorentz Transform Type

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Invariant $\mathrm{Tr}[\Lambda^{\mu'}{}_\nu]\rightarrow$
$-\infty,..,(-4),..,-2,..,(0),..,+2,..,(+4),...,+\infty$
Trace identifies CPT Symmetry in the Lorentz Transform

$\mathrm{Trace}[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
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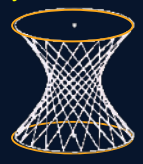

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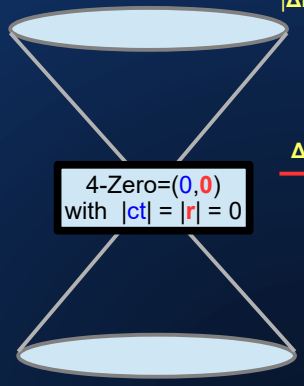
There are 6 classes of 4-Vector which are unchanged by continuous Lorentz Transforms, ie. the Boosts and Rotations:
4-TimeLike,Future
4-TimeLike,Past
4-Null,Future = 4-LightLike,Future
4-Null,Past = 4-LightLike,Past
4-SpaceLike
4-Zero

4-TimeLike=(ct,r) →Future with |ct| > |r|

4-Null=(ct,r)=(|r|,r) →Future with |ct| = |r|

Δt

|Δr/Δt|=c



Δr

4-Zero=(0,0) with |ct| = |r| = 0

4-SpaceLike=(ct,r) with |ct| < |r|

-c

LightCone

4-TimeLike=(-ct,r) →Past with |ct| > |r|

4-Null=(-ct,r)=(-|r|,r) →Past with |ct| = |r|

4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\partial^\mu R^\nu=\eta^{\mu\nu}$ →Diag[1,-1,-1,-1] =Diag[1,$-\delta^{jk}$] Minkowski Metric

$\partial_\nu[R^{\mu'}]$ $=\partial R^{\mu'}/\partial R^\nu=\Lambda^{\mu'}{}_\nu$ Lorentz Transform

4-Gradient
$\partial=(\partial_t/c,-\nabla)=\partial/\partial R_\mu$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

$\eta_{\mu\nu}$

SpaceTime Dim
$Tr[\eta^{\mu\nu}] = 4 = \Lambda_{\mu\nu}\Lambda^{\mu\nu}$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$
$\Delta\mathbf{R}\cdot\Delta\mathbf{R}=(c\Delta t)^2-\Delta\mathbf{r}\cdot\Delta\mathbf{r}=(c\Delta\tau)^2$
$d\mathbf{R}\cdot d\mathbf{R}=(cdt)^2-d\mathbf{r}\cdot d\mathbf{r}=(cd\tau)^2$

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$=\gamma(\partial_t+(dx/dt)\partial_x+(dy/dt)\partial_y+(dz/dt)\partial_z)$
$=\gamma d/dt = d/d\tau$

$Tr[B^{\mu'}{}_\nu]$
$2[1+\gamma]=\{4..+\infty\}$
Lorentz Boost

Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\Delta\mathbf{R} = \gamma(c,\mathbf{u})\cdot(c\Delta t,\Delta\mathbf{r})$
$= \gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{r})$
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SRQM Diagram

4-Velocity
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$\partial_\nu[R^{\mu'}] = \partial R^{\mu'}/\partial R^\nu = \Lambda^{\mu'}{}_\nu$
$\Lambda^\mu{}_\nu = (\Lambda^{-1})_\nu{}^\mu$
$\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta = \eta_{\alpha\beta}$
$Det[\Lambda^\mu{}_\nu]=\pm1$ $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^\mu{}_\nu\Lambda_\mu{}^\nu$
$Tr[\Lambda^\mu{}_\nu]=\{-\infty..+\infty\}$
=Lorentz Transform Type

Each of the 4-Vector Classes stays within its own class under a continuous Lorentz Transformation, {Rotations & Boosts}.

Space-reversal, a discrete transform, also maintains the various classes.

Time-reversal obviously discrete flips Future:Past 4-TimeLike & 4-Null

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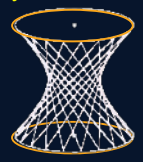

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The Lorentz transformation can also be derived empirically.
In order to achieve this, it's necessary to write down coordinate transformations that include experimentally testable parameters.
For instance, let there be given a single "preferred" inertial frame (t,x,y,z) in which the speed of light is constant, isotropic, and independent of the velocity of the source.
It is also assumed that Einstein synchronization and synchronization by slow clock transport are equivalent in this frame. Then assume another frame (t,x,y,z)'=(t',x',y',z') in relative motion, in which clocks and rods have the same internal constitution as in the preferred frame.
The following relations, however, are left undefined:

a(v): differences in time measurements,
b(v): differences in measured longitudinal lengths,
d(v): differences in measured transverse lengths,
ε(v): depends on the clock synchronization procedure in the moving frame,

then the transformation formula (assumed to be linear) between those frames are given by:

$t' = a(v)\,(t + \varepsilon(v)\,x)$
$x' = b(v)\,(x - vt)$
$y' = d(v)\,y$
$z' = d(v)\,z$

4-Position R'^μ
$\mathbf{R}'=(ct',r')=(ct',x',y',z')=$
$(\gamma ct - \gamma\beta x, -\gamma\beta ct + \gamma x, y, z)$
$(\gamma ct - \gamma xv/c, -\gamma vt + \gamma x, y, z)$

Lorentz x-Boost Transform $\Lambda^{\mu'}{}_\nu \to B^{\mu'}{}_\nu =$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \gamma & -\beta\gamma & 0 & 0 \\ x & -\beta\gamma & \gamma & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array}$$

4-Position R^μ
$\mathbf{R}=(ct,r)=(ct,x,y,z)$

ε(v) depends on the synchronization convention and is not determined experimentally, it obtains the value $(-v/c^2)$ by using Einstein synchronization in both frames.
The ratio between b(v) and d(v) is determined by the Michelson–Morley experiment.
The ratio between a(v) and b(v) is determined by the Kennedy–Thorndike experiment.
a(v) alone is determined by the Ives–Stilwell experiment.
In this way, they have been determined with great precision to { a(v) = b(v) = γ and d(v) = 1 }, which converts the above transformation into the Lorentz transformation.

4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$
$\Delta\mathbf{R}\cdot\Delta\mathbf{R}=(c\Delta t)^2-\Delta\mathbf{r}\cdot\Delta\mathbf{r}=(c\Delta\tau)^2$
$d\mathbf{R}\cdot d\mathbf{R}=(cdt)^2-d\mathbf{r}\cdot d\mathbf{r}=(cd\tau)^2$

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\partial^\mu R^\nu=\eta^{\mu\nu}$
$\to \mathrm{Diag}[1,-1,-1,-1]$
$=\mathrm{Diag}[1,-\delta^{jk}]$
Minkowski Metric

$\partial_\nu[R^{\mu'}]$
$=\partial R^{\mu'}/\partial R^\nu=\Lambda^{\mu'}{}_\nu$
Lorentz Transform

$\eta_{\mu\nu}$

SpaceTime Dim
$\mathrm{Tr}[\eta^{\mu\nu}] = 4 = \Lambda_{\mu\nu}\Lambda^{\mu\nu}$

4-Gradient
$\partial=(\partial_t/c,-\nabla)=\partial/\partial R_\mu$
$\to(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$=\gamma(\partial_t+(dx/dt)\partial_x+(dy/dt)\partial_y+(dz/dt)\partial_z)$
$=\gamma d/dt = d/d\tau$

Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\Delta\mathbf{R} = \gamma(c,\mathbf{u})\cdot(c\Delta t,\Delta\mathbf{r})$
$= \gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{r})$
$= c^2\Delta t_o = c^2\Delta\tau$

Continuity of 4-Velocity Flow
$\partial\cdot\mathbf{U}=0$

ProperTime Differential
$d\tau =(1/\gamma)dt$
=Time Dilation

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed
$\mathbf{U}\cdot\mathbf{U}=c^2$

SRQM Diagram

$\Lambda^\mu{}_\alpha(\Lambda^{-1})^\alpha{}_\nu = \Lambda^{\mu\alpha}(\Lambda^{-1})_{\alpha\nu} = \eta^\mu{}_\nu = \delta^\mu{}_\nu$

SR:Lorentz Transform
$\partial_\nu[R^{\mu'}] = \partial R^{\mu'}/\partial R^\nu = \Lambda^{\mu'}{}_\nu$
$\Lambda^\mu{}_\nu = (\Lambda^{-1})_\nu{}^\mu$
$\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta = \eta_{\alpha\beta}$
$\mathrm{Det}[\Lambda^\mu{}_\nu]=\pm1$ $\quad \Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^\mu{}_\nu\Lambda_\mu{}^\nu$
$\mathrm{Tr}[\Lambda^\mu{}_\nu]=\{-\infty..+\infty\}$
=Lorentz Transform Type

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

The value of LightSpeed (c) was 1st empirically measured by Ole Rømer to be finite using the timing of Jovian moon eclipses. His estimate was 1 Earth-Orbit-Diameter/22 minutes, about 75% of the actual value of (c).

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
Time·Space Dimension = 4D = (1+3)D

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
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$\partial\cdot\mathbf{R} = Tr[\eta^{\mu\nu}] = \Lambda^{\mu\beta}\Lambda_{\mu\beta} = 4$
The SpaceTime Dimension Relations

Tensor Invariants include: {Trace, InnerProduct, Determinant, etc.}
4-Divergence[4-Position] , Trace[Minkowski Metric] , and
the InnerProduct[any of the Lorentz Transforms]
give the Dimension of SR SpaceTime as 4D.

Minkowski Metric Trace Invariant	4-Divergence of 4-Position	Lorentz Transform Inner Prod Invariant
$Trace[\eta^{\mu\nu}]$	$\partial\cdot\mathbf{R}$	$\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$
$= Tr[\eta^{\mu\nu}]$	$= \partial^{\mu}\cdot R^{\nu}$	$\eta^{\alpha\beta}\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}\eta^{\alpha\beta}$
$= \eta_{\mu\nu}[\eta^{\mu\nu}]$	$= \partial^{\mu}\eta_{\mu\nu}R^{\nu}$	$\eta^{\alpha\beta}\Lambda^{\mu}{}_{\alpha}\eta_{\mu\nu}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}\eta^{\alpha\beta}$
$= \eta_{\mu}{}^{\mu}$	$= \eta_{\mu\nu}\partial^{\mu}R^{\nu}$	$(\eta^{\alpha\beta}\Lambda^{\mu}{}_{\alpha})(\eta_{\mu\nu}\Lambda^{\nu}{}_{\beta}) = \eta_{\alpha\beta}\eta^{\alpha\beta}$
$= \delta_{\mu}{}^{\mu}$	$= \eta_{\mu\nu}\eta^{\mu\nu}$	$\Lambda^{\mu\beta}\Lambda_{\mu\beta} = \eta_{\alpha\beta}\eta^{\alpha\beta} = Tr[\eta^{\mu\nu}]$
$= (1+1+1+1)$	$= Tr[\eta^{\mu\nu}]$	$\Lambda^{\mu\beta}\Lambda_{\mu\beta} = 4$
$= 4$	$= 4$	$= 4$

Minkowski Metric Trace Invariant

$Tr[\eta^{\mu\nu}]=\eta_{\nu}{}^{\nu}=(1) - (-1) - (-1) - (-1)= 4$

General Tensor Trace Invariant

$Tr[T^{\mu\nu}]=T_{\nu}{}^{\nu}=(T_0{}^0+T_1{}^1+T_2{}^2+T_3{}^3)$
$=(T^{00}-T^{11}-T^{22}-T^{33})=T$

4-Tensor

$$T^{\mu\nu} = \begin{bmatrix} T^{00}, & T^{01}, & T^{02}, & T^{03} \\ T^{10}, & T^{11}, & T^{12}, & T^{13} \\ T^{20}, & T^{21}, & T^{22}, & T^{23} \\ T^{30}, & T^{31}, & T^{32}, & T^{33} \end{bmatrix}$$

Minkowski Metric $\eta^{\mu\nu}$ →

$$\begin{bmatrix} +1, & 0, & 0, & 0 \\ 0, & -1, & 0, & 0 \\ 0, & 0, & -1, & 0 \\ 0, & 0, & 0, & -1 \end{bmatrix}$$

4-Displacement
$\mathbf{\Delta R}=(c\Delta t,\mathbf{\Delta r})$
$\mathbf{dR}=(cdt,\mathbf{dr})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$
$\mathbf{\Delta R}\cdot\mathbf{\Delta R}=(c\Delta t)^2-\mathbf{\Delta r}\cdot\mathbf{\Delta r}=(c\Delta\tau)^2$
$\mathbf{dR}\cdot\mathbf{dR}=(cdt)^2-\mathbf{dr}\cdot\mathbf{dr}=(c d\tau)^2$

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\partial^{\mu}R^{\nu}=\eta^{\mu\nu}$
$\rightarrow Diag[1,-1,-1,-1]$
$=Diag[1,-\delta^{jk}]$
Minkowski Metric

$\partial_{\nu}[R^{\mu'}]$
$=\partial R^{\mu'}/\partial R^{\nu}=\Lambda^{\mu'}{}_{\nu}$
Lorentz Transform

$\eta_{\mu\nu}$

SpaceTime Dim
$Tr[\eta^{\mu\nu}] = 4 = \Lambda_{\mu\nu}\Lambda^{\mu\nu}$

4-Gradient
$\partial=(\partial_t/c,-\nabla)=\partial/\partial R_{\mu}$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$=\gamma(\partial_t+(dx/dt)\partial_x+(dy/dt)\partial_y+(dz/dt)\partial_z)$
$=\gamma d/dt = d/d\tau$

Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\mathbf{\Delta R} = \gamma(c,\mathbf{u})\cdot(c\Delta t,\mathbf{\Delta r})$
$= \gamma(c^2\Delta t - \mathbf{u}\cdot\mathbf{\Delta r})$
$= c^2\Delta t_o = c^2\Delta\tau$

Continuity of 4-Velocity Flow
$\partial\cdot\mathbf{U}=0$

ProperTime Differential
$d\tau =(1/\gamma)dt$
=Time Dilation

SRQM Diagram

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed
$\mathbf{U}\cdot\mathbf{U}=c^2$

Conservation:Non-Divergence of Minkowksi Metric

$\partial\cdot\eta^{\mu\nu}$

$= \partial^{\sigma}\cdot\eta^{\mu\nu}$	
$= \partial^{\sigma}\eta_{\sigma\mu}\eta^{\mu\nu}$	$= \partial^{\sigma}\eta_{\sigma\mu}\eta^{\mu\nu}$
$= \partial_{\mu}\eta^{\mu\nu}$	$= \partial^{\sigma}\eta_{\alpha}{}^{\nu}$
	$= \partial^{\sigma}\delta_{\alpha}{}^{\nu}$
$= 0^{\nu}$	$= 0^{\nu}$

SR : Minkowski Time·Space is 4D

(1+3)D = 4D

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

$\eta_{\mu\nu}$ ·

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
Lorentz Scalar (Dot) Product ($\eta_{\mu\nu}$ = ·)

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
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The Tensor Invariant Lorentz Scalar Product (LSP) is the SR 4D (Dot=·) Product. It is used to make Invariant Lorentz Scalars from two 4-Vectors.

$\mathbf{A}\cdot\mathbf{B} = A^\mu\cdot B^\nu = A^\mu\eta_{\mu\nu}B^\nu = A_\nu B^\nu = A^\mu B_\mu = (a^0b^0 - \mathbf{a}\cdot\mathbf{b}) = (a^0_\circ b^0_\circ)$

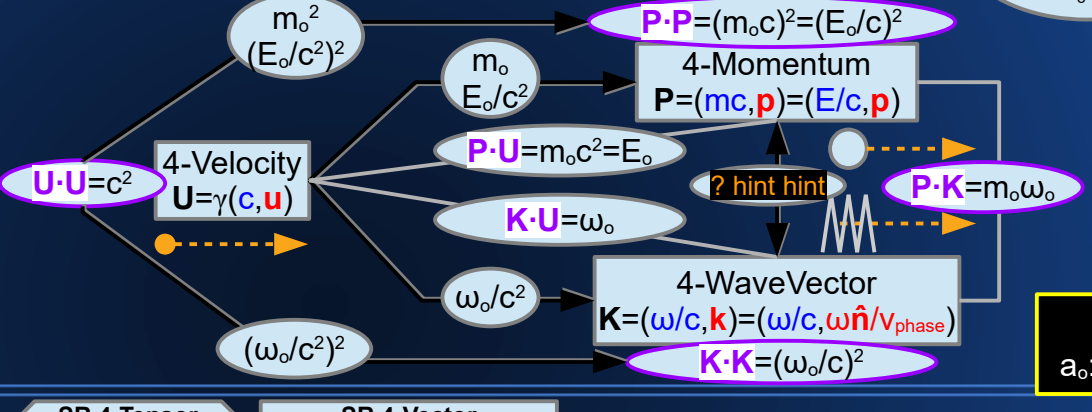
$\mathbf{A}\cdot\mathbf{A} = A^\mu\cdot A^\nu = A^\mu\eta_{\mu\nu}A^\nu = A_\nu A^\nu = A^\mu A_\mu = (a^0a^0 - \mathbf{a}\cdot\mathbf{a}) = (a^0_\circ)^2$

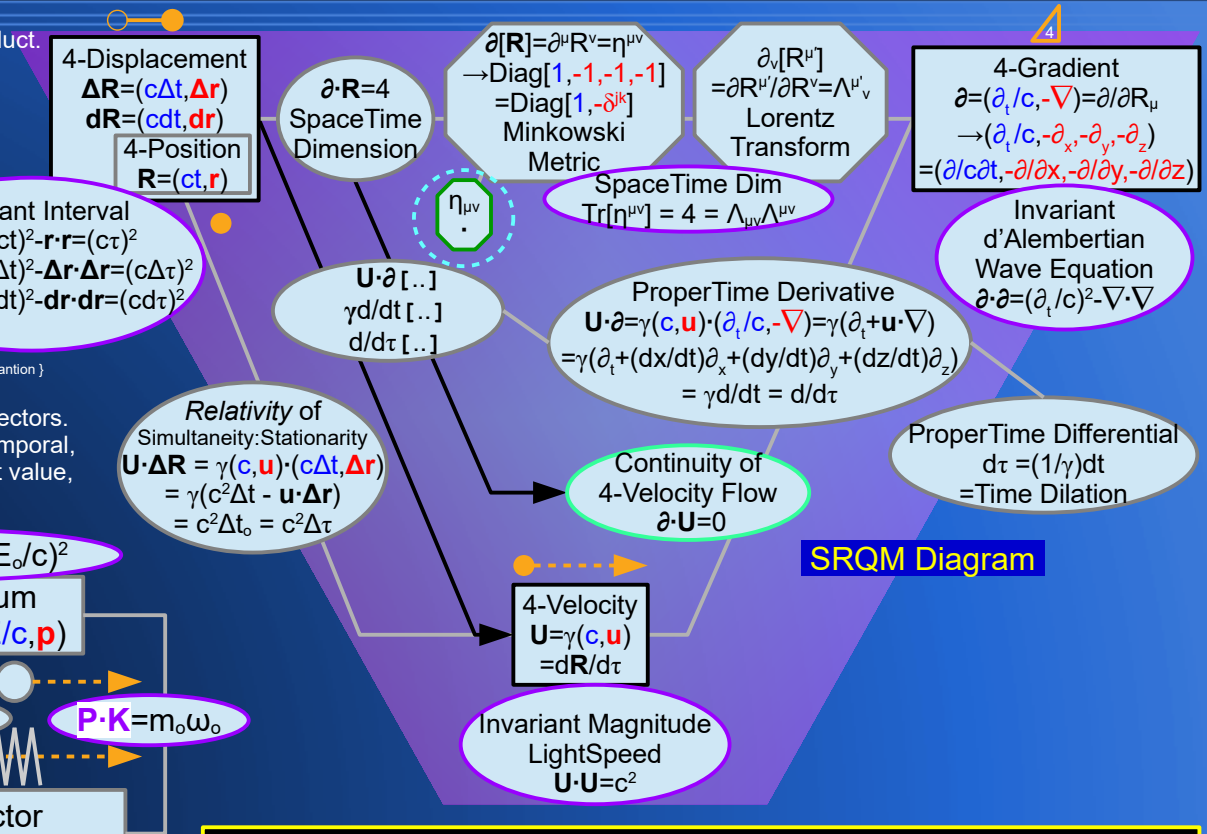
$\eta_{\mu\nu}\cdot = \eta_{\mu\nu}\cdot\hat{\mathbf{e}}_\mu\hat{\mathbf{e}}_\nu$ →Diag[+1,-1,-1,-1]{ Cartesian basis } with $\hat{\mathbf{e}}_\mu$ and $\hat{\mathbf{e}}_\nu$ as basis co-vectors

$\mathbf{A} = A^\mu\hat{\mathbf{e}}_\mu \rightarrow A^\mu$ { Cartesian basis }

($\eta_{\mu\nu}$) is itself just the lowered-index form of the SR Minkowski Metric ($\eta^{\mu\nu}$), with individual components [$\eta_{\mu\mu}$] = 1/[$\eta^{\mu\mu}$], else 0. In Cartesian basis, this gives { $\eta_{\mu\nu} = \eta^{\mu\nu}$ }{ Cartesian : no summantion }

The LSP is used in just about every relation between any two interesting 4-Vectors. It also gives the Invariant Magnitude of a single 4-Vector. If the 4-Vector is temporal, then the spatial component can be set to zero, giving the rest-frame invariant value, aka. the (o)bserver rest value ("naught" = $_\circ$).





a^0 or a_0: $(0)^{th}$ = temporal component (can *relativistically* vary)
$a_\circ$: (o)bserver's rest-frame "naught" **Invariant** value (does not vary)

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(0,2)-Tensor $T_{\mu\nu}$

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(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or $S_\circ$
Lorentz Scalar

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0_\circ)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
4-Velocity U, SpaceTime <Event> Motion

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
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4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})=(\gamma c,\gamma\mathbf{u})=(\mathbf{U}\cdot\partial)\mathbf{R}=\gamma(\partial_t+\mathbf{u}\cdot\nabla)\mathbf{R}=(d/d\tau)\mathbf{R}=$
$=d\mathbf{R}/d\tau=(dt/dt)(d\mathbf{R}/d\tau)=(dt/d\tau)(d\mathbf{R}/dt)=\gamma(d\mathbf{R}/dt)=\gamma(c\dot{t},\dot{\mathbf{r}})=\gamma(c,\mathbf{u})=U^\alpha$

4-Velocity **U** is the ProperTime Derivative ($d/d\tau$) of the 4-Position **R** or of the 4-Displacement **ΔR**.

4-Velocity is the SR 4-Vector that describes the motion of <Events> through SpaceTime.
(a) For an un-accelerated observer, the 4-Velocity **U** is a constant along the WorldLine at all points.
(b) For an accelerated observer, the 4-Velocity **U** is still tangent to the WorldLine at each point, but changes direction as the WorldLine bends thru SpaceTime.

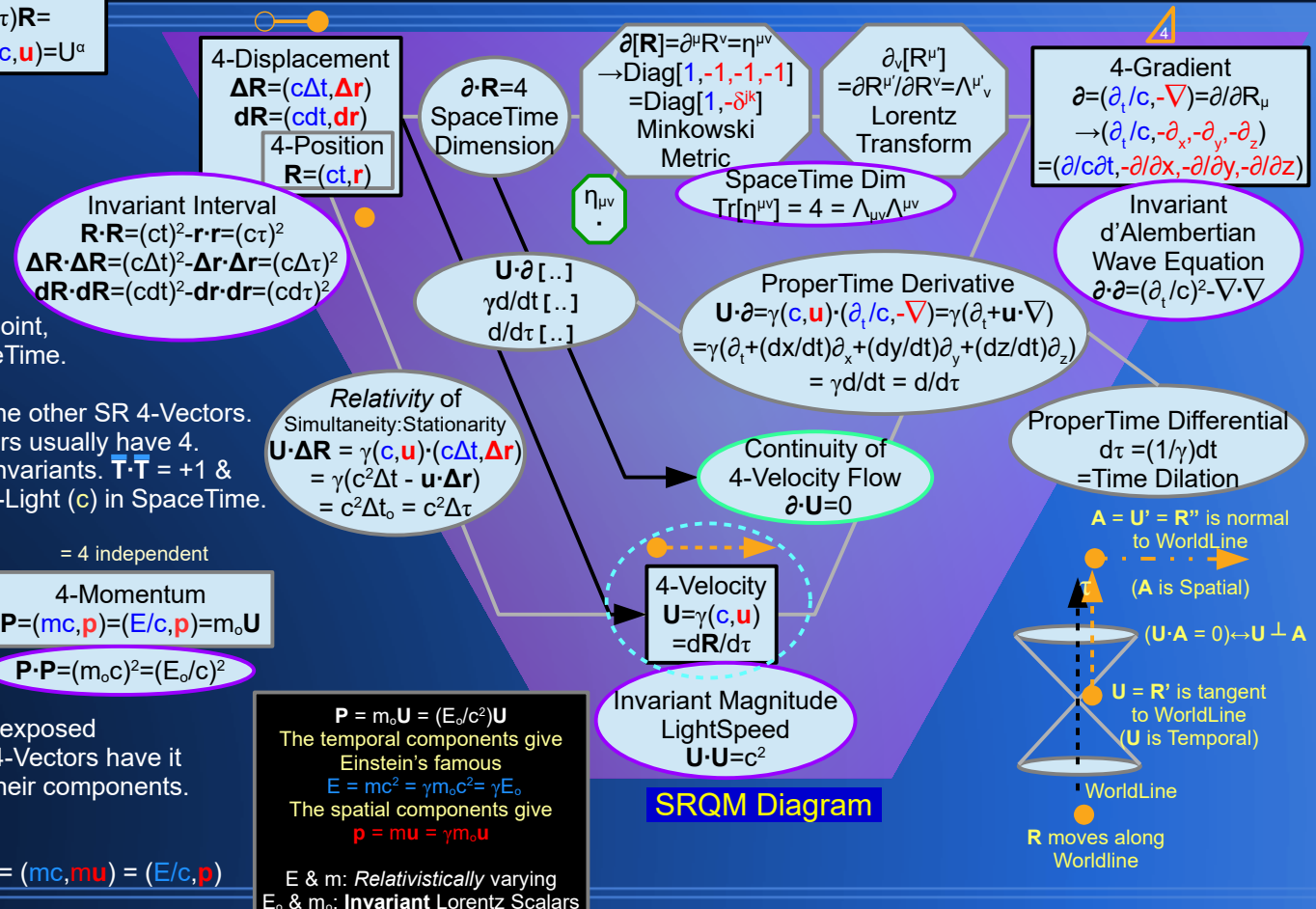
The 4-UnitTemporal $\bar{\mathbf{T}}$ & 4-Velocity **U** are unlike most of the other SR 4-Vectors. They have 3 independent components, whereas the others usually have 4. This is due to the constraints placed by the LSP Tensor Invariants. $\bar{\mathbf{T}}\cdot\bar{\mathbf{T}}=+1$ & $\mathbf{U}\cdot\mathbf{U}=c^2$ have constant magnitudes, giving the Speed-of-Light (c) in SpaceTime.

Components:
3 independent + 0 independent → 3 independent + 1 independent = 4 independent

4-UnitTemporal $\bar{\mathbf{T}}=\gamma(1,\boldsymbol{\beta})$ —c→ 4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$ —m_o E_o/c^2→ 4-Momentum $\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})=m_o\mathbf{U}$

$\bar{\mathbf{T}}\cdot\bar{\mathbf{T}}=+1$ | $\mathbf{U}\cdot\mathbf{U}=c^2$ | $\mathbf{P}\cdot\mathbf{P}=(m_oc)^2=(E_o/c)^2$

They also usually have the Relativistic Gamma factor (γ) exposed in component form, whereas most of the other temporal 4-Vectors have it absorbed into the Lorentz 4-Scalar factor that goes into their components.
4-UnitTemporal $\bar{\mathbf{T}}=T^\alpha=\gamma(1,\boldsymbol{\beta})=(\gamma,\gamma\boldsymbol{\beta})=\mathbf{U}/c$
4-Velocity $\mathbf{U}=U^\alpha=\gamma(c,\mathbf{u})=(\gamma c,\gamma\mathbf{u})=c\bar{\mathbf{T}}$
4-Momentum $\mathbf{P}=P^\alpha=(mc,\mathbf{p})=m_o\mathbf{U}=\gamma m_o(c,\mathbf{u})=m(c,\mathbf{u})=(mc,m\mathbf{u})=(E/c,\mathbf{p})$



P = m_o**U** = (E_o/c^2)**U**
The temporal components give Einstein's famous
$E = mc^2 = \gamma m_oc^2 = \gamma E_o$
The spatial components give
$\mathbf{p} = m\mathbf{u} = \gamma m_o\mathbf{u}$

E & m: *Relativistically* varying
E_o & m_o: **Invariant** Lorentz Scalars

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Relativistic Gamma $\gamma = 1/\sqrt{[\,1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}\,]}$, $\boldsymbol{\beta} = \mathbf{u}/c$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

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SRQM Diagram:
The Basis of Classical SR Physics
4-Velocity U, SpaceTime <Event> Motion

4-Vector SRQM Interpretation of QM

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$=d\mathbf{R}/d\tau=(dt/dt)(d\mathbf{R}/d\tau)=(dt/d\tau)(d\mathbf{R}/dt)=\gamma(d\mathbf{R}/dt)=\gamma(c\dot{t},\dot{\mathbf{r}})=\gamma(c,\mathbf{u})=U^\alpha$
$=c\bar{\mathbf{T}} = c\,\Lambda^{\mu'}{}_{\nu}\bar{\mathbf{T}}_{\bar{o}}$

Components: 0 independent

4-"Rest"UnitTemporal $\bar{\mathbf{T}}_{\bar{o}}=(1,\mathbf{0})_{rest}$ — $\bar{\mathbf{T}}_{\bar{o}}\cdot\bar{\mathbf{T}}_{\bar{o}}=+1$

Components: 3 independent

Inverse Boost Lorentz Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow B^{\mu'}{}_{\nu}=$

γ	$\gamma\beta_j$
$\gamma\beta^i$	$(\gamma-1)\beta^i\beta_j/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\delta^i_j$

Components: 3 independent + 0 independent → 3 independent + 1 independent = 4 independent

4-UnitTemporal $\bar{\mathbf{T}}=\gamma(1,\boldsymbol{\beta})$ — $\bar{\mathbf{T}}\cdot\bar{\mathbf{T}}=+1$ → c → 4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$ — $\mathbf{U}\cdot\mathbf{U}=c^2$ → m_o E_o/c^2 → 4-Momentum $\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})=m_o\mathbf{U}$ — $\mathbf{P}\cdot\mathbf{P}=(m_oc)^2=(E_o/c)^2$

4-Displacement $\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$, $d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position $\mathbf{R}=(ct,\mathbf{r})$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$
$\Delta\mathbf{R}\cdot\Delta\mathbf{R}=(c\Delta t)^2-\Delta\mathbf{r}\cdot\Delta\mathbf{r}=(c\Delta\tau)^2$
$d\mathbf{R}\cdot d\mathbf{R}=(cdt)^2-d\mathbf{r}\cdot d\mathbf{r}=(c d\tau)^2$

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\partial^\mu R^\nu=\eta^{\mu\nu}$ $\rightarrow$Diag[1,-1,-1,-1] =Diag[1,$-\delta^{jk}$] Minkowski Metric

$\partial_\nu[R^\mu]=\partial R^\mu/\partial R^\nu=\Lambda^{\mu'}{}_{\nu}$ Lorentz Transform

$\eta_{\mu\nu}$

SpaceTime Dim $Tr[\eta^{\mu\nu}] = 4 = \Lambda_{\mu\nu}\Lambda^{\mu\nu}$

4-Gradient $\partial=(\partial_t/c,-\nabla)=\partial/\partial R_\mu$ $\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$ $=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Invariant d'Alembertian Wave Equation $\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
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$=\gamma d/dt = d/d\tau$

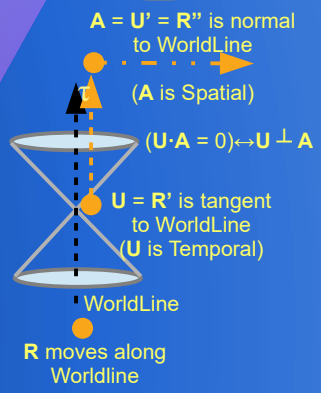
Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\Delta\mathbf{R} = \gamma(c,\mathbf{u})\cdot(c\Delta t,\Delta\mathbf{r})$
$= \gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{r})$
$= c^2\Delta t_o = c^2\Delta\tau$

Continuity of 4-Velocity Flow $\partial\cdot\mathbf{U}=0$

ProperTime Differential $d\tau=(1/\gamma)dt$ =Time Dilation

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})=d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed $\mathbf{U}\cdot\mathbf{U}=c^2$



4-Velocity **U** can also be derived by an Inverse Lorentz Boost on the 4-"Rest"UnitTemporal $\bar{\mathbf{T}}_{\bar{o}}$.

$\bar{\mathbf{T}} = \bar{T}^\mu = \Lambda^{\mu'}{}_{\nu}\,(1,\mathbf{0}) = \Lambda^{\mu'}{}_{\nu}\,\bar{T}_{\bar{o}}{}^{\nu} = \Lambda^{\mu'}{}_{0}\,\bar{T}_{\bar{o}}{}^{0} + \Lambda^{\mu'}{}_{1}\,\bar{T}_{\bar{o}}{}^{1} + \Lambda^{\mu'}{}_{2}\,\bar{T}_{\bar{o}}{}^{2} + \Lambda^{\mu'}{}_{3}\,\bar{T}_{\bar{o}}{}^{3} = \Lambda^{\mu'}{}_{0} + 0^{\mu'} + 0^{\mu'} + 0^{\mu'} = (\Lambda^{0'}{}_{0},\Lambda^{i'}{}_{0}) = (\gamma,\gamma\beta^i) = \gamma(1,\boldsymbol{\beta})$

SRQM Diagram

$\mathbf{U} = c\bar{\mathbf{T}} = c\gamma(1,\boldsymbol{\beta}) = \gamma(c,\mathbf{u})$ = 4-Velocity

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Relativistic Gamma $\gamma = 1/\sqrt{[\,1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}\,]}$, $\boldsymbol{\beta} = \mathbf{u}/c$

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

SRQM Diagram:

The Basis of Classical SR Physics

4-Velocity |Magnitude| = Invariant Speed-of-Light (c)

A Tensor Study of Physical 4-Vectors

4-Vector SRQM Interpretation of QM

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4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})=(\gamma c,\gamma\mathbf{u})=(\mathbf{U}\cdot\partial)\mathbf{R}=\gamma(\partial_t+\mathbf{u}\cdot\nabla)\mathbf{R}=(d/d\tau)\mathbf{R}=$
$=d\mathbf{R}/d\tau=(dt/dt)(d\mathbf{R}/d\tau)=(dt/d\tau)(d\mathbf{R}/dt)=\gamma(d\mathbf{R}/dt)=\gamma(c\dot{t},\dot{\mathbf{r}})=\gamma(c,\mathbf{u})=U^\alpha$

The Lorentz Scalar Product of the 4-Velocity leads to the Invariant |Magnitude| Speed-of-Light (c), one the main fundamental SR physical constants of physics.

U·U
$= \gamma(c,\mathbf{u})\cdot\gamma(c,\mathbf{u}) = \gamma^2(c^2 - \mathbf{u}\cdot\mathbf{u})$
$= [1/(1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta})](c^2 - \mathbf{u}\cdot\mathbf{u}) = [1/(1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta})]c^2(1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta})$
$= c^2$: Invariant |Magnitude| Speed-of-Light (c)

Alt Derivation:
U·U
$= d\mathbf{R}/d\tau\cdot d\mathbf{R}/d\tau$
$= (d\mathbf{R}\cdot d\mathbf{R})/(d\tau)^2$
$= (cd\tau)^2/(d\tau)^2$
$= (c)^2$

(c) is the unique maximum speed of SR causality, which all massless particles (RestMass $m_o=0$), ex. the photon, travel at temporally & spatially. Massive particles can travel at (c) only temporally.

$\mathbf{P} = (E/c,\mathbf{p}) = (E_o/c^2)\mathbf{U} = (E_o/c^2)\gamma(c,\mathbf{u}) = (E/c,\mathbf{p}=E\mathbf{u}/c^2)$
$\mathbf{P}\cdot\mathbf{P} = (m_oc)^2 = (E/c)^2 - \mathbf{p}\cdot\mathbf{p} = (E/c)^2 - (E/c)^2(\mathbf{u}\cdot\mathbf{u}/c^2) = (E/c)^2[1-\beta^2]$
From this eqn:
$(|\boldsymbol{\beta}|=1) \leftrightarrow (|\mathbf{u}|=c) \leftrightarrow (m_o=0)$: Massless objects always spatially-move at speed (c)

This fundamental constant Lorentz Invariant (c) provides an extra constraint on the components of 4-Velocity **U**, making it have only 3 independent components (**u**). This allows it to construct new 4-Vectors related to 4-Velocity by multiplying by other Lorentz Scalars.
(Lorentz Scalar)*(4-Velocity) = (New 4-Vector)

Components: 3 independent

$\mathbf{P} = (E/c,\mathbf{p}) = (E_o/c^2)\mathbf{U}$

$\mathbf{K} = (\omega/c,\mathbf{k}) = (\omega_o/c^2)\mathbf{U}$

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$
$\mathbf{U}\cdot\mathbf{U}=c^2$

m_o, E_o/c^2 → 4-Momentum $\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$; $\mathbf{P}\cdot\mathbf{P}=(m_oc)^2=(E_o/c)^2$

+1 independent

ω_o/c^2 → 4-WaveVector $\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$; $\mathbf{K}\cdot\mathbf{K}=(\omega_o/c)^2$

= 4 independent

The newly made 4-Vectors thus have { 3 + 1 = 4 } independent components.

4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$
$\Delta\mathbf{R}\cdot\Delta\mathbf{R}=(c\Delta t)^2-\Delta\mathbf{r}\cdot\Delta\mathbf{r}=(c\Delta\tau)^2$
$d\mathbf{R}\cdot d\mathbf{R}=(cdt)^2-d\mathbf{r}\cdot d\mathbf{r}=(cd\tau)^2$

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\partial^\mu R^\nu=\eta^{\mu\nu}$
$\rightarrow$Diag[1,-1,-1,-1]
=Diag[1,$-\delta^{jk}$]
Minkowski Metric

$\partial_\nu[R^\mu]$
$=\partial R^\mu/\partial R^\nu=\Lambda^{\mu'}{}_\nu$
Lorentz Transform

$\eta_{\mu\nu}$

SpaceTime Dim
$Tr[\eta^{\mu\nu}] = 4 = \Lambda_{\mu\nu}\Lambda^{\mu\nu}$

4-Gradient
$\partial=(\partial_t/c,-\nabla)=\partial/\partial R_\mu$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$=\gamma(\partial_t+(dx/dt)\partial_x+(dy/dt)\partial_y+(dz/dt)\partial_z)$
$=\gamma d/dt = d/d\tau$

ProperTime Differential
$d\tau=(1/\gamma)dt$
=Time Dilation

Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\Delta\mathbf{R} = \gamma(c,\mathbf{u})\cdot(c\Delta t,\Delta\mathbf{r})$
$= \gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{r})$
$= c^2\Delta t_o = c^2\Delta\tau$

Continuity of 4-Velocity Flow
$\partial\cdot\mathbf{U}=0$

SRQM Diagram

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed
$\mathbf{U}\cdot\mathbf{U}=c^2$

An interesting thing to note is that all <events> move at the Speed-of-Light (c) in 4D SpaceTime. Massive at-rest particles simply travel at (c) temporally as $\mathbf{U}_o = (c,\mathbf{0})$, while massless photons move at (c) spatially also (in vacuum) as $\mathbf{U}_c \sim (c,c\hat{\mathbf{n}})$. Magnitude $\sqrt{[\mathbf{U}\cdot\mathbf{U}]} = (c)$

If (c) was not a constant, but varied somehow, then all 4-Vectors made from the 4-Velocity would have more than 4 independent components, which is not observed. It seems a very strong, compelling argument against variable light-speed theories.

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(0,0)-Tensor S or S_o
Lorentz Scalar

Lorentz Factor:
Relativistic Gamma $\gamma = 1/\sqrt{[1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: $\boldsymbol{\beta} = \mathbf{u}/c$
$\gamma' = d\gamma/dt = \gamma^3\boldsymbol{\beta}\cdot\mathbf{a}/c = \gamma^3\mathbf{u}\cdot\mathbf{a}/c^2$

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The Basis of Classical SR Physics
Relativity of Simultaneity:Time-Delay
(Simultaneity ↔ Same-Time Occurrence ↔ Δt=0)
(Time-Delay ↔ Different-Time Occurrence ↔ Δt≠0)

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Relativity of Simultaneity:Time-Delay
$\mathbf{U}\cdot\Delta\mathbf{X} = \gamma(c,\mathbf{u})\cdot(c\Delta t,\Delta\mathbf{x}) = \gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{x})$
$= c^2\Delta t_o = c^2\Delta\tau$

If Lorentz Scalar ($\mathbf{U}\cdot\Delta\mathbf{X} = 0 = c^2\Delta\tau$),
then the ProperTime displacement ($\Delta\tau$) is zero,
and the <Event>'s separation ($\Delta\mathbf{X} = \mathbf{X}_2 - \mathbf{X}_1$) is orthogonal
to the worldline at **U**.

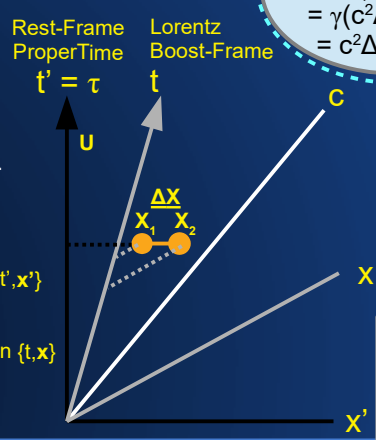
<Event>'s $\mathbf{X}_1$ and $\mathbf{X}_2$ are therefore simultaneous ($\Delta\tau = 0$)
for the observer on this worldline at **U**.

Examining the equation we get Boost Frame $\gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{x}) = 0$.
The coordinate time difference between the events is ($\Delta t = \mathbf{u}\cdot\Delta\mathbf{x}/c^2$)
The condition for simultaneity in an alternate reference frame
(moving at 3-velocity **u** wrt. the worldline **U**) is $\Delta t = 0$,
which implies ($\mathbf{u}\cdot\Delta\mathbf{x}$) = 0.

This condition can be met by:
($|\mathbf{u}| = 0$), the alternate observer is not moving wrt. the events,
i.e. is on worldline **U** or on a worldline parallel to **U**.
($|\Delta\mathbf{x}| = 0$), the events are at the same spatial location (co-local).
($\mathbf{u}\cdot\Delta\mathbf{x} = 0 = |\mathbf{u}||\Delta\mathbf{x}|\cos[\theta]$), the alternate observer's motion is
perpendicular (orthogonal, θ=90°) to the spatial separation **Δx**
of the events in that frame.

If none of these conditions is met,
then the events will not be simultaneous
in the alternate reference-frame.

This can be shown on a Minkowski Diagram.



4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$
$\Delta\mathbf{R}\cdot\Delta\mathbf{R}=(c\Delta t)^2-\Delta\mathbf{r}\cdot\Delta\mathbf{r}=(c\Delta\tau)^2$
$d\mathbf{R}\cdot d\mathbf{R}=(cdt)^2-d\mathbf{r}\cdot d\mathbf{r}=(cd\tau)^2$

$\partial\cdot\mathbf{R}=4$
SpaceTime Dimension

$\partial[\mathbf{R}]=\partial^\mu R^\nu=\eta^{\mu\nu}$
$\rightarrow$Diag[1,-1,-1,-1]
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$\partial=(\partial_t/c,-\nabla)=\partial/\partial R_\mu$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$

$\mathbf{U}\cdot\partial$ [..]
$\gamma d/dt$ [..]
$d/d\tau$ [..]

ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$=\gamma(\partial_t+(dx/dt)\partial_x+(dy/dt)\partial_y+(dz/dt)\partial_z)$
$= \gamma d/dt = d/d\tau$

Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\Delta\mathbf{R} = \gamma(c,\mathbf{u})\cdot(c\Delta t,\Delta\mathbf{r})$
$= \gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{r})$
$= c^2\Delta t_o = c^2\Delta\tau$

Continuity of 4-Velocity Flow
$\partial\cdot\mathbf{U}=0$

ProperTime Differential
$d\tau = (1/\gamma)dt$
=Time Dilation

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed
$\mathbf{U}\cdot\mathbf{U}=c^2$

SRQM Diagram

Realizing that Simultaneity (no-delay) is not an invariant concept was a breakthrough that lead Einstein to Special Relativity (SR).

Temporal Ordering:

Simultaneity (=same time occurrence) is Relative
Space-like Separated Events:
Can appear in any temporal order,
depending on one's reference frame. (Boost)

Causality is Absolute → Invariant Proper Time
Time-like Separated Events:
All observers agree on 1D causal ordering.
Causality is an invariant concept.

Spatial Ordering:

Stationarity (=same place occurrence) is Relative
Time-like Separated Events:
Can appear in any spatial order,
depending on one's reference frame. (Boost)

Topology is Absolute → Invariant Proper Length
Space-like Separated Events:
All observers agree on topology=3D spatial ordering.
Topology/topological-extension is an invariant concept.

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
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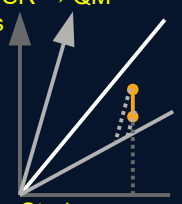

A Tensor Study of Physical 4-Vectors

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4-Vector SRQM Interpretation of QM

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Relativity of Stationarity:Space-Motion
$\mathbf{U}\cdot\Delta\mathbf{X} = \gamma(c,\mathbf{u})\cdot(c\Delta t,\Delta\mathbf{x}) = \gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{x})$
$= c^2\Delta t_o = c^2\Delta\tau$

Let <Event>'s $\mathbf{X}_1$ and $\mathbf{X}_2$ be local ($\Delta\mathbf{x}' = \mathbf{0}$) for the observer on worldline at $\mathbf{U}$.

This has equation $(\mathbf{U}\cdot\Delta\mathbf{X}) = \gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{x}) = c^2\Delta\tau \neq 0$.

To be stationary/motionless in the Rest-Frame is $\Delta\mathbf{x}' = \mathbf{0}$.

In a Boosted Frame, using $\gamma\beta = \sqrt{[\gamma^2 - 1]}$ and $\Delta t = \gamma\Delta\tau$:

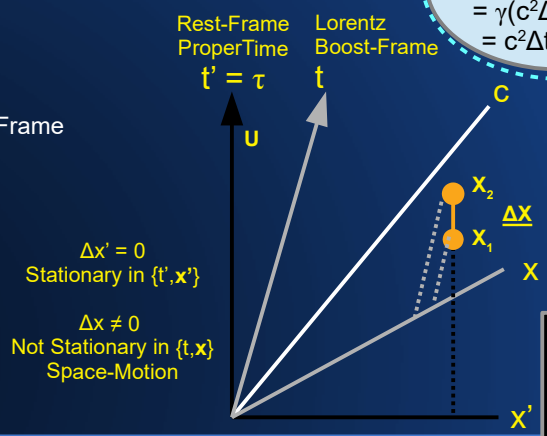
$\gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{x}) = c^2\Delta\tau$
$\gamma c^2\Delta t - c^2\Delta\tau = \gamma\mathbf{u}\cdot\Delta\mathbf{x}$
$\gamma^2 c^2\Delta\tau - c^2\Delta\tau = \gamma\mathbf{u}\cdot\Delta\mathbf{x}$
$(\gamma^2 - 1)c^2\Delta\tau = \gamma\mathbf{u}\cdot\Delta\mathbf{x}$
$(\gamma^2 - 1)c\Delta\tau = \gamma\boldsymbol{\beta}\cdot\Delta\mathbf{x}$
$\sqrt{[\gamma^2 - 1]}c\Delta\tau = \hat{\mathbf{n}}\cdot\Delta\mathbf{x}$

If $\mathbf{u} = 0$, then $\gamma = 1$, then $\hat{\mathbf{n}}\cdot\Delta\mathbf{x} = 0$, which is RestFrame
If $\mathbf{u} > 0$, then $\gamma > 1$, then $\hat{\mathbf{n}}\cdot\Delta\mathbf{x} \neq 0$

So, in any Boosted Frame, $\Delta\mathbf{x} \neq 0$

If this condition is met, then the events will not be stationary in the alternate reference-frame.

This can be shown on a Minkowski Diagram.



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$\mathrm{Tr}[\eta^{\mu\nu}] = 4 = \Lambda_{\mu\nu}\Lambda^{\mu\nu}$

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Invariant Magnitude LightSpeed
$\mathbf{U}\cdot\mathbf{U}=c^2$

SRQM Diagram

Realizing that Stationarity (no-motion) is not an invariant concept leads to a duality of Time and Space, via SR Lorentz Time·Space Boosts.

Temporal Ordering:

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The ProperTime Derivative (d/dτ) = (U·∂)

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4-Velocity U^μ
$\mathbf{U}=d\mathbf{R}/d\tau=\gamma(c,\mathbf{u})=(u^\mu)$

4-Gradient ∂^μ
$\partial=\partial/\partial R_\mu=(\partial_t/c,-\nabla)=(\partial^\mu)$

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$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$=\gamma(\partial_t+(dx/dt)\partial_x+(dy/dt)\partial_y+(dz/dt)\partial_z)$
$=\gamma d/dt = d/d\tau$

The derivation shows that the ProperTime Derivative (d/dτ) is an Invariant Lorentz Scalar. Therefore, all observers must agree on its magnitude, regardless of their frame-of-reference. (d/dτ) is used to derive some of the physical 4-Vectors: 4-Velocity, 4-Acceleration, 4-Force, 4-Torque, etc.

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$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$=\gamma(\partial_t+(dx/dt)\partial_x+(dy/dt)\partial_y+(dz/dt)\partial_z)$
$=\gamma d/dt = d/d\tau$

ProperTime Differential
$d\tau=(1/\gamma)dt$
=Time Dilation

Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\Delta\mathbf{R}=\gamma(c,\mathbf{u})\cdot(c\Delta t,\Delta\mathbf{r})$
$=\gamma(c^2\Delta t-\mathbf{u}\cdot\Delta\mathbf{r})$
$=c^2\Delta t_o=c^2\Delta\tau$

Continuity of 4-Velocity Flow
$\partial\cdot\mathbf{U}=0$

SRQM Diagram

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

4-Acceleration
$\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

$E_o/c^2=m_o$

4-AngularMomentum
$M^{\alpha\beta}=R^\alpha P^\beta-R^\beta P^\alpha=\mathbf{R}\wedge\mathbf{P}$

0 $-cn^j$	0 $-cn$
$+cn^i$ $\varepsilon^{ij}{}_k\|^k$	$+cn^T$ $\mathbf{r}\wedge\mathbf{p}$

4-Tensor Anti-symmetric

4-Momentum
$\mathbf{P}=(E/c,\mathbf{p})=(mc,\mathbf{p})$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

4-Force
$\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed
$\mathbf{U}\cdot\mathbf{U}=c^2$

4-Torque
$T^{\alpha\beta}$
$=R^\alpha F^\beta-R^\beta F^\alpha=\mathbf{R}\wedge\mathbf{F}$
$=d/d\tau[M^{\alpha\beta}]$

4-Tensor Anti-symmetric

The ProperTime Derivative can be used to make new tensors from existing tensors, as it is taking the derivative of an existing tensor by a Lorentz Scalar: the ProperTime τ.

$f=f[t,x,y,z]$ $df=dt(\partial f/\partial t)+dx(\partial f/\partial x)+dy(\partial f/\partial y)+dz(\partial f/\partial z)=(d\mathbf{R}\cdot\partial)[f]$

$df/dt=(\partial f/\partial t)+dx/dt\,(\partial f/\partial x)+dy/dt\,(\partial f/\partial y)+dz/dt\,(\partial f/\partial z)$
$df/dt=(\partial f/\partial t)+u_x(\partial f/\partial x)+u_y(\partial f/\partial y)+u_z(\partial f/\partial z)$
$df/dt=(\partial f/\partial t)+\mathbf{u}\cdot\nabla[f]$

$d/dt=(\partial/\partial t)+\mathbf{u}\cdot\nabla$

$\gamma(d/dt)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)=(d/d\tau)=(\mathbf{U}\cdot\partial)=(d\mathbf{R}/d\tau\cdot\partial)$ = Lorentz Invariant

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Lorentz Factor = Relativistic Gamma γ
$=1/\sqrt{[1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: $\boldsymbol{\beta}=\mathbf{u}/c$

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
ProperTime Derivative in SR:
4-Tensors, 4-Vectors, and 4-Scalars

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
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The ProperTime Derivative
$\mathbf{U}\cdot\partial = \gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla) = \gamma(\partial_t+\mathbf{u}\cdot\nabla) = \gamma d/dt = d/d\tau$

4-Vectors & 4-Tensors (acted on by ProperTime Derivative $d/d\tau$):

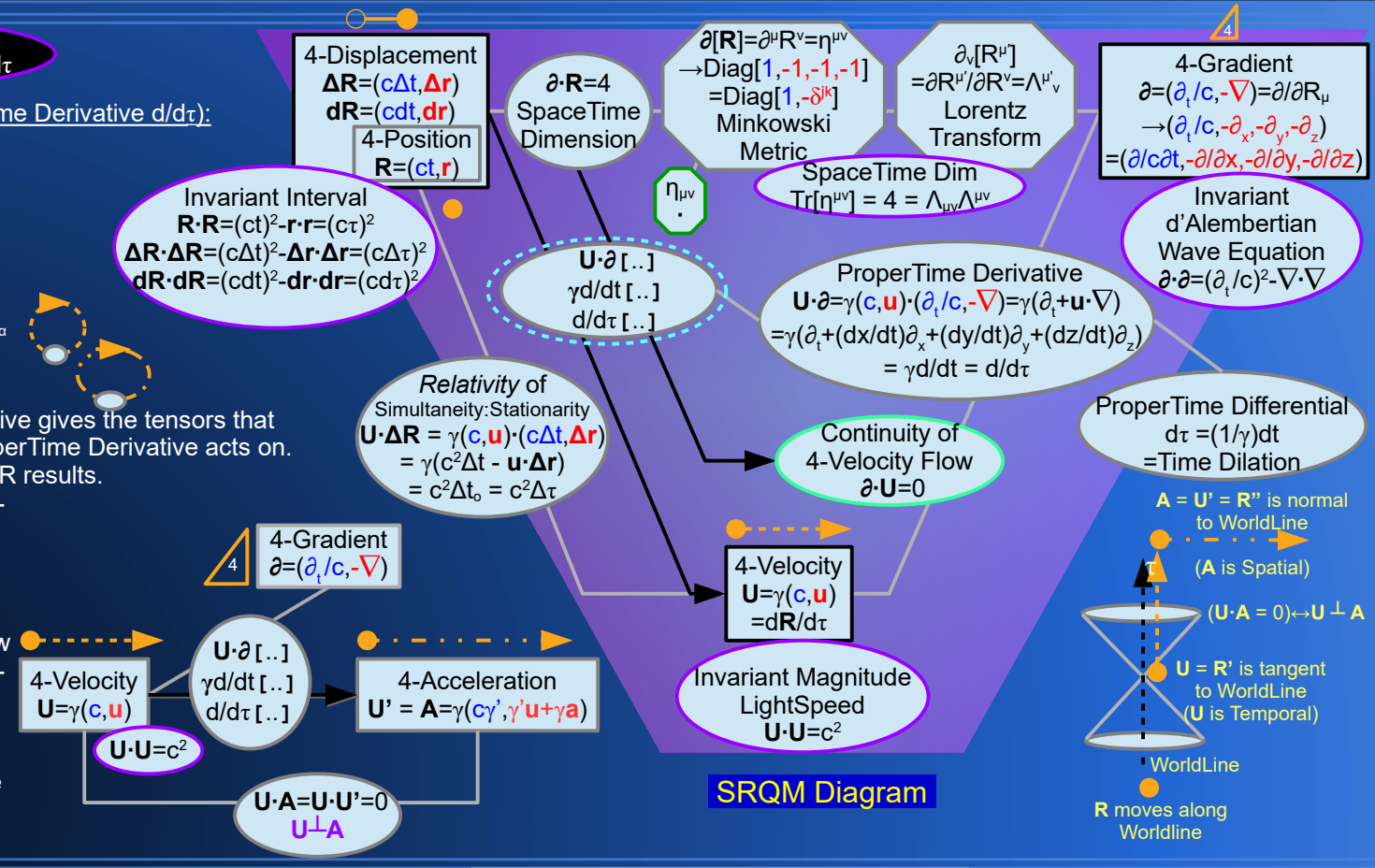
4-Position $\mathbf{R}$ = <Event>
4-Velocity $\mathbf{U} = d\mathbf{R}/d\tau$
4-Acceleration $\mathbf{A} = d\mathbf{U}/d\tau$
...
4-Momentum $\mathbf{P} = m_o\mathbf{U}$
4-Force $\mathbf{F} = d\mathbf{P}/d\tau$
...
4-AngularMomentum $M^{\alpha\beta} = \mathbf{R} \wedge \mathbf{P} = R^\alpha P^\beta - R^\beta P^\alpha$
4-Torque $T^{\alpha\beta} = \mathbf{R} \wedge \mathbf{F} = R^\alpha F^\beta - R^\beta F^\alpha = dM^{\alpha\beta}/d\tau$

As shown from the list, the ProperTime Derivative gives the tensors that are the change in status of the tensor that ProperTime Derivative acts on. It can also act on Scalar Values to give deep SR results.

$\partial\cdot\mathbf{R} = 4$: SpaceTime Dimension is 4
$d/d\tau(\partial\cdot\mathbf{R}) = d/d\tau(4) = 0$
$d/d\tau(\partial\cdot\mathbf{R}) = d/d\tau[\partial]\cdot\mathbf{R} + \partial\cdot\mathbf{U} = 0$
...
$\partial\cdot\mathbf{U} = 0$: Conservation of the SR 4-Velocity Flow

---

$\mathbf{U}\cdot\mathbf{U} = c^2$: Tensor Invariant of 4-Velocity
$d/d\tau[\mathbf{U}\cdot\mathbf{U}] = d/d\tau[c^2] = 0$
$d/d\tau[\mathbf{U}\cdot\mathbf{U}] = d/d\tau[\mathbf{U}]\cdot\mathbf{U} + \mathbf{U}\cdot d/d\tau[\mathbf{U}] = 2(\mathbf{U}\cdot\mathbf{A}) = 0$
$\mathbf{U}\cdot\mathbf{A} = \mathbf{U}\cdot\mathbf{U}' = 0$: The 4-Velocity $\mathbf{U}$ is SpaceTime orthogonal ($\perp$) to it's own 4-Acceleration $\mathbf{A}=\mathbf{U}'$



SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

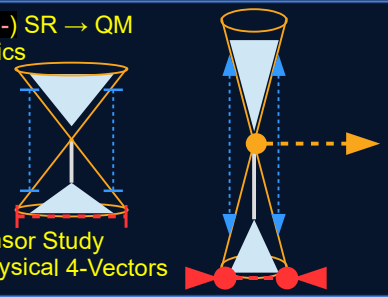
SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

4-Spin Precession: $d\mathbf{S}/d\tau = -(\mathbf{S}\cdot\mathbf{A})\mathbf{U}/c^2$
from $d[\mathbf{S}\cdot\mathbf{U}]/d\tau = d\mathbf{S}/d\tau\cdot\mathbf{U}+\mathbf{S}\cdot\mathbf{A} = 0$,
since $(\mathbf{S}\cdot\mathbf{U}) = 0$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics



A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
ProperTime Differential (dτ) → Time Dilation & Length Contraction

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

There are several ways to derive Time Dilation.

The ProperTime Derivative
$\mathbf{U}\cdot\partial = \gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla) = \gamma(\partial_t+\mathbf{u}\cdot\nabla) = \gamma d/dt = d/d\tau$

ProperTime Differential (Lorentz 4-Scalar): $d\tau = (1/\gamma)dt$

$\mathbf{dR}\cdot\mathbf{dR}=(cd\tau)^2$ ← $d\tau^2$ — $\mathbf{U}\cdot\mathbf{U}=c^2$

4-Differential $\mathbf{dR}=(cdt,\mathbf{dr})$ ← $d\tau$ — 4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})=d\mathbf{R}/d\tau$

Take the temporal component of the 4-Vector relation.
$dt = \gamma d\tau = \gamma dt_o$
$\Delta t = \gamma\Delta\tau = \gamma\Delta t_o$: ←Time Dilation→

The coordinate time Δt measured by an observer is "dilated", compared to the ProperTime as measured by a clock moving with the object. This has the effect that moving objects appear to age more slowly than at-rest objects. The effect is reciprocal as well. Since velocity is relative, each observer will see the other as ageing more slowly, similarly to the effect that each will appear smaller to the other when seen at a distance.

Now multiply both sides by the moving-frame speed $v=|\mathbf{v}|$
$v\Delta t = \gamma v\Delta\tau$
$v\Delta t$ = distance L_o the moving clock travels wrt. frame, which is a proper (fixed-to-frame) displacement length.
$L_o = \gamma L$
$L = (1/\gamma)L_o$: →Length Contraction← {in spatial **v** direction}

SRQM Diagram

4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\partial^\mu R^\nu=\eta^{\mu\nu}$
$\rightarrow$Diag[1,-1,-1,-1]
=Diag[1,$-\delta^{jk}$]
Minkowski Metric

$\partial_\nu[R^\mu]$
$=\partial R^\mu/\partial R^\nu=\Lambda^{\mu'}{}_\nu$
Lorentz Transform

4-Gradient
$\partial=(\partial_t/c,-\nabla)=\partial/\partial R_\mu$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

$\eta_{\mu\nu}$

SpaceTime Dim
$Tr[\eta^{\mu\nu}] = 4 = \Lambda_{\mu\nu}\Lambda^{\mu\nu}$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$
$\Delta\mathbf{R}\cdot\Delta\mathbf{R}=(c\Delta t)^2-\Delta\mathbf{r}\cdot\Delta\mathbf{r}=(c\Delta\tau)^2$
$d\mathbf{R}\cdot d\mathbf{R}=(cdt)^2-d\mathbf{r}\cdot d\mathbf{r}=(cd\tau)^2$

$\mathbf{U}\cdot\partial$ [..]
$\gamma d/dt$ [..]
$d/d\tau$ [..]

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$

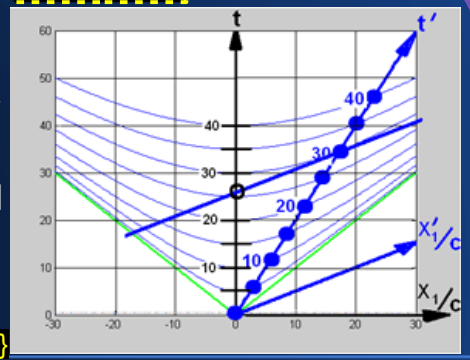
ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$=\gamma(\partial_t+(dx/dt)\partial_x+(dy/dt)\partial_y+(dz/dt)\partial_z)$
$= \gamma d/dt = d/d\tau$

Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\Delta\mathbf{R} = \gamma(c,\mathbf{u})\cdot(c\Delta t,\Delta\mathbf{r})$
$= \gamma(c^2\Delta t - \mathbf{u}\cdot\Delta\mathbf{r})$
$= c^2\Delta t_o = c^2\Delta\tau$

Continuity of 4-Velocity Flow
$\partial\cdot\mathbf{U}=0$

ProperTime Differential
$d\tau = (1/\gamma)dt$
=Time Dilation

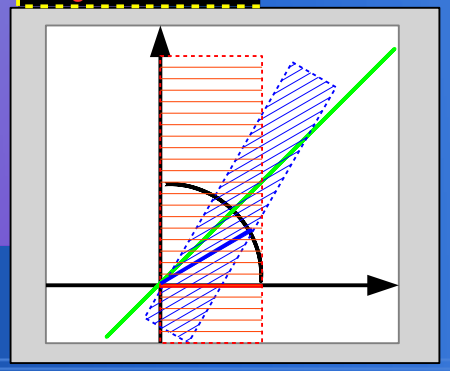
Time Dilation



4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed
$\mathbf{U}\cdot\mathbf{U}=c^2$

Length Contraction



Red and Blue lengths equal in the moving frame, ruler moves with it; blue appears contracted in the ProperTime frame

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Relativity: Time Dilation (←| clock moving |→), Length Contraction (|→ruler moving←|)
Invariants: Proper Time (| clock at rest |) , Proper Length (| ruler at rest |)

(+,-,-,-) SR → QM Physics

4

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
4-Gradient ∂, SR 4-Vector Function:Operator

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
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4-Gradient
$\partial=\partial^\mu=\partial/\partial R_\mu=(\partial^\mu)=(\partial_t/c,-\nabla)$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,\ -\partial/\partial x,\ -\partial/\partial y,\ -\partial/\partial z)$

4

Gradient 4D One-Form
$\partial_\mu=\partial/\partial R^\mu=(\partial_\mu)=(\partial_t/c,+\nabla)$
$\rightarrow(\partial_t/c,\partial_x,\partial_y,\partial_z)$
$=(\partial/c\partial t,\ \partial/\partial x,\ \partial/\partial y,\ \partial/\partial z)$

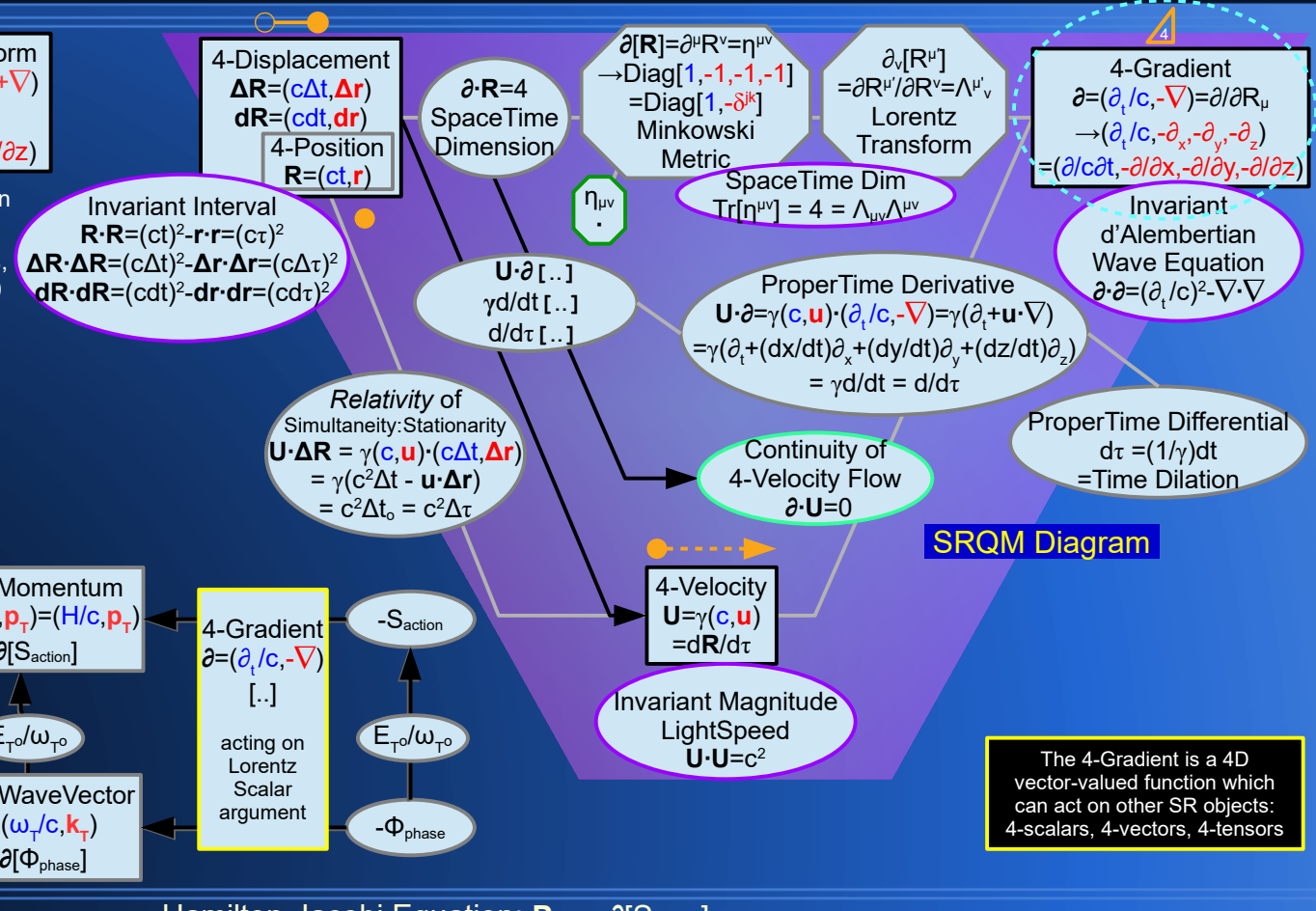
The 4-Gradient $(\partial^\mu)=(\partial_t/c,-\nabla)=(\eta^{\mu\nu}\partial_\nu)$ is the index-raised version of the more natural SR Gradient One-Form $(\partial_\mu)=(\partial_t/c,\nabla)$.
It is the 4D version of the partial derivative function of calculus, one partial for each dimensional direction, just as the Del (∇) is the 3D version of the partial derivative function.

The 4-Gradient is a 4-Vector function that can act on other 4-Scalars, 4-Vectors, or 4-Tensors. The 4-Gradient tells how things change wrt. (1-time,**3-space**) = 4D (Time·Space).
It is instrumental in creating the ProperTime Derivative
$\mathbf{U}\cdot\partial = \gamma d/dt = d/d\tau$.

The 4-Gradient plays a major role in advanced physics, showing how SR waves are formed, creating the Hamilton-Jacobi equations, the Euler-Lagrange equations, Conservation Equations ($\partial\cdot[..]=0$), Maxwell's Equations, the Lorenz Gauge, the d'Alembertian, etc.
It gives the Dimension of SpaceTime, the Minkowski Metric,and the Lorentz Transformations.

In QM, it provides the Schrödinger relations.
$P = (E/c,\mathbf{p}) = i\hbar\partial = i\hbar(\partial_t/c,-\nabla)$

The 4-Gradient is fundamental in connecting SR to QM.



The 4-Gradient is a 4D vector-valued function which can act on other SR objects: 4-scalars, 4-vectors, 4-tensors

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Hamilton-Jacobi Equation: $\mathbf{P_T} = -\partial[S_{action}]$
SR Plane-Wave Equation: $\mathbf{K_T} = -\partial[\Phi_{phase}]$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
Invariant d'Alembertian Wave Equation (∂·∂)

4-Vector SRQM Interpretation of QM

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SciRealm@aol.com
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The Lorentz Scalar Product Invariant of the 4-Gradient gives the Invariant d'Alembertian Wave Equation, describing SR wave motion. It is seen, for example, in the SR Maxwell Equation for EM light waves.

Lorenz Gauge= Conservation of EM Potential: $\partial\cdot\mathbf{A}=0$

4-Gradient $\partial=(\partial_t/c,-\nabla)$

$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$ d'Alembertian

$(\partial\cdot\partial)\mathbf{A}-\partial(\partial\cdot\mathbf{A})=\mu_o\mathbf{J}$ Maxwell EM Wave Eqn

4-(EM)VectorPotential $\mathbf{A}=A^\mu=(\varphi/c,\mathbf{a})=(\varphi_o/c^2)\mathbf{U}$, $\mathbf{A}_{EM}=A_{EM}{}^\mu=(\varphi_{EM}/c,\mathbf{a}_{EM})$

μ_o

4-CurrentDensity $\mathbf{J}=J^\mu=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})=\rho_o\mathbf{U}=qn_o\mathbf{U}=q\mathbf{N}$

Importantly, the d'Alembertian is fully from basic SR rules, with no quantum axioms required. However, it will be seen again in the Klein-Gordon RQM wave equation. Its solution provides for the introduction of SR 4-WaveVector $\mathbf{K}$ which can also be given by the negative 4-Gradient of a Lorentz Scalar Phase Φ.

4-WaveVector $\mathbf{K}=(\omega_o/c^2)\mathbf{U}=(\omega/c,\mathbf{k})=-\partial[\Phi_{phase}]=\partial[\mathbf{K}\cdot\mathbf{R}]$ = Solution to d'Alembertian

The usual mathematical (complex) plane-wave solutions apply in SR:
$f_{wave}=(a)*e^{[\pm i(\mathbf{K}\cdot\mathbf{R})]}$, with (a)mplitude possibly {4-Scalar S, 4-Vector V^μ, 4-Tensor $T^{\mu\nu}$}
{KG wave, EM wave , Grav wave}

Invariant Phase $\mathbf{K}\cdot\mathbf{R}=(\omega/c,\mathbf{k})\cdot(ct,\mathbf{r})=(\omega t-\mathbf{k}\cdot\mathbf{r})=-\Phi_{phase,plane}$

4-Position $\mathbf{R}=(ct,\mathbf{r})$ — $\mathbf{R}\cdot\mathbf{R}=(c\tau)^2$

$\mathbf{U}\cdot\partial[..]$, $\gamma d/dt[..]$, $d/d\tau[..]$

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$ — $\mathbf{U}\cdot\mathbf{U}=c^2$

ω_o/c^2

4-WaveVector $\mathbf{K}=(\omega/c,\mathbf{k})$ — $\mathbf{K}\cdot\mathbf{K}=(\omega_o/c)^2$

$..[\mathbf{K}\cdot\mathbf{R}]$, $..\int[\mathbf{K}\cdot d\mathbf{R}]$, $..[-\Phi_{phase}]$

4-Gradient $\partial=(\partial_t/c,-\nabla)$ — $\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla=(\partial_\tau/c)^2$

4-Displacement $\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$, $d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position $\mathbf{R}=(ct,\mathbf{r})$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$
$\Delta\mathbf{R}\cdot\Delta\mathbf{R}=(c\Delta t)^2-\Delta\mathbf{r}\cdot\Delta\mathbf{r}=(c\Delta\tau)^2$
$d\mathbf{R}\cdot d\mathbf{R}=(cdt)^2-d\mathbf{r}\cdot d\mathbf{r}=(c d\tau)^2$

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\partial^\mu R^\nu=\eta^{\mu\nu}$ →Diag[1,-1,-1,-1] =Diag[1,$-\delta^{jk}$] Minkowski Metric

$\partial_\nu[R^\mu]=\partial R^\mu/\partial R^\nu=\Lambda^\mu{}_\nu$ Lorentz Transform

$\eta_{\mu\nu}$

SpaceTime Dim $Tr[\eta^{\mu\nu}]=4=\Lambda_{\mu\nu}\Lambda^{\mu\nu}$

4-Gradient $\partial=(\partial_t/c,-\nabla)=\partial/\partial R_\mu$ →$(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$ $=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Invariant d'Alembertian Wave Equation $\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$

$\mathbf{U}\cdot\partial[..]$, $\gamma d/dt[..]$, $d/d\tau[..]$

ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$=\gamma(\partial_t+(dx/dt)\partial_x+(dy/dt)\partial_y+(dz/dt)\partial_z)$
$=\gamma d/dt=d/d\tau$

Relativity of Simultaneity:Stationarity
$\mathbf{U}\cdot\Delta\mathbf{R}=\gamma(c,\mathbf{u})\cdot(c\Delta t,\Delta\mathbf{r})$
$=\gamma(c^2\Delta t-\mathbf{u}\cdot\Delta\mathbf{r})$
$=c^2\Delta t_o=c^2\Delta\tau$

Continuity of 4-Velocity Flow $\partial\cdot\mathbf{U}=0$

ProperTime Differential $d\tau=(1/\gamma)dt$ =Time Dilation

SRQM Diagram

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})=d\mathbf{R}/d\tau$

Invariant Magnitude LightSpeed $\mathbf{U}\cdot\mathbf{U}=c^2$

SR is the "natural" 4D arena for the description of waves, using the d'Alembertian
$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla=(\partial_\tau/c)^2$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$Trace[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
Continuity of 4-Velocity Flow ($\partial\cdot\mathbf{U}=0$)

4-Vector SRQM Interpretation of QM

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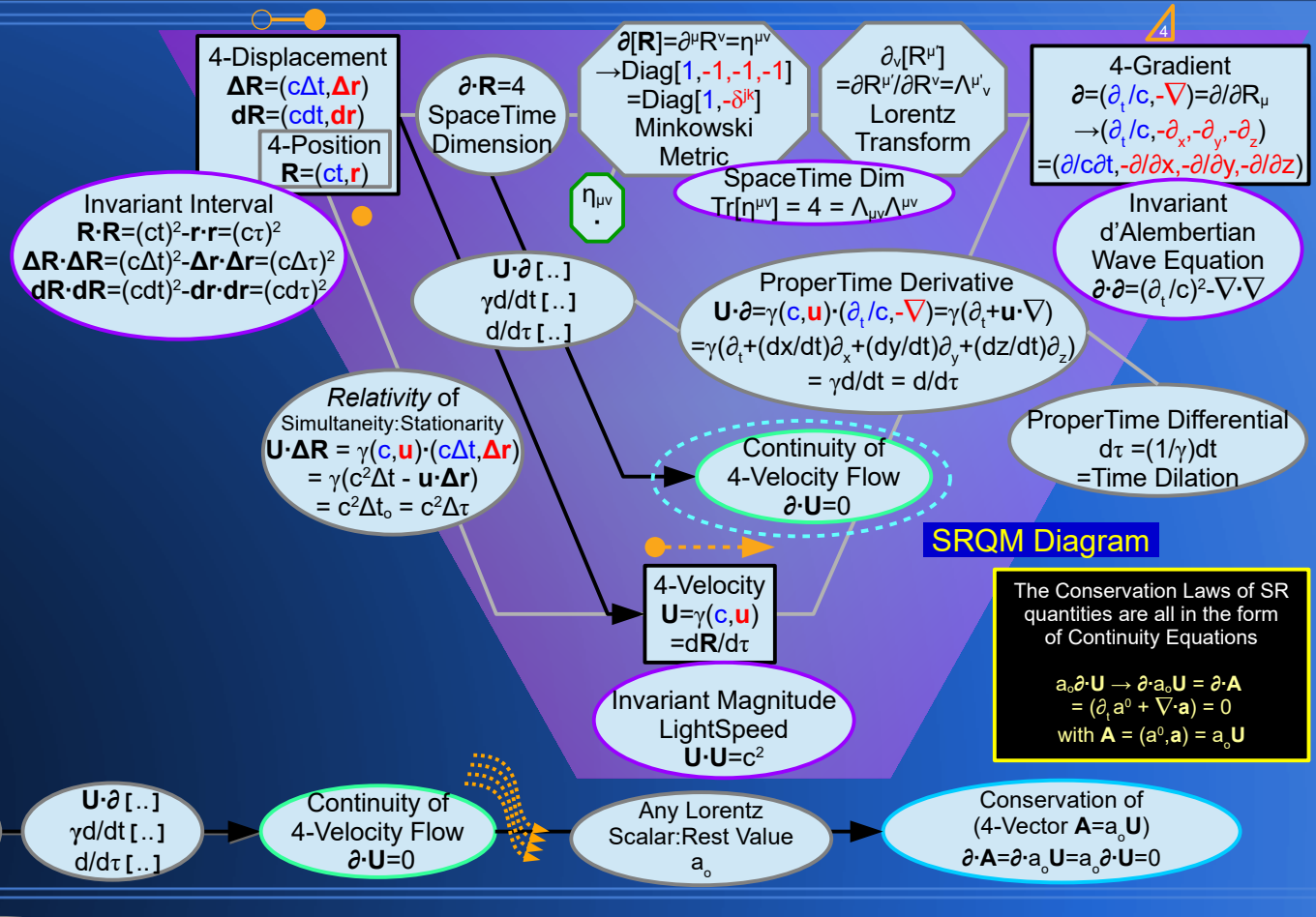
Continuity of 4-Velocity Flow $\partial\cdot\mathbf{U}=0$
This leads to all the SR Conservation Laws.

$\partial\cdot\mathbf{R} = 4$
$d/d\tau(\partial\cdot\mathbf{R}) = d/d\tau(4) = 0$
$d/d\tau(\partial\cdot\mathbf{R}) = d/d\tau(\partial)\cdot\mathbf{R} + \partial\cdot d/d\tau(\mathbf{R}) = 0$
$d/d\tau(\partial\cdot\mathbf{R}) = d/d\tau[\partial]\cdot\mathbf{R} + \partial\cdot\mathbf{U} = 0$
$\partial\cdot\mathbf{U} = -d/d\tau[\partial]\cdot\mathbf{R}$
$\partial\cdot\mathbf{U} = -(\mathbf{U}\cdot\partial)[\partial]\cdot\mathbf{R}$
$\partial\cdot\mathbf{U} = -(U_\nu\partial^\nu)[\partial_\mu]R^\mu$
$\partial\cdot\mathbf{U} = -U_\nu\partial^\nu\partial_\mu R^\mu$
$\partial\cdot\mathbf{U} = -U_\nu\partial_\mu\partial^\nu R^\mu$: I believe this is legit, partials commute
$\partial\cdot\mathbf{U} = -U_\nu\partial_\mu\eta^{\nu\mu}$
$\partial\cdot\mathbf{U} = -U_\nu(0^\nu)$
$\partial\cdot\mathbf{U} = 0$
Conservation of the 4-Velocity Flow
(4-Velocity Flow-Field)

All of the Physical Conservation Laws are in the form of a 4-Divergence ($\partial\cdot[\ ..\ ] = 0$), which is a Lorentz Invariant Scalar equation, a continuity equation.

These are local continuity equations which basically say that the temporal change of a quantity ∂_t is balanced by the flow of that quantity $\nabla\cdot$ in-to or out-of a local region.

Conservation of Charge, continuity eqn:
$\rho_o\partial\cdot\mathbf{U} = \partial\cdot\rho_o\mathbf{U} = \partial\cdot\mathbf{J} = (\partial_t\rho + \nabla\cdot\mathbf{j}) = 0$



SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
<Event> Substantiation

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Now focus on a few more of the main SR 4-Vectors.

4-Position R^μ
$\mathbf{R}=(ct,\mathbf{r})\in$<Event>

4-Velocity U^μ
$\mathbf{U}=d\mathbf{R}/d\tau=\gamma(c,\mathbf{u})$

4-Gradient ∂^μ
$\partial=\partial/\partial R_\mu=(\partial_t/c,-\nabla)$

4-Momentum P^μ
$\mathbf{P}=(E/c,\mathbf{p})=(mc,\mathbf{p})=(mc,m\mathbf{u})$
$=(E_o/c^2)\mathbf{U}=(m_o)\mathbf{U}$

4-WaveVector K^μ
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$
$=(1/c\mathrm{T},\hat{\mathbf{n}}/\lambda)=(\omega_o/c^2)\mathbf{U}$

4-CurrentDensity:ChargeFlux J^μ
$\mathbf{J}=(\rho c,\mathbf{j})=(\rho c,\rho\mathbf{u})=(\rho_o)\gamma(c,\mathbf{u})$
$=(\rho_o)\mathbf{U}=(qn_o)\mathbf{U}=(q)\mathbf{N}$

4-(Dust)NumberFlux N^μ
$\mathbf{N}=(nc,\mathbf{n})=(nc,n\mathbf{u})=(n_o)\gamma(c,\mathbf{u})$
$=(n_o)\mathbf{U}$

q

<Event> Location

<Event> Motion

<Event> Alteration

<Event> Substantiation (particle:mass m_o)

<Event> Substantiation (wave:phase oscillation ω_o)

<Event> Substantiation (charge Q or q or ρ_o)

<Event> Substantiation (dust:number N or n_o)

4-Displacement
$\mathbf{\Delta R}=(c\Delta t,\mathbf{\Delta r})$
$\mathbf{dR}=(cdt,\mathbf{dr})$
4-Position
$\mathbf{R}=(ct,\mathbf{r})$

<Event> Location

SRQM Diagram

<Event> Alteration

4-Gradient
$\partial=(\partial_t/c,-\nabla)=\partial/\partial R_\mu$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Lorentz Scalar

Substantiatied 4-Vector
Something moving at <Event>

<Event> Substantiation

Lorentz Scalar

<Event> Motion

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

These 4-Vectors give more of the main classical results of Special Relativity, including SR concepts like:
SR Particles and Waves, Matter-Wave Dispersion
Einstein's $E = mc^2 = \gamma m_o c^2 = \gamma E_o$, Rest Mass, Rest Energy
Conservation of Charge (Q), Conservation of Particle Number (N), Continuity Equations

Motion of various Lorentz Scalars leads to the "Substantiation" of the various physical SR 4-Vectors from the 4-Velocity.

Lorentz 4-Scalar a_o
4-Vector $\mathbf{A} = A^\mu = (a^0,\mathbf{a}) = a_o\mathbf{U} = a_o\gamma(c,\mathbf{u}) = a(c,\mathbf{u}) = (ac,a\mathbf{u})$

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(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
4-Momentum, Einstein's $E = mc^2$, $\mathbf{p} = m\mathbf{u}$

4-Vector SRQM Interpretation of QM

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4-Position R=(ct,**r**)
4-Gradient $\partial=(\partial_t/c,-\nabla)$
4-Velocity U = $\gamma(c,\mathbf{u})$

4-Momentum P = $(E/c,\mathbf{p}) = m_o\mathbf{U} = \gamma m_o(c,\mathbf{u}) = m(c,\mathbf{u}) = (mc,m\mathbf{u})$

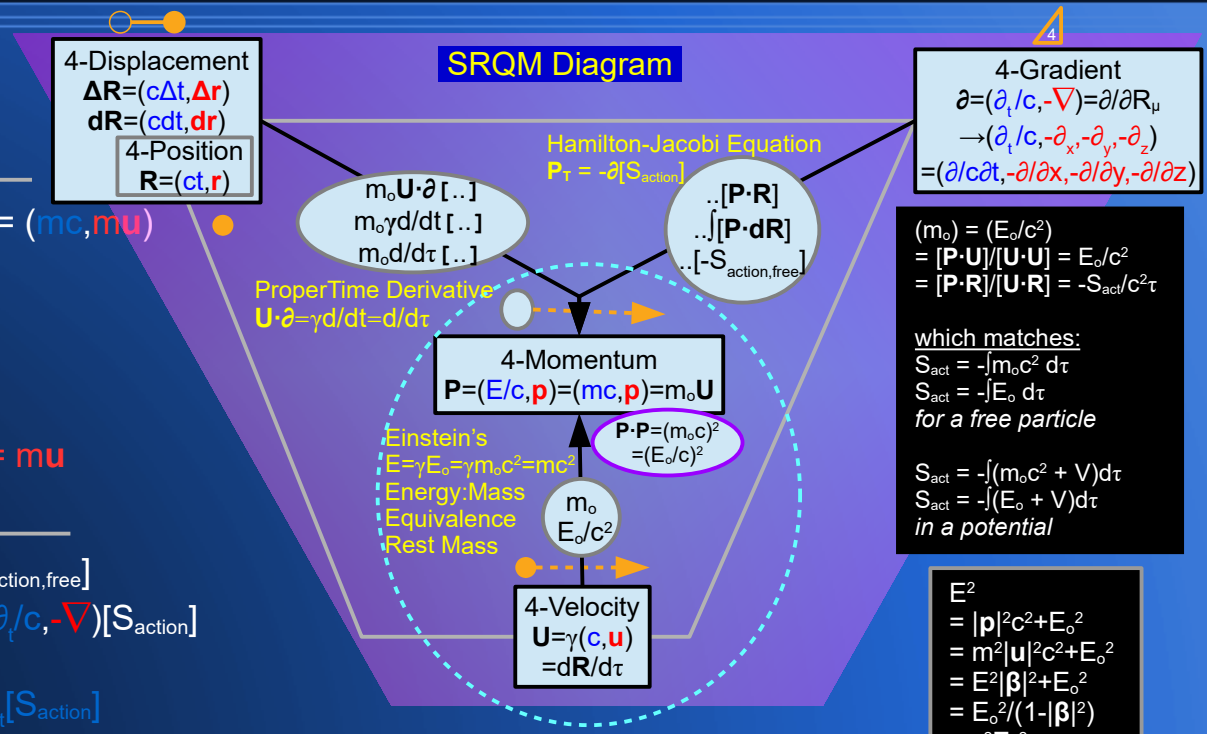
Temporal part: {energy}
$E = \gamma E_o = \gamma m_o c^2 = mc^2$
$E = m_o c^2 + (\gamma-1)m_o c^2$
(rest) + (kinetic)

Spatial part: **{3-momentum}**
$\mathbf{p} = E\mathbf{u}/c^2 = \gamma E_o\mathbf{u}/c^2 = \gamma m_o\mathbf{u} = m\mathbf{u}$

4-Momentum P = $(E/c,\mathbf{p}) = -\partial[S_{action,free}] = -(\partial_t/c,-\nabla)[S_{action,free}]$
4-TotalMomentum $\mathbf{P}_T = (E_T/c=H/c,\mathbf{p}_T) = -\partial[S_{action}] = -(\partial_t/c,-\nabla)[S_{action}]$

Temporal part: {energy}
$E = -\partial_t[S_{action,free}] : E_T = H = -\partial_t[S_{action}]$

Spatial part: **{3-momentum}**
$\mathbf{p} = +\nabla[S_{action,free}] : \mathbf{p}_T = +\nabla[S_{action}]$



$(m_o) = (E_o/c^2)$
$= [\mathbf{P}\cdot\mathbf{U}]/[\mathbf{U}\cdot\mathbf{U}] = E_o/c^2$
$= [\mathbf{P}\cdot\mathbf{R}]/[\mathbf{U}\cdot\mathbf{R}] = -S_{act}/c^2\tau$

which matches:
$S_{act} = -\int m_o c^2\, d\tau$
$S_{act} = -\int E_o\, d\tau$
for a free particle

$S_{act} = -\int(m_o c^2 + V)d\tau$
$S_{act} = -\int(E_o + V)d\tau$
in a potential

E^2
$= |\mathbf{p}|^2c^2+E_o^2$
$= m^2|\mathbf{u}|^2c^2+E_o^2$
$= E^2|\boldsymbol{\beta}|^2+E_o^2$
$= E_o^2/(1-|\boldsymbol{\beta}|^2)$
$= \gamma^2E_o^2$

$E = \gamma E_o$

$(\mathbf{P}\cdot\mathbf{P}) = (E/c)^2 - (\mathbf{p}\cdot\mathbf{p}) = (m_oc)^2$
$E^2 = (|\mathbf{p}|c)^2 + (m_oc^2)^2$
$E^2 = (|\mathbf{p}|c)^2 + (E_o)^2$: Einstein Mass:Energy

Relativistic Energy(E):Mass(m) vs **Invariant** Rest Energy(E_o):Mass(m_o)

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(0,0)-Tensor S or S_o
Lorentz Scalar

$E = \gamma E_o = \gamma m_o c^2 = mc^2$

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SRQM Diagram:
The Basis of Classical SR Physics
4-WaveVector, $|u * v_{phase}| = |v_{group} * v_{phase}| = c^2$

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4-Position R=(ct,**r**)
4-Gradient $\partial=(\partial_t/c,-\nabla)$
4-Velocity U = $\gamma(c,\mathbf{u})$

4-WaveVector K = $(\omega/c,\mathbf{k}) = (\omega_o/c^2)\mathbf{U} = \gamma(\omega_o/c^2)(c,\mathbf{u})$
$= (\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$

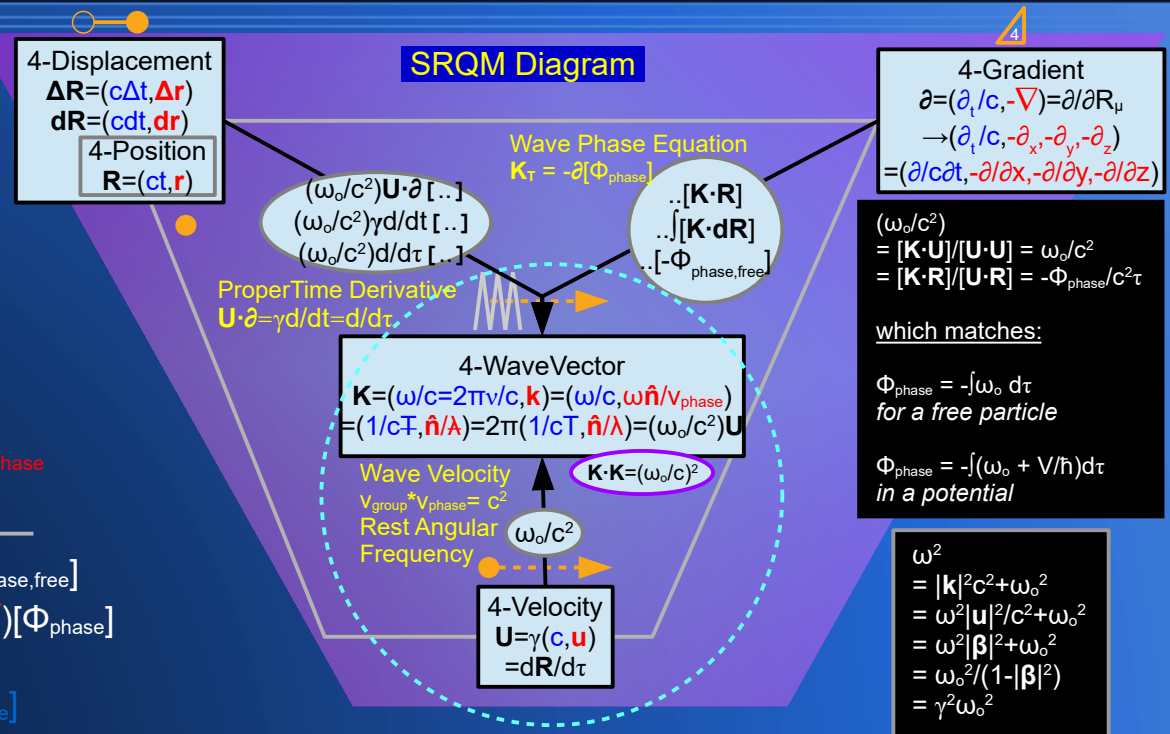
Temporal part: $\omega = \gamma\omega_o$
{angular frequency}

Spatial part: $\mathbf{k} = \gamma(\omega_o/c^2)\mathbf{u} = (\omega/c^2)\mathbf{u} = \omega\hat{\mathbf{n}}/v_{phase}$
{3-wavevector} $|u * v_{phase}| = c^2 = |v_{group} * v_{phase}|$

4-WaveVector K = $(\omega/c,\mathbf{k}) = -\partial[\Phi_{phase,free}] = -(\partial_t/c,-\nabla)[\Phi_{phase,free}]$
4-TotalWaveVector $\mathbf{K_T} = (\omega_T/c,\mathbf{k_T}) = -\partial[\Phi_{phase}] = -(\partial_t/c,-\nabla)[\Phi_{phase}]$

Temporal part: $\omega = -\partial_t[\Phi_{phase,free}] : \omega_T = -\partial_t[\Phi_{phase}]$
{angular frequency}

Spatial part: $\mathbf{k} = +\nabla[\Phi_{phase,free}] : \mathbf{k_T} = +\nabla[\Phi_{phase}]$
{3-wavevector}



$(\mathbf{K}\cdot\mathbf{K}) = (\omega/c)^2 - (\mathbf{k}\cdot\mathbf{k}) = (\omega_o/c)^2$
$\omega^2 = (|\mathbf{k}|c)^2 + (\omega_o)^2$: Matter-Wave Dispersion Relation
Relativistic AngFreq(ω) vs **Invariant** Rest AngFreq(ω_o)

$\omega = \gamma\omega_o$

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(0,0)-Tensor S or S_o
Lorentz Scalar

$v_{phase} = \omega/|\mathbf{k}| = \omega/k = E/p = mc^2/mu = c^2/u = c/\beta$
$v_{group} = \partial\omega/\partial|\mathbf{k}| = \partial\omega/\partial k = \partial E/\partial p = pc^2/E = |\mathbf{u}| = u = c\beta$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
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SRQM Diagram:
The Basis of Classical SR Physics
4-CurrentDensity, Charge Conservation

4-Vector SRQM Interpretation of QM

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4-Position R=(ct,r)
4-Gradient $\partial=(\partial_t/c,-\nabla)$
4-Velocity U = $\gamma(c,u)$

4-CurrentDensity J = $(\rho c,j) = \rho_o U = \gamma\rho_o(c,u) = \rho(c,u)$
4-ChargeFlux J

Temporal part: $\rho = \gamma\rho_o$
{charge-density}

Spatial part: $j = \gamma\rho_o u = \rho u$
{3-current-density}

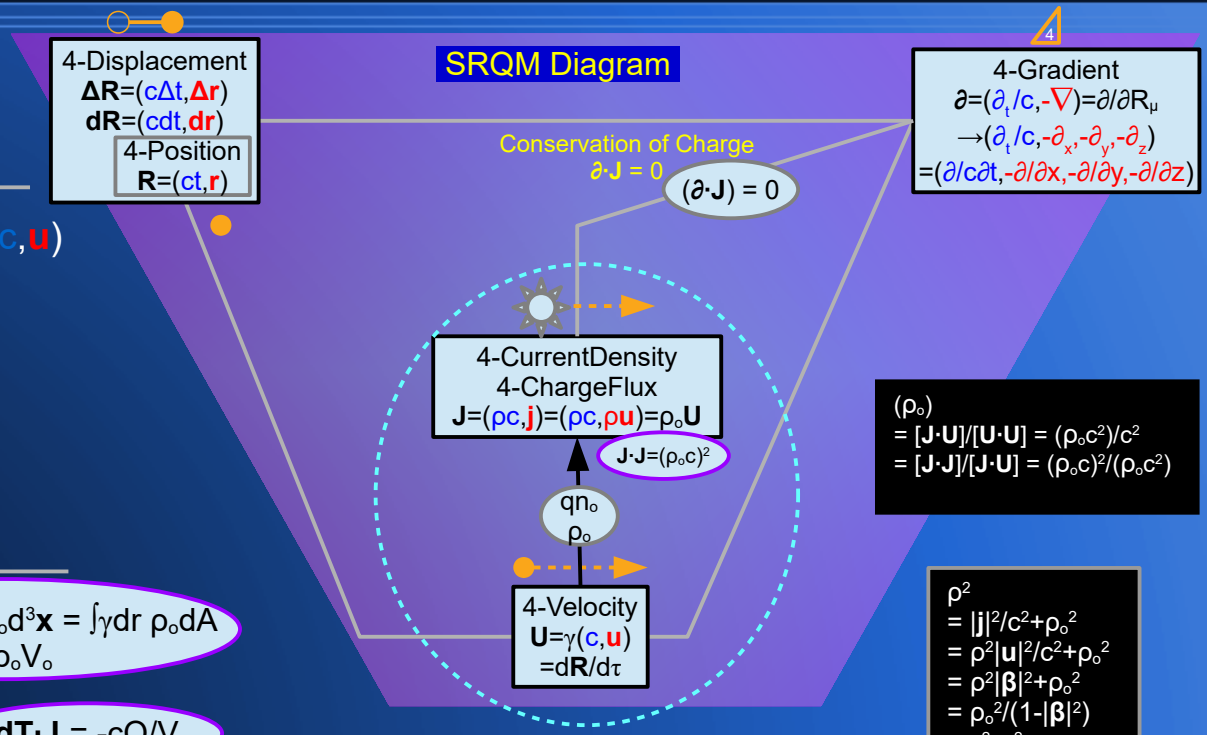
Conservation of Charge (Q)

$Q = \int\rho d^3\mathbf{x} = \int\gamma\rho_o d^3\mathbf{x} = \int\gamma dr\, \rho_o dA \rightarrow \rho_o V_o$

$\partial\cdot J = (\partial_t/c,-\nabla)\cdot(\rho c,j) = (\partial_t\rho + \nabla\cdot j) = 0$

$\int dT\cdot J = -cQ/V_o$

Continuity Equation:Noether's Theorem
The temporal change in charge density is balanced by the spatial change in current density.
Charge is neither created nor destroyed
It just moves around as charge currents...



(ρ_o)
$= [\mathbf{J}\cdot\mathbf{U}]/[\mathbf{U}\cdot\mathbf{U}] = (\rho_o c^2)/c^2$
$= [\mathbf{J}\cdot\mathbf{J}]/[\mathbf{J}\cdot\mathbf{U}] = (\rho_o c)^2/(\rho_o c^2)$

ρ^2
$= |\mathbf{j}|^2/c^2+\rho_o^2$
$= \rho^2|\mathbf{u}|^2/c^2+\rho_o^2$
$= \rho^2|\boldsymbol{\beta}|^2+\rho_o^2$
$= \rho_o^2/(1-|\boldsymbol{\beta}|^2)$
$= \gamma^2\rho_o^2$

$\rho = \gamma\rho_o$

$(\mathbf{J}\cdot\mathbf{J}) = (\rho c)^2 - (\mathbf{j}\cdot\mathbf{j}) = (\rho_o c)^2$
$\rho^2 = (|\mathbf{j}|/c)^2 + (\rho_o)^2$

Relativistic ChargeDensity(ρ) vs **Invariant** Rest ChargeDensity(ρ_o)

$\rho = \gamma\rho_o$

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(0,0)-Tensor S or S_o
Lorentz Scalar

Rest Volume
$V_o = \int\gamma d^3\mathbf{x} = \int\gamma dr\, dA$
emphasizing linear contraction along direction dr

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
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(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
4-(Dust)NumberFlux, Particle # Conservation

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4-Position R=(ct,**r**)
4-Gradient ∂=(∂_t/c,-∇)
4-Velocity U = γ(c,**u**)

4-NumberFlux N = (nc,**n**) = n_o**U** = γn_o(c,**u**) = n(c,**u**)
aka. **4-ParticleFlux:4-DustFlux**

Temporal part: n = γn_o
{number-density}

Spatial part: **n** = γn_o**u** = n**u**
{3-number-flux}

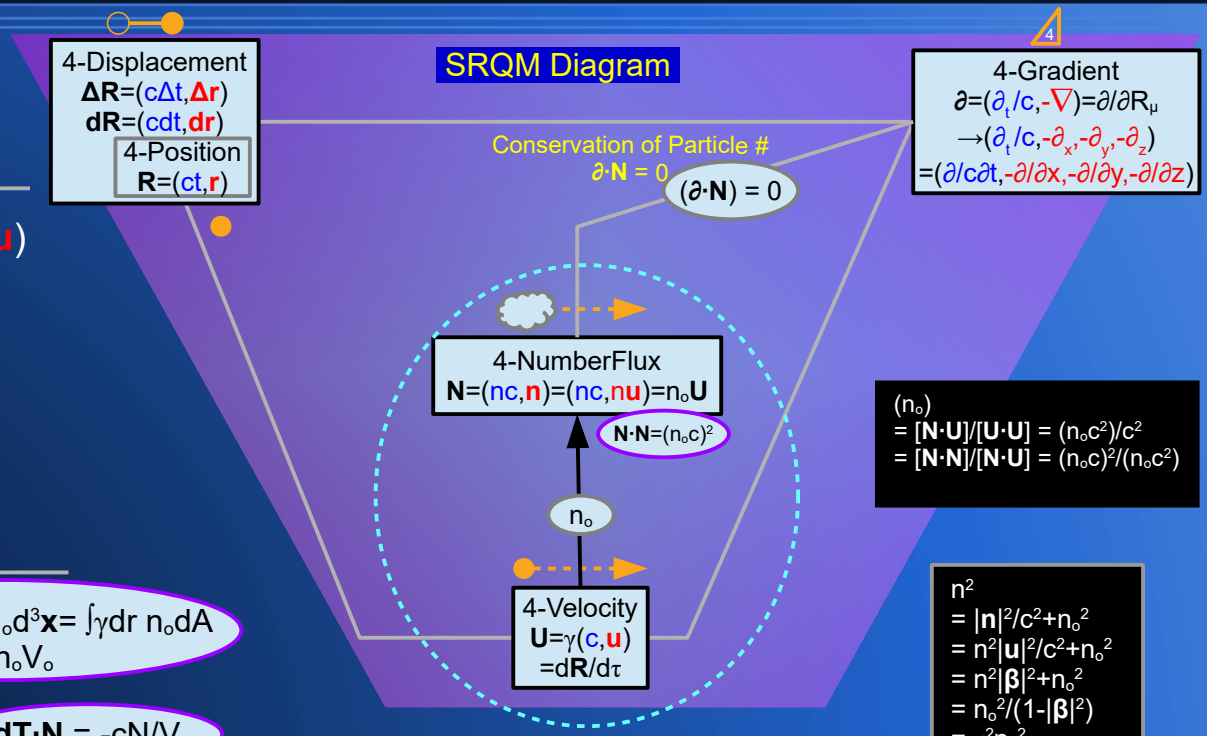
Conservation of Particle # (N)

N = ∫nd^3**x** = ∫γ$n_o d^3$**x**= ∫γdr n_odA
→ $n_o V_o$

∂·**N** = (∂_t/c,-∇)·(nc,**n**) = (∂_tn + ∇·**n**) = 0

∫**dT·N** = -cN/V_o

Continuity Equation:Noether's Theorem
The temporal change in number density is balanced by the spatial change in number-flux.
Particle # is neither created nor destroyed
It just moves around as number currents...



(n_o)
= [N·U]/[U·U] = ($n_o c^2$)/c^2
= [N·N]/[N·U] = (n_oc)²/($n_o c^2$)

n^2
= |**n**|²/c^2+n_o^2
= n^2|**u**|²/c^2+n_o^2
= n^2|**β**|²+n_o^2
= n_o^2/(1-|**β**|²)
= $\gamma^2 n_o^2$

n = γn_o

(**N·N**) = (nc)² - (**n·n**) = (n_oc)²
n^2 = (|**n**|/c)² + (n_o)²
Relativistic NumberDensity(n) vs **Invariant** Rest NumberDensity(n_o)

n = γn_o

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V_o = ∫γd^3**x** = ∫γdr dA
emphasizing linear contraction along direction dr

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V·V = $V^\mu\eta_{\mu\nu}V^\nu$ = [(v^0)² - **v·v**] = ($v^0{}_o$)²
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

Lorentz Transforms $\Lambda^{\mu'}{}_{\nu} = \partial_{\nu}[X^{\mu'}]$

(Continuous) vs (Discrete)

(Proper Det=+1) vs (Improper Det=-1)

4-Vector SRQM Interpretation of QM

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The main idea that makes a generic 4-Vector into an SR 4-Vector is that it must transform correctly according to an SR Lorentz Transformation { $\Lambda^{\mu'}{}_{\nu} = \partial X^{\mu'}/\partial X^{\nu} = \partial_{\nu}[X^{\mu'}]$}, which is basically any linear, unitary or antiunitary, transform (Determinant[$\Lambda^{\mu'}{}_{\nu}$] = ±1) which leaves the Invariant Interval unchanged.
The SR continuous transforms (variable with some parameter) have {Det = +1, Proper} and include:
"Rotation" {a mixing of space-space coordinates} and "(Velocity) Boost" {a mixing of time-space coordinates}.
The SR discrete transforms can be {Det = +1, Proper} or {Det = -1, Improper} and include:
"Space Parity-Inversion" {reversal of the all space coordinates} , "Time-Reversal" {reversal of the temporal coordinate} ,
"Identity" {no change}, various single dimension "Flips", "Fixed Rotations", and combinations of all of these discrete transforms.

SR:Lorentz Transform

$\partial_{\nu}[R^{\mu'}] = \partial R^{\mu'}/\partial R^{\nu} = \Lambda^{\mu'}{}_{\nu}$

$\Lambda^{\mu}{}_{\nu} = (\Lambda^{-1})_{\nu}{}^{\mu} : \Lambda^{\mu}{}_{\alpha}(\Lambda^{-1})^{\alpha}{}_{\nu} = \eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$

$\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$

Det[$\Lambda^{\mu}{}_{\nu}$]=±1 $\quad \Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$

Tr[$\Lambda^{\mu}{}_{\nu}$]={-∞..+∞}
=Lorentz Transform Type

Typical Lorentz Boost Transformation, for a linear-velocity frame-shift x̂-Boost:

$A^{\nu} = (a^t, a^x, a^y, a^z)$

$A^{\mu'} = (a^t, a^x, a^y, a^z)'$
$= B^{\mu'}{}_{\nu}A^{\nu}$
$= (\gamma a^t - \gamma\beta a^x, -\gamma\beta a^t + \gamma a^x, a^y, a^z)$

Continuous: Boost depends on variable parameter β, with $\gamma=1/\sqrt{[1-\beta^2]}$

Lorentz x̂-Boost Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow B^{\mu'}{}_{\nu}$ =

	t	x	y	z
t	γ	-βγ	0	0
x	-βγ	γ	0	0
y	0	0	1	0
z	0	0	0	1

Det[$B^{\mu'}{}_{\nu}$]= +1, Proper
$\gamma^2 - \beta^2\gamma^2 = +1$

x̂-Boosted 4-Vector
$\mathbf{A'}=A^{\mu'}=\Lambda^{\mu'}{}_{\nu}A^{\nu}\rightarrow B^{\mu'}{}_{\nu}A^{\nu}=(a^{0'},\mathbf{a'})$
$\rightarrow(\gamma a^t - \gamma\beta a^x, -\gamma\beta a^t + \gamma a^x, a^y, a^z)$

Proper: preserves orientation of basis

4-Vector
$\mathbf{A}=A^{\nu}=(a^0,\mathbf{a})$
$\rightarrow(a^t, a^x, a^y, a^z)$

Lorentz Parity-Inversion Transformation:

$A^{\nu} = (a^t, a^x, a^y, a^z)$

$A^{\mu'} = (a^t, a^x, a^y, a^z)'$
$= P^{\mu'}{}_{\nu}A^{\nu}$
$= (a^t, -a^x, -a^y, -a^z)$

Discrete: Parity has no variable parameters

Lorentz Parity Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow P^{\mu'}{}_{\nu}$ =

	t	x	y	z
t	1	0	0	0
x	0	-1	0	0
y	0	0	-1	0
z	0	0	0	-1

Det[$P^{\mu'}{}_{\nu}$]= -1, Improper
$(+1)(-1)^3 = -1$

Parity-Inversed 4-Vector
$\mathbf{A'}=A^{\mu'}=\Lambda^{\mu'}{}_{\nu}A^{\nu}\rightarrow P^{\mu'}{}_{\nu}A^{\nu}=(a^{0'},\mathbf{a'})$
$\rightarrow(a^t, -a^x, -a^y, -a^z)$

Improper: reverses orientation of basis

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(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

Lorentz Transforms $\Lambda^{\mu'}{}_{\nu} = \partial_{\nu}[X^{\mu'}]$

Proper Lorentz Transforms (Det=+1): Continuous: (Boost) vs (Rotation)

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

$\beta = v/c$: dimensionless Velocity Beta Factor { $\beta=(0..1)$, with speed-of-light (c) at ($\beta=1$) }
$\gamma = 1/\sqrt{[1-\beta^2]} = 1/\sqrt{[1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: dimensionless Lorentz Relativistic Gamma Factor { $\gamma=(1..\infty)$ }

Lorentz Transforms:
Lambda (Λ) for Lorentz
"B" (B) for Boost
"R" (R) for Rotation

Proper Transforms
Determinant = +1

{ $\cos^2 + \sin^2 = +1$ }

{ $\gamma^2 - \beta^2\gamma^2 = +1$ }
{ $\cosh^2 - \sinh^2 = +1$ }

ζ = rapidity = hyperbolic angle
$\gamma = \cosh[\zeta] = 1/\sqrt{[1-\beta^2]}$
$\beta\gamma = \sinh[\zeta]$
$\beta = \tanh[\zeta]$

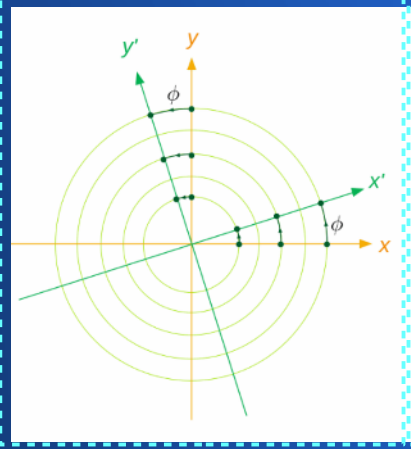
4-Vector
$\mathbf{A}=A^{\nu}=(a^0,\mathbf{a})$

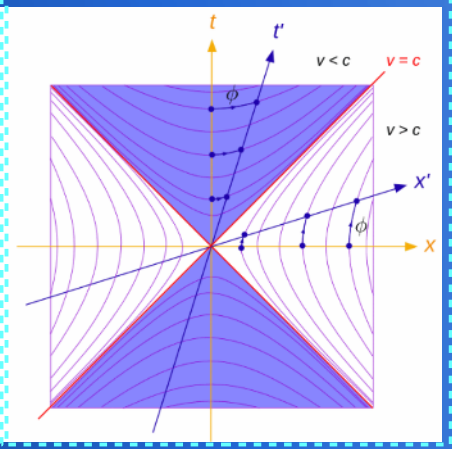
Space-Space: Lorentz Rotation Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow R^{\mu'}{}_{\nu}$ — $Tr[R^{\mu'}{}_{\nu}]=\{0..4\}$, $Det[R^{\mu'}{}_{\nu}]=+1$

Time-Space: Lorentz Boost Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow B^{\mu'}{}_{\nu}$ — $Tr[B^{\mu'}{}_{\nu}]=\{4..Infinity\}$, $Det[B^{\mu'}{}_{\nu}]=+1$

Rotated 4-Vector
Circularly-Rotated
$\mathbf{A'}=A^{\mu'}=R^{\mu'}{}_{\nu}A^{\nu}=(a^{0'},\mathbf{a'})$

Boosted 4-Vector
Hyperbolically-Rotated
$\mathbf{A'}=A^{\mu'}=B^{\mu'}{}_{\nu}A^{\nu}=(a^{0'},\mathbf{a'})$





Typical Lorentz Boost Transform (symmetric):

for a linear-velocity frame-shift (x,t)-Boost in the x̂-direction:

$\Lambda^{\mu'}{}_{\nu} \rightarrow B^{\mu'}{}_{\nu}[\boldsymbol{\zeta}] = e^{\wedge}-(\boldsymbol{\zeta}\cdot\mathbf{K}) =$

$$\begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cosh[\zeta] & -\sinh[\zeta] & 0 & 0 \\ -\sinh[\zeta] & \cosh[\zeta] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = e^{\wedge}(\zeta_x \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$A^{\nu} = (a^t, a^x, a^y, a^z)$
$A^{\mu'} = (a^t, a^x, a^y, a^z)' = B^{\mu'}{}_{\nu}A^{\nu} = (\gamma a^t - \gamma\beta a^x, -\gamma\beta a^t + \gamma a^x, a^y, a^z)$

Typical Lorentz Rotation Transform (non-symmetric):

for an angular-displacement frame-shift (x,y)-Rotation about the ẑ-direction:

$\Lambda^{\mu'}{}_{\nu} \rightarrow R^{\mu'}{}_{\nu}[\boldsymbol{\theta}] = e^{\wedge}(\boldsymbol{\theta}\cdot\mathbf{J}) =$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[\theta] & -\sin[\theta] & 0 \\ 0 & \sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = e^{\wedge}(\theta_z \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

SR:Lorentz Transform
$\partial_{\nu}[R^{\mu'}] = \partial R^{\mu'}/\partial R^{\nu} = \Lambda^{\mu'}{}_{\nu}$
$\Lambda^{\mu}{}_{\nu} = (\Lambda^{-1})_{\nu}{}^{\mu} : \Lambda^{\mu}{}_{\alpha}(\Lambda^{-1})^{\alpha}{}_{\nu} = \eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$
$\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$
$Det[\Lambda^{\mu}{}_{\nu}]=\pm1$ — $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$
$Tr[\Lambda^{\mu}{}_{\nu}]=\{-\infty..+\infty\}$
=Lorentz Transform Type

$A^{\nu} = (a^t, a^x, a^y, a^z)$
$A^{\mu'} = (a^t, a^x, a^y, a^z)' = R^{\mu'}{}_{\nu}A^{\nu} = (a^t, \cos[\theta]a^x - \sin[\theta]a^y, \sin[\theta]a^x + \cos[\theta]a^y, a^z)$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

The Lorentz Rotation $R^{\mu'}{}_{\nu}$ is a 4D rotation matrix. It simply adds the time component, which remains unchanged (1), to the standard 3D rotation matrix.

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

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A Tensor Study of Physical 4-Vectors

Lorentz Transforms $\Lambda^{\mu'}{}_{\nu} = \partial_{\nu}[X^{\mu'}]$

Proper Lorentz Transforms (Det=+1): (Boost) vs (Rotation) vs (Identity)

4-Vector SRQM Interpretation of QM

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General Lorentz Boost Transform (symmetric,continuous):
for a linear-velocity frame-shift (Boost)
in the **v**/c=**β**=(β^1,β^2,β^3)-direction:
$\Lambda^{\mu'}{}_{\nu} \rightarrow B^{\mu'}{}_{\nu} =$

$$\begin{bmatrix} \gamma & -\gamma\beta_j \\ -\gamma\beta^i & (\gamma-1)\beta^i\beta_j/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\delta^i{}_j \end{bmatrix}$$

$\Lambda_{\mu\nu}\Lambda^{\mu\nu} = 4$

$\Lambda^{\mu'}{}_{\nu} = \begin{bmatrix} \Lambda^{0'}{}_{0}, \Lambda^{0'}{}_{j} \\ \Lambda^{i'}{}_{0}, \Lambda^{i'}{}_{j} \end{bmatrix}$

$\mathrm{Tr}[\eta^{\mu'}{}_{\nu}]=4$
$\mathrm{Det}[\eta^{\mu'}{}_{\nu}]=+1$

4-Vector
$\mathbf{A}=A^{\nu}=(a^0,\mathbf{a})$

No mixing
Lorentz Identity Transform
$\Lambda^{\mu'}{}_{\nu} \rightarrow \eta^{\mu'}{}_{\nu} = \delta^{\mu'}{}_{\nu} = I_{(4)}$

Identical 4-Vector
Un-Rotated
$\mathbf{A'}=A^{\mu'}=\eta^{\mu'}{}_{\nu}A^{\nu}=(a^{0'},\mathbf{a'})=\mathbf{A}$

Space-Space
Lorentz Rotation Transform
$\Lambda^{\mu'}{}_{\nu} \rightarrow R^{\mu'}{}_{\nu}$

$\mathrm{Tr}[R^{\mu'}{}_{\nu}]=\{0..4\}$
$\mathrm{Det}[R^{\mu'}{}_{\nu}]=+1$

Rotated 4-Vector
Circularly-Rotated
$\mathbf{A'}=A^{\mu'}=R^{\mu'}{}_{\nu}A^{\nu}=(a^{0'},\mathbf{a'})$

Time-Space
Lorentz Boost Transform
$\Lambda^{\mu'}{}_{\nu} \rightarrow B^{\mu'}{}_{\nu}$

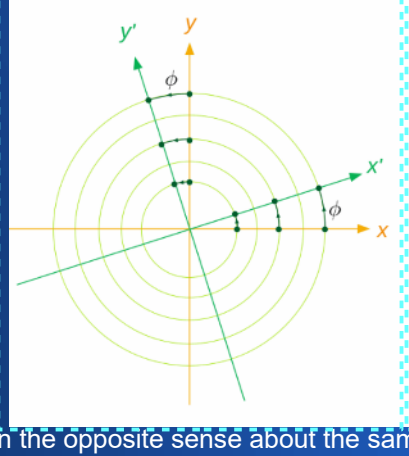
$\mathrm{Tr}[B^{\mu'}{}_{\nu}]=\{4..\mathrm{Infinity}\}$
$\mathrm{Det}[B^{\mu'}{}_{\nu}]=+1$

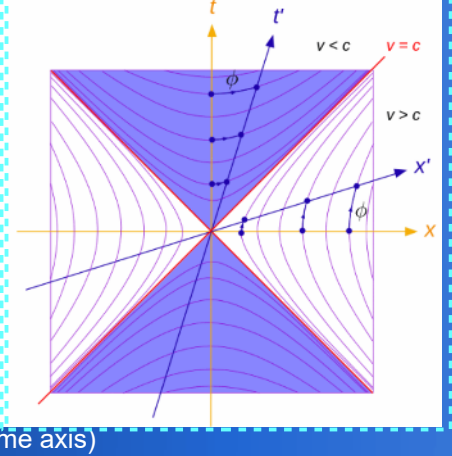
Boosted 4-Vector
Hyperbolically-Rotated
$\mathbf{A'}=A^{\mu'}=B^{\mu'}{}_{\nu}A^{\nu}=(a^{0'},\mathbf{a'})$

General Lorentz Rotation Transform (non-symmetric,continuous):
for an angular-displacement frame-shift (Rotation)
angle θ about the $\hat{\mathbf{n}}$=(n^1,n^2,n^3)-direction:
$\Lambda^{\mu'}{}_{\nu} \rightarrow R^{\mu'}{}_{\nu} =$

$$\begin{bmatrix} 1 & 0_j \\ 0^i & (\delta^i{}_j-n^in_j)\cos(\theta)-(\varepsilon^i{}_{jk}n^k)\sin(\theta)+n^in_j \end{bmatrix}$$

The Lorentz Identity Transform is the limit of both the Rotation and Boost Transfoms when the respective "rotation angle" is 0





Lorentz Identity Transform (symmetric,"discrete:continuous"):
for a non-frame-shift (Identity)
in any direction
$\Lambda^{\mu'}{}_{\nu} \rightarrow \eta^{\mu'}{}_{\nu} = \delta^{\mu'}{}_{\nu} = \mathrm{Diag}[1,\delta^i{}_j] = I_{(4)} =$

$$\begin{bmatrix} 1 & 0_j \\ 0^i & \delta^i{}_j \end{bmatrix}$$

SR:Lorentz Transform

$\partial_{\nu}[R^{\mu'}] = \partial R^{\mu'}/\partial R^{\nu} = \Lambda^{\mu'}{}_{\nu}$

$\Lambda^{\mu}{}_{\nu} = (\Lambda^{-1})_{\nu}{}^{\mu} : \Lambda^{\mu}{}_{\alpha}(\Lambda^{-1})^{\alpha}{}_{\nu} = \eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$

$\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$

$\mathrm{Tr}[\Lambda^{\mu}{}_{\nu}]=\{-\infty..+\infty\}$
=Lorentz Transform Type

$\mathrm{Det}[\Lambda^{\mu}{}_{\nu}]=\pm1$

$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4$

β = v/c: dimensionless Velocity Beta Factor { β=(0..1), with speed-of-light (c) at (β=1) }
$\gamma = 1/\sqrt{[1-\beta^2]} = 1/\sqrt{[1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: dimensionless Lorentz Relativistic Gamma Factor { γ=(1..∞) }
Identity transformation for zero relative motion:boost/rotation: $B[0] = R[0] = I_{(4)}$
Proper Transformation = positive unit determinant: det[B] = det[R] = det[η] = +1.
Inverses: $B(v)^{-1} = B(-v)$ (relative motion in the opposite direction), and $R(\theta,\hat{\mathbf{n}})^{-1} = R(-\theta,\hat{\mathbf{n}})$ (rotation in the opposite sense about the same axis)
Matrix symmetry: B is symmetric (equals transpose, $B=B^T$), while R is nonsymmetric but orthogonal (transpose equals inverse, $R^T = R^{-1}$)

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(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

The Lorentz Rotation $R^{\mu'}{}_{\nu}$ (Tr={0..4}) meets the Lorentz Boost $B^{\mu'}{}_{\nu}$ (Tr={4..∞}) at the 4D Identity $I_{(4)} = \delta^{\mu'}{}_{\nu}$ (Tr={4})

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

Lorentz Transforms $\Lambda^{\mu'}{}_{\nu} = \partial_{\nu}[X^{\mu'}]$

Discrete (non-continuous) (Parity-Inversion) vs (Time-Reversal) vs (Identity)

4-Vector SRQM Interpretation of QM

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SciRealm@aol.com
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General Lorentz Parity-Inversion (Space-Reversal) Transform:
$\Lambda^{\mu'}{}_{\nu} \rightarrow P^{\mu'}{}_{\nu}$ (Improper,symmetric,discrete)
$= \begin{bmatrix} 1 & 0_j \\ 0^i & -\delta^i_j \end{bmatrix}$

General Lorentz Time-Reversal Transform:
$\Lambda^{\mu'}{}_{\nu} \rightarrow T^{\mu'}{}_{\nu}$ (Improper,symmetric,discrete)
$= \begin{bmatrix} -1 & 0_j \\ 0^i & \delta^i_j \end{bmatrix}$

General Lorentz Identity Transform:
$\Lambda^{\mu'}{}_{\nu} \rightarrow \eta^{\mu'}{}_{\nu} = \delta^{\mu'}{}_{\nu} = I_{(4)}$ (Proper,symmetric,discrete)
$= \begin{bmatrix} 1 & 0_j \\ 0^i & \delta^i_j \end{bmatrix}$

$\Lambda_{\mu\nu}\Lambda^{\mu\nu} = 4$

$\Lambda^{\mu'}{}_{\nu} = \begin{bmatrix} \Lambda^{0'}{}_{0}, \Lambda^{0'}{}_{j} \\ \Lambda^{i'}{}_{0}, \Lambda^{i'}{}_{j} \end{bmatrix}$

SR:Lorentz Transform
$\partial_{\nu}[R^{\mu'}] = \partial R^{\mu'}/\partial R^{\nu} = \Lambda^{\mu'}{}_{\nu}$
$\Lambda^{\mu}{}_{\nu} = (\Lambda^{-1})_{\nu}{}^{\mu} : \Lambda^{\mu}{}_{\alpha}(\Lambda^{-1})^{\alpha}{}_{\nu} = \eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$
$\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$
$\mathrm{Det}[\Lambda^{\mu}{}_{\nu}]=\pm1$ $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$
$\mathrm{Tr}[\Lambda^{\mu}{}_{\nu}]=\{-\infty..+\infty\}$
=Lorentz Transform Type

4-Vector
$\mathbf{A}=A^{\nu}=(a^0,\mathbf{a})$

No mixing	Time	Space	TimeSpace
$\mathrm{Tr}[\eta^{\mu'}{}_{\nu}]= +4$	$\mathrm{Tr}[T^{\mu'}{}_{\nu}]= +2$	$\mathrm{Tr}[P^{\mu'}{}_{\nu}]= -2$	$\mathrm{Tr}[(PT)^{\mu'}{}_{\nu}]= -4$
Lorentz Identity Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow\eta^{\mu'}{}_{\nu}=\delta^{\mu'}{}_{\nu}=I_{(4)}$ $= \begin{bmatrix} 1 & 0_j \\ 0^i & \delta^i_j \end{bmatrix}$	Lorentz Time-Reversal Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow T^{\mu'}{}_{\nu}$ $= \begin{bmatrix} -1 & 0_j \\ 0^i & \delta^i_j \end{bmatrix}$	Lorentz Parity-Inversion Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow P^{\mu'}{}_{\nu}$ $= \begin{bmatrix} 1 & 0_j \\ 0^i & -\delta^i_j \end{bmatrix}$	Lorentz ComboPT Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow(PT)^{\mu'}{}_{\nu} = -I_{(4)}$ $= \begin{bmatrix} -1 & 0_j \\ 0^i & -\delta^i_j \end{bmatrix}$
$\mathrm{Det}[\eta^{\mu'}{}_{\nu}]$ $=+1$	$\mathrm{Det}[T^{\mu'}{}_{\nu}]=\mathrm{Det}[P^{\mu'}{}_{\nu}]$ $= -1$		$\mathrm{Det}[(PT)^{\mu'}{}_{\nu}]$ $=+1$
Identical 4-Vector $\mathbf{A'}=A^{\mu'}=\eta^{\mu'}{}_{\nu}A^{\nu}=(a^{0'},\mathbf{a'})$ $=(a^0,\mathbf{a})=\mathbf{A}$	Time-Reversed 4-Vector $\mathbf{A'}=A^{\mu'}=T^{\mu'}{}_{\nu}A^{\nu}=(a^{0'},\mathbf{a'})$ $=(-a^0,\mathbf{a})$	Parity-Inverted 4-Vector $\mathbf{A'}=A^{\mu'}=P^{\mu'}{}_{\nu}A^{\nu}=(a^{0'},\mathbf{a'})$ $=(a^0,-\mathbf{a})$	Combo PT'd 4-Vector $\mathbf{A'}=A^{\mu'}=(PT)^{\mu'}{}_{\nu}A^{\nu}=(a^{0'},\mathbf{a'})$ $=(-a^0,-\mathbf{a})$
Lorentz Identity Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow\eta^{\mu'}{}_{\nu}=\delta^{\mu'}{}_{\nu}$	Lorentz Time-Reversal Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow T^{\mu'}{}_{\nu}$	Lorentz Parity-Inversion Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow P^{\mu'}{}_{\nu}$	Lorentz ComboPT Transform $\Lambda^{\mu'}{}_{\nu}\rightarrow(PT)^{\mu'}{}_{\nu}$

Original 4-Vector
$\mathbf{A}=A^{\nu}=(a^0,\mathbf{a})$

Both the Parity-Inversion (P) and Time-Reversal (T) have a Determinant of -1, which is an improper transform.
However, combinations (PP), (TT), (PT) have overall Determinant of +1, which is proper.
Classical SR Time Reversal neglects spin and charge. When included, there is also a Charge-Conjugation(C) transform.
Then one gets (CC),(PP),(TT),({PT}{PT}), & permutations of (CPT) transforms all leading back to the Identity ($I_{(4)}$).

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(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Note that the Trace of Discrete Lorentz Transforms goes in steps from {-4,-2,2,4}. As we will see in a bit, this is a major hint for SR antimatter and CPT Symmetry.

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Lorentz Transforms $\Lambda^{\mu'}{}_{\nu} = \partial_{\nu}[X^{\mu'}]$

Discrete & Fixed Rotation → Particle Exchange

Lorentz Coordinate-Flip Transforms

4-Vector SRQM Interpretation of QM

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Lorentz Flip-t Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow Ft^{\mu'}{}_{\nu} = T^{\mu'}{}_{\nu}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & -1 & 0 & 0 & 0 \\ x & 0 & 1 & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array}$$

$Tr[Ft^{\mu'}{}_{\nu}] = 2$
$Det[Ft^{\mu'}{}_{\nu}] = -1$

Lorentz Flip-x Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow Fx^{\mu'}{}_{\nu} =$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array}$$

$Tr[Fx^{\mu'}{}_{\nu}] = 2$
$Det[Fx^{\mu'}{}_{\nu}] = -1$

Lorentz Flip-y Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow Fy^{\mu'}{}_{\nu} =$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & 1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array}$$

$Tr[Fy^{\mu'}{}_{\nu}] = 2$
$Det[Fy^{\mu'}{}_{\nu}] = -1$

Lorentz Flip-z Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow Fz^{\mu'}{}_{\nu} =$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & 1 & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & -1 \end{array}$$

$Tr[Fz^{\mu'}{}_{\nu}] = 2$
$Det[Fz^{\mu'}{}_{\nu}] = -1$

$Tr[R^{\mu'}{}_{\nu}] = 2+2\cos[\theta] = \{0..4\}$
$Det[R^{\mu'}{}_{\nu}] = \cos[\theta]^2 + \sin[\theta]^2 = +1$

Lorentz z-Rotation Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow R^{\mu'}{}_{\nu} =$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & \cos[\theta] & -\sin[\theta] & 0 \\ y & 0 & \sin[\theta] & \cos[\theta] & 0 \\ z & 0 & 0 & 0 & 1 \end{array}$$

$Tr[Fxy^{\mu'}{}_{\nu}] = 2+2\cos[\pi] = 0$
$Det[Fxy^{\mu'}{}_{\nu}] = \cos[\pi]^2 + \sin[\pi]^2 = +1$

Lorentz Flip-xy Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow Fxy^{\mu'}{}_{\nu} =$ Exchange

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array}$$

SR:Lorentz Transform

$\partial_{\nu}[R^{\mu'}] = \partial R^{\mu'}/\partial R^{\nu} = \Lambda^{\mu'}{}_{\nu}$

$\Lambda^{\mu}{}_{\nu} = (\Lambda^{-1})_{\nu}{}^{\mu} : \Lambda^{\mu}{}_{\alpha}(\Lambda^{-1})^{\alpha}{}_{\nu} = \eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$

$\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$

$Det[\Lambda^{\mu}{}_{\nu}] = \pm 1$ $\quad \Lambda_{\mu\nu}\Lambda^{\mu\nu} = 4 = \Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$

$Tr[\Lambda^{\mu}{}_{\nu}] = \{-\infty..+\infty\}$
=Lorentz Transform Type

Any single Lorentz Flip Transform is Improper, with a Determinant of -1. However, pairwise combinations are Proper, with a Determinant of +1. All single flips have Trace of 2.

The combination of any two Spatial Flips is the equivalent of a Spatial Rotation by (π) about the associated rotational axis.
$\sin(\pi) = 0$, $\cos(\pi) = -1$
Since this is a Proper transform, it is also the equivalent of a particle location exchange.

The combination of all three Spatial Flips, Flip-xyz, gives the Lorentz Parity Transform, which is again Improper, with Trace of -2

The Flip-t is the standard Lorentz Time-Reversal, Improper.

$Tr[Fxyz^{\mu'}{}_{\nu}] = -2$
$Det[Fxyz^{\mu'}{}_{\nu}] = -1$

Lorentz Parity Transform $\Lambda^{\mu'}{}_{\nu} \rightarrow P^{\mu'}{}_{\nu} = Fxyz^{\mu'}{}_{\nu}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & 0 & -1 & 0 & 0 \\ y & 0 & 0 & -1 & 0 \\ z & 0 & 0 & 0 & -1 \end{array}$$

Lorentz Transform $\partial_{\nu}[R^{\mu'}] = \Lambda^{\mu'}{}_{\nu}$
$[\Lambda^{0'}{}_{0}, \Lambda^{0'}{}_{j}]$ temporal-spatial-mixed
$[\Lambda^{i'}{}_{0}, \Lambda^{i'}{}_{j}]$ components

InnerProd$[\Lambda^{\mu'}{}_{\nu}]$
All Lorentz Trans.
$\Lambda_{\mu\nu}\Lambda^{\mu\nu} = 4 = \Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$

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(0,0)-Tensor S or S_o
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SRQM Lorentz Transforms $\Lambda^{\mu'}{}_{\nu} = \partial_{\nu}[X^{\mu'}]$

Lorentz Transform Connection Map

4-Vector SRQM Interpretation of QM

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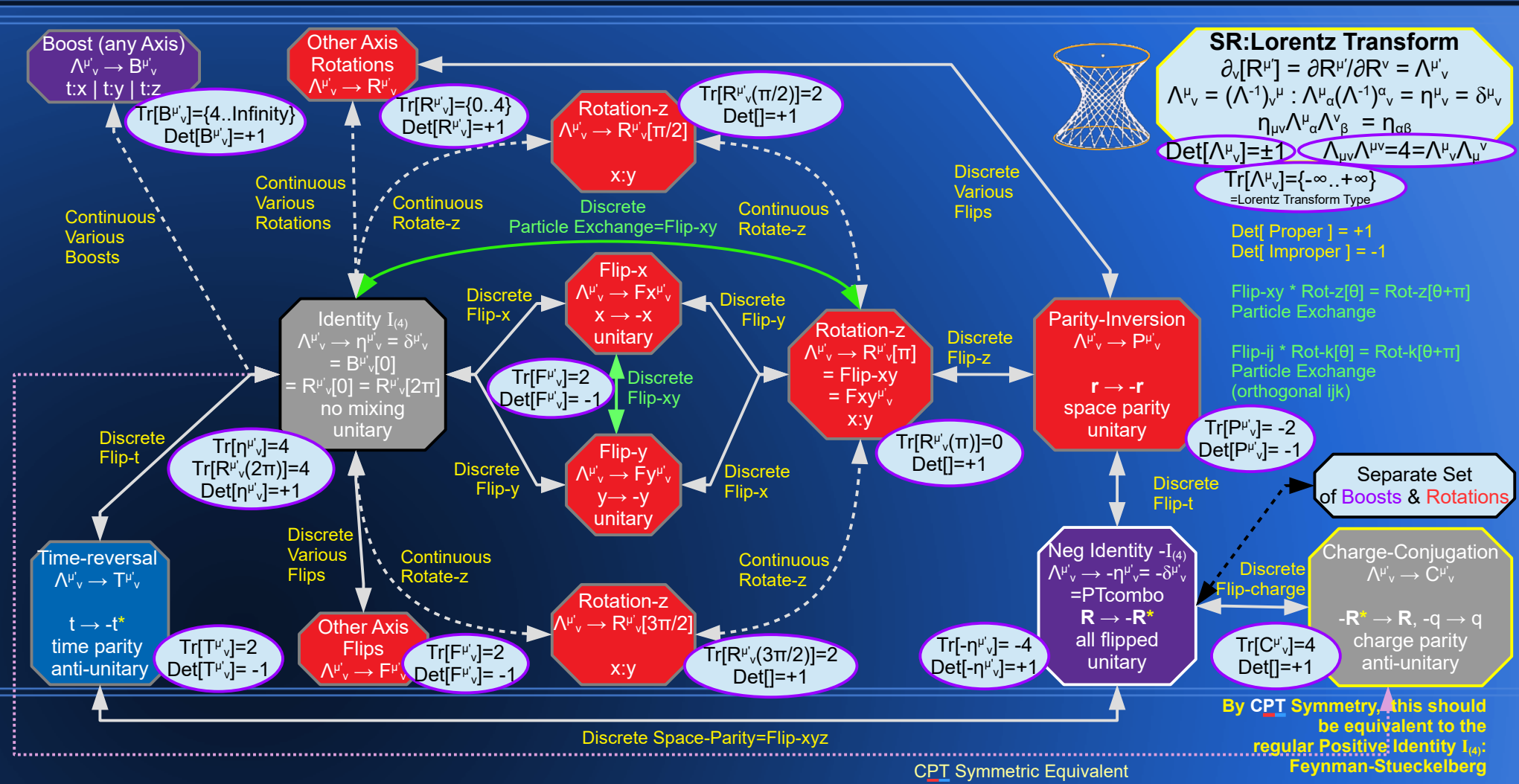

SR:Lorentz Transform

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$\Lambda^{\mu}{}_{\nu} = (\Lambda^{-1})_{\nu}{}^{\mu} : \Lambda^{\mu}{}_{\alpha}(\Lambda^{-1})^{\alpha}{}_{\nu} = \eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$

$\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$

$Det[\Lambda^{\mu}{}_{\nu}]=\pm 1$ $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$

$Tr[\Lambda^{\mu}{}_{\nu}]=\{-\infty..+\infty\}$
=Lorentz Transform Type

Det[Proper] = +1
Det[Improper] = -1

Flip-xy * Rot-z[θ] = Rot-z[θ+π]
Particle Exchange

Flip-ij * Rot-k[θ] = Rot-k[θ+π]
Particle Exchange
(orthogonal ijk)

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

4-Vector SRQM Interpretation of QM

SRQM Lorentz Transforms $\Lambda^{\mu'}{}_{\nu} = \partial_{\nu}[X^{\mu'}]$

Lorentz Transform Connection Map – Discrete Transforms

CPT, Big-Bang, (Matter↔AntiMatter), Arrow(s)-of-Time

SciRealm.org
John B. Wilson
SciRealm@aol.com
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Examine all possible combinations of **Discrete Lorentz Transformations** which are Linear (Determinant of ±1).

A lot of the standard SR texts only mention (P)arity-Inverse and (T)ime-Reversal. However, there are many others, including (F)lips and (R)otations of a fixed amount. However, the (T)imeReversal and Combo(P)arity(T)ime take one into a separate section of the chart. Taking into account all possible discrete Lorentz Transformations fills in the rest of the chart. The resulting interpretation is that there is **CPT Symmetry** (Charge:Parity:Time) and **Dual** Time·Space (with reversed timeflow). In other words, one can go from the Identity Transform (all +1) to the Negative Identity Transform (all -1) by doing a Combo PT Lorentz Transform or by Negating the Charge (Matter↔Antimatter). The Feynman-Stueckelberg **CPT** Interpretation (AntiMatter moving spacetime-backward = NormalMatter moving spacetime-forward) aligns with this as a Dual-Universe "AntiMatter" Side.

This is similar to Dirac's prediction of AntiMatter, but without the formal need of Quantum Mechanics, or Spin. In fact, it is more general than Dirac's work, which was about the electron. This is from general Lorentz Transforms for any kind of particle:event.

SR:Lorentz Transform

$\partial_{\nu}[R^{\mu'}] = \partial R^{\mu'}/\partial R^{\nu} = \Lambda^{\mu'}{}_{\nu}$

$\Lambda^{\mu}{}_{\nu} = (\Lambda^{-1})_{\nu}{}^{\mu} : \Lambda^{\mu}{}_{\alpha}(\Lambda^{-1})^{\alpha}{}_{\nu} = \eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$

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$\mathrm{Tr}[\Lambda^{\mu}{}_{\nu}]=\{-\infty..+\infty\}$

=Lorentz Transform Type

Tao – I Ching – YinYang fantastic metaphors for SR SpaceTime...

Tao: "Flow of the Universe" "way, path, route, road"

I Ching: "Book of Changes" "Transformations"

YinYang: "Positive/Negative" "complementary opposites"

Matter-AntiMatter
Dual balance along Temporal
Binary Spatial states
for 3 units:dimensions
Discrete Lorentz
Transform (1,1)-Tensor
{ octagon representation }
Pair production (+ -)
in little circles (• •)

t	x	y	z	Discrete **NormalMatter** (NM) Lorentz Transform Type	Trace : Determinant
+1	**+1**	**+1**	**+1**	NM-Minkowski-Identity : AM-Flip-txyz=AM-ComboPT	Tr = +4 : Det = +1 Proper
+1	+1	+1	-1	NM-Flip-z	Tr = +2 : Det = -1 Improper
+1	+1	-1	+1	NM-Flip-y	Tr = +2 : Det = -1 Improper
+1	+1	-1	-1	NM-Flip-yz=NM-Rotate-yz(π)	Tr = 0 : Det = +1 Proper
+1	-1	+1	+1	NM-Flip-x	Tr = +2 : Det = -1 Improper
+1	-1	+1	-1	NM-Flip-xz=NM-Rotate-xz(π)	Tr = 0 : Det = +1 Proper
+1	-1	-1	+1	NM-Flip-xy=NM-Rotate-xy(π)	Tr = 0 : Det = +1 Proper
+1	-1	-1	-1	NM-Flip-xyz=NM-ParityInverse:AM-Flip-t=AM-TimeReversal	Tr = -2 : Det = -1 Improper
-1	+1	+1	+1	AM-Flip-xyz=AM-ParityInverse:NM-Flip-t=NM-TimeReversal	Tr = +2 : Det = -1 Improper
-1	+1	+1	-1	AM-Flip-xy=AM-Rotate-xy(π)	Tr = 0 : Det = +1 Proper
-1	+1	-1	+1	AM-Flip-xz=AM-Rotate-xz(π)	Tr = 0 : Det = +1 Proper
-1	+1	-1	-1	AM-Flip-x	Tr = -2 : Det = -1 Improper
-1	-1	+1	+1	AM-Flip-yz=AM-Rotate-yz(π)	Tr = 0 : Det = +1 Proper
-1	-1	+1	-1	AM-Flip-y	Tr = -2 : Det = -1 Improper
-1	-1	-1	+1	AM-Flip-z	Tr = -2 : Det = -1 Improper
-1	**-1**	**-1**	**-1**	AM-Minkowski-Identity : NM-Flip-txyz=NM-ComboPT	Tr = -4 : Det = +1 Proper
t	x	y	z	Discrete **AntiMatter** (AM) Lorentz TransformType	Trace : Determinant

Combo (P)arityInverse & (T)imeReversal

take

NormalMatter
↕↕
AntiMatter

SRQM Lorentz Transforms $\Lambda^{\mu'}{}_{\nu} = \partial_{\nu}[X^{\mu'}]$

Lorentz Transform Connection Map – Discrete Transforms

CPT, Big-Bang, (Matter↔AntiMatter), Arrow(s)-of-Time

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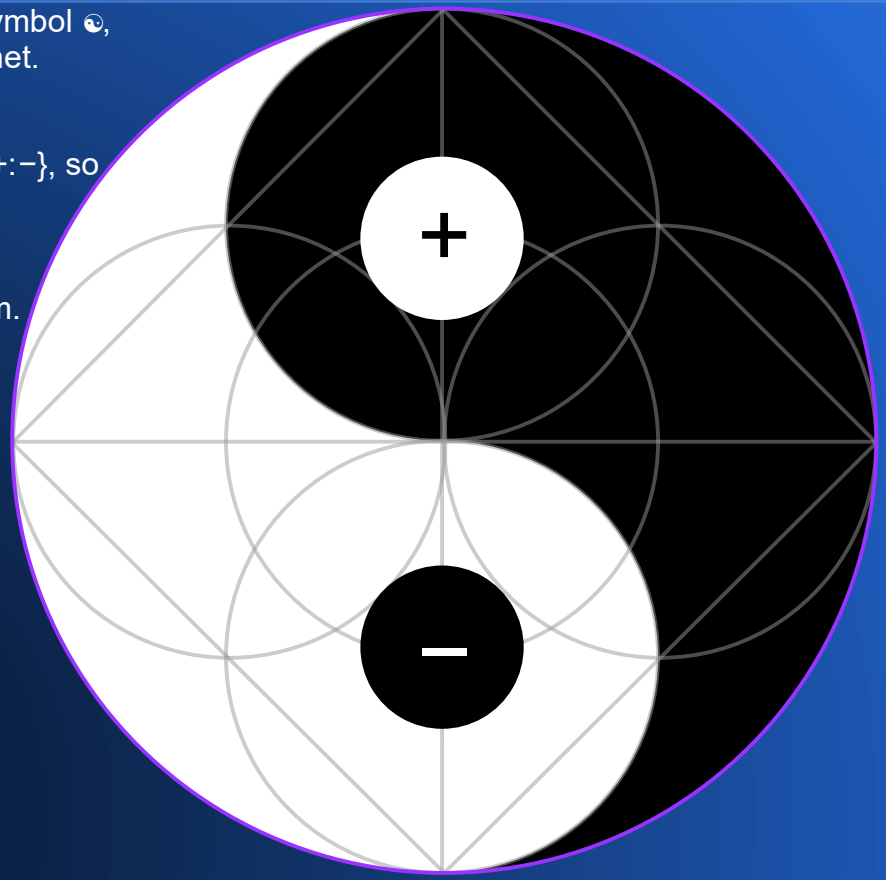
I ran across another variation of the YinYang symbol ☯,
also known as the T'ai chi symbol, on the internet.
I like the {1+3=4}-level symmetries.
There are 8 total circles,
with an overall even balance of {white:black}, {+:−}, so
4D in the two dual realms.

It also reminds me a bit of a Penrose Diagram,
which is an extension of the Minkowski Diagram.



Tao – I Ching – YinYang
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(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

4-Vector SRQM Interpretation of QM

SRQM Lorentz Transforms $\Lambda^{\mu'}{}_{\nu} = \partial_{\nu}[X^{\mu'}]$

Lorentz Transform Connection Map – Trace Identification

CPT, Big-Bang, (Matter↔AntiMatter), Arrow(s)-of-Time

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All Lorentz Transforms have Tensor Invariants: Determinant = ±1 and InnerProduct = 4.
However, one can use the Tensor Invariant Trace to Identify CPT Symmetry & AntiMatter

Tr[NM-Rotate] = {0...+4} Tr[NM-Identity] = +4 Tr[NM-Boost] = {+4...+∞}
Tr[AM-Rotate] = {0….-4} Tr[AM-Identity] = -4 Tr[AM-Boost] = {-4.....-∞}

Discrete NormalMatter (NM) Lorentz Transform Type

NM-Minkowski-Identity : AM-Flip-txyz=AM-ComboPT=AM-NegateIdentity

NM-Flip-t=NM-TimeReversal, NM-Flip-x, NM-Flip-y, NM-Flip-z
AM-Flip-xyz=AM-ParityInverse

NM-Flip-xy=NM-Rotate-xy(π),NM-Flip-xz=NM-Rotate-xz(π),NM-Flip-yz=NM-Rotate-yz(π)

AM-Flip-xy=AM-Rotate-xy(π), AM-Flip-xz=AM-Rotate-xz(π), AM-Flip-yz=AM-Rotate-yz(π)

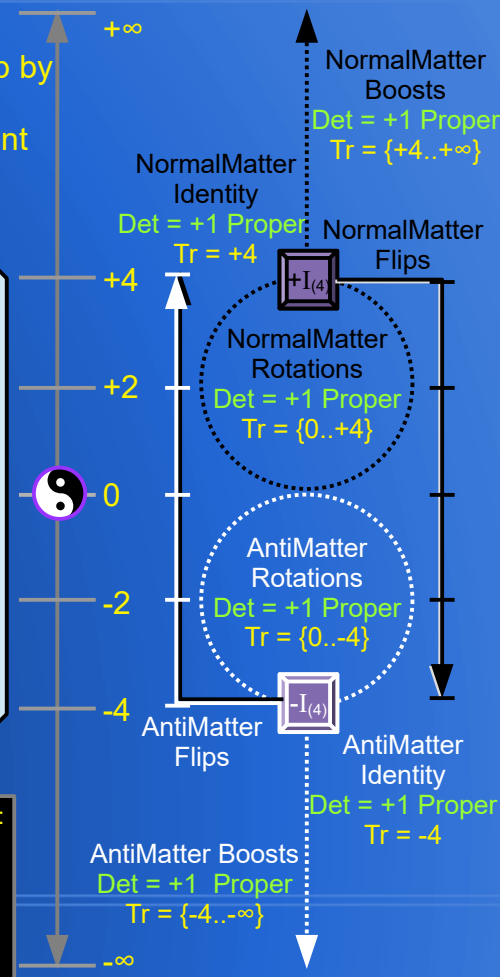
NM-Flip-xyz=NM-ParityInverse
AM-Flip-t=AM-TimeReversal, AM-Flip-x, AM-Flip-y, AM-Flip-z

AM-Minkowski-Identity : NM-Flip-txyz=NM-ComboPT=NM-NegateIdentity

Discrete AntiMatter (AM) Lorentz TransformType

Trace : Determinant
Tr = +4 : Det = +1 Proper
Tr = +2 : Det = -1 Improper
Tr = 0 : Det = +1 Proper
Tr = 0 : Det = +1 Proper
Tr = -2 : Det = -1 Improper
Tr = -4 : Det = +1 Proper
Trace : Determinant

Line up by Trace Invariant values



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$\mathrm{Det}[\Lambda^{\mu}{}_{\nu}]=\pm1$ $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$

$\mathrm{Tr}[\Lambda^{\mu}{}_{\nu}]=\{-\infty..+\infty\}$ =Lorentz Transform Type

Two interesting properties of (1,1)-Tensors, of which the Lorentz Transform is an example:
Trace = Sum (Σ) of EigenValues : Determinant = Product (Π) of EigenValues
As 4D Tensors, each Lorentz Transform has 4 EigenValues (EV's).
Create an Anti-Transform which has all EigenValue Tensor Invariants negated.
Σ[-(EV's)] = -Σ[EV's]: The Anti-Transform has negative Trace of the Transform.
Π[-(EV's)] = $(-1)^4$ Π[EV's] = Π[EV's]: The Anti-Transform has equal Determinant.

The Trace Invariant identifies a "Dual" Negative-Side for all Lorentz Transforms.

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Lorentz Transforms $\Lambda^{\mu'}{}_{\nu} = \partial_{\nu}[X^{\mu'}]$

Lorentz Transform Connection Map - Interpretations

CPT, Big-Bang, (Matter↔AntiMatter), Arrow(s)-of-Time

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
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Based on the Lorentz Transform properties of the last few pages, here is interesting observation about Lorentz Transforms: They all have Determinant of {±1}, and Inner Product of {+4 = 4D}, but the Trace varies depending on the particular Transform.

The Trace of the Identity is at {+4}. Assume this applies to normal matter particles.
The Trace of normal-matter particle Rotations varies continuously from {0..+4}
The Trace of the normal-matter particle Boosts varies continuously from {+4..+Infinity (+∞)}
So, one can think of Trace = {+4} being the connection point between normal-matter Rotations and Boosts.

Now, various Flip Transforms (inc. the Time Reversal and Parity Transforms, and their combination as PT transform), take the Trace in discrete steps from {-4,-2,0,+2,+4}. Applying a bit of symmetry:

The Trace of the Negative Identity is at {-4}. Assume this applies to anti-matter particles.
The Trace of anti-matter particle Rotations varies continuously from {0..-4}
The Trace of the anti-matter particle Boosts varies continuously from {-4..-Infinity (-∞)}
So, one can think of Trace = {-4} being the connection point between anti-matter Rotations and Boosts.

This observation would be in agreement with the CPT Theorem:(Feynman-Stueckelberg) idea that (normal/anti)-matter particles moving backward in SpaceTime are CPT symmetrically equivalent to (anti/normal)-matter particles moving forward in SpaceTime.

Now, scale this up to Universe size: The Baryon Asymmetry problem (aka. The Matter-AntiMatter Asymmetry Problem). If the Universe was created as a huge chunk of energy, and matter-creating energy is always transformed into matter-antimatter mirrored pairs, then *where is all the antimatter?* Turns out this is directly related to the Arrow-of-Time Problem as well.

Answer: It is temporally on the "Other/Dual-Side" of the Big-Bang! The antimatter created at the Big-Bang is travelling in the negative-time (-t) direction from the Big-Bang creation point, and the normal matter is travelling in the positive-time direction (+t). **Universal CPT Symmetry**. So, what happened "before" the Big-Bang? It "is" the AntiMatter Dual to our normal matter universe! Pair-production is creation of AM-NM mirrored pairs within SpaceTime. The Big-Bang is the creation of SpaceTime itself.

This also resolves the Arrow-of-Time Problem. If all known physical microscopic processes are time-symmetric, why is the flow of Time experienced as uni-directional??? {see Wikipedia "CPT Symmetry","CP Violation","Andrei Sakharov"}

Answer: Time flow on This-Side of the Universe is (+t) direction, while time flow on the Dual-Side of the Universe is (-t) direction. The math all works out. Time flow is bi-directional, but on opposite sides of the BB/Origin-Singularity. **Universal CPT Symmetry**!

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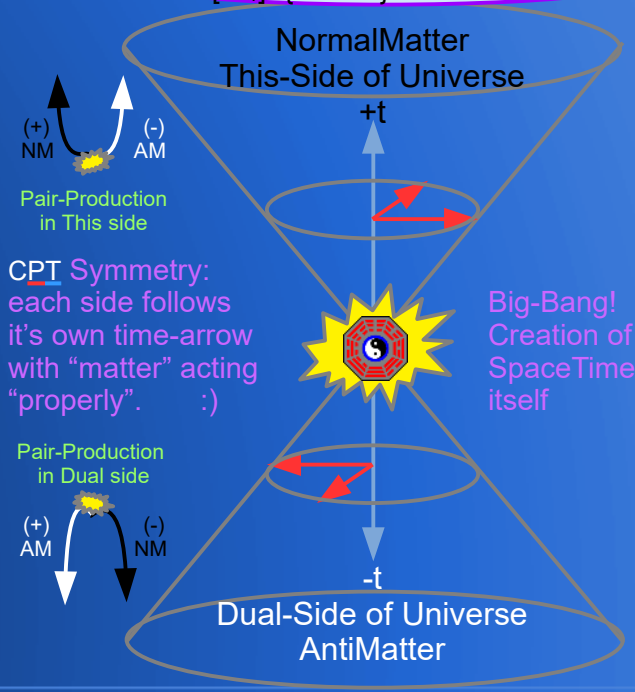

This gives total CPT Symmetry to all of the possible Lorentz Transforms (AM=AntiMatter , NM=NormalMatter):
Trace Various (AM_Flips) : Trace Various (NM_Flips)
-Infinity...(AM_Boosts)...(AM_Identity=-4)...(AM_Rotations)...0...(NM_Rotations)...(+4=NM_Identity)...(NM_Boosts)...+Infinity

This solves the:
Baryon (Matter-AntiMatter) Asymmetry Problem
& Arrow(s)-of-Time Problem (+t / -t)

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Lorentz Transforms $\Lambda^{\mu'}{}_{\nu} = \partial_{\nu}[X^{\mu'}]$

Lorentz Transform Connection Map – Interpretations 2

CPT, Big-Bang, (Matter↔AntiMatter), Arrow(s)-of-Time

Black Holes ↔ White Holes

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
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This idea of Universal CPT Symmetry also gives a Universal Dimensional Symmetry as well.

Consider the well-known "balloon" analogy of the universe expansion. The "spatial" coordinates are on the surface of the balloon, and the expansion is in the (+t) direction. There is symmetry in the (+/-) directions of the spatial coordinates, but the time flow is always uni-directional, (+t), as the balloon gets bigger→inflates.

By allowing a "Dual-Side", it provides a universal dimensional symmetry. One now has (+/-) symmetry for temporal (t) directions.

The "center" of the Universe is, literally, the Big Bang Singularity. It is the "center = zero = origin" point of both time and space directions. There are some people who prefer to say the BB is after inflation, but I am simply referring the "Origin:Singularity".

The expansion gives time-flow always AWAY FROM the Big Bang singularity in both the Normal-Side (+t) and the Dual-Side (-t). All spatial coordinates expand in both the (+/-) directions on both temporal sides of the singularity.

Note that this gives an unusual interpretation of what came "before" the Big Bang.
The "past" on either side extends only to the BB singularity, not beyond. Time flow is always away from this creation singularity.

This is also in accord with known black hole physics, in that all matter entering a BH event horizon ends at the BH singularity. Time and space coordinates both come to a stop at either type of singularity, from the point of view of an observer that is in the spacetime but not at one of these singularities.

So, the Big Bang is a "starting" singularity, and black holes are "ending" singularities. This also provides for idea of "white holes" actually just being black holes on the Dual-Side. White hole = time-reversed black hole. Always confusion about stuff coming out. This way, the mass is still attractive. Time-flow is simply reversed on the alternate side so stuff still goes INTO the hole… which makes way more sense than stuff that can only come out of the "massive→attractive" white hole.

So, Universal CPT Symmetry = Universal Dimensional Symmetry.

And, going even further, I suspect this is the reason there is a duality in Metric conventions.
In other words, physicists have wondered why one can use Metric signature {+,-,-,-} or {-,+,+,+}.
I submit that one of these metrics applies to the Normal Matter side, while the other complementarily applies to the Dual side.
This would allow correct causality conditions to apply on either side.
Again, this is similar to the Dirac prediction of antimatter based on a duality of possible solutions.

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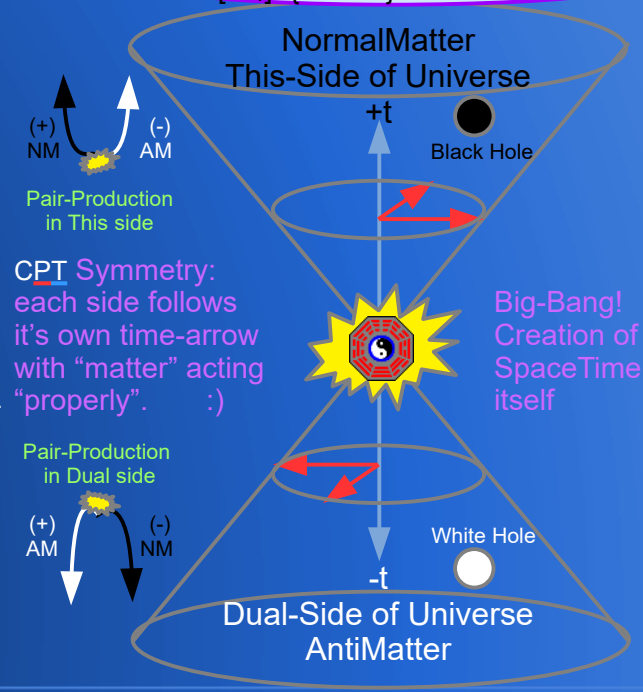

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This solves the:
Baryon (Matter-AntiMatter) Asymmetry Problem
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SRQM Study: Model SpaceTimes

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SciRealm@aol.com
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Model SpaceTimes Klein Geometry G/H	$\Lambda < 0$	$\Lambda = 0$	$\Lambda > 0$
Lorentzian pseudo-Riemannian	Anti de Sitter SO(3,2)/SO(3,1)	Minkowski ISO(3,1)/SO(3,1) $ds^2 = (cdt)^2 - \mathbf{dx \cdot dx}$	De Sitter SO(4,1)/SO(3,1)
Riemannian	Hyperbolic SO(4,1)/SO(4)	Euclidean ISO(4)/SO(4) $ds^2 = (cdt)^2 + \mathbf{dx \cdot dx}$	Spherical SO(5)/SO(4)

A Klein geometry is a pair (G,H) where G is a Lie group and H is a closed Lie subgroup of G such that the (left) coset space X:=G/H is connected.

G acts transitively on the homogeneous space X.

We may think of H↪G as the stabilizer subgroup of a point in X.

d is the dimension of the SpaceTime, which is 4D for our universe.

Geometric Context	**Gauge Group**	**Stabilizer Subgroup**	**Local Model Space**	**Local Geometry**	**Global Geometry**	**Differential Cohomology**	**First Order Formulation of Gravity**
Differential geometry	Lie group/algebraic group G	subgroup (monomorphism) H↪G	quotient (“coset space”) G/H	Klein geometry	Cartan geometry	Cartan connection	
Examples:	Euclidean group Iso(d)	rotation group O(d)	Cartesian space $\mathbb{R}^d$	Euclidean geometry	Riemannian geometry	Affine connection	Euclidean gravity
**********Fits known observational data**	Poincaré group Iso(d−1,1)	Lorentz group O(d−1,1)	Minkowski spacetime $\mathbb{R}^{d-1,1}$	Lorentzian geometry	Pseudo-Riemannian geometry	Spin connection	Einstein gravity
	anti de Sitter group O(d−1,2)	O(d−1,1)	anti de Sitter spacetime AdS^d				AdS gravity
	de Sitter group O(d,1)	O(d−1,1)	de Sitter spacetime dS^d				de Sitter gravity
	linear algebraic group	parabolic subgroup/ Borel subgroup	flag variety	Parabolic geometry			
	conformal group O(d,t+1)	conformal parabolic subgroup	Möbius space $S^{d,t}$		Conformal geometry	Conformal connection	Conformal gravity

Classical Transforms: Venn Diagram

Full Galilean = Galilean + Translations

(10) (6) (4)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Transformations

(# of independent parameters = # continuous symmetries = # Lie Dimensions)

Galilean Transformation Group aka. Inhomogeneous Galilean Transformation

Lie group of all affine isometries of Classical:Euclidean Time + Space (preserve quadratic form $\delta_{\mu\nu}$)

General Linear,Affine Transform $X^{\mu'} = G^{\mu'}{}_{\nu}X^{\nu} + \Delta X^{\mu'}$ with $\mathrm{Det}[G^{\mu'}{}_{\nu}] = \pm 1$

(6+4=10)

Galilean Transform $G^{\mu'}{}_{\nu}$

(3+3=6) 4-Tensor {mixed type-(1,1)}

Discrete

Time-reversal
$G^{\mu'}{}_{\nu} \rightarrow T^{\mu'}{}_{\nu}$
(0)
$t \rightarrow -t^*$
time parity
anti-unitary

SpatialFlipCombos
$G^{\mu'}{}_{\nu} \rightarrow F^{\mu'}{}_{\nu}$
(0)
{x|y|z} → -{x|y|z}
unitary

Parity-Inversion
$G^{\mu'}{}_{\nu} \rightarrow P^{\mu'}{}_{\nu}$
(0)
r → -r
space parity
unitary

Identity $I_{(4)}$
$G^{\mu'}{}_{\nu} \rightarrow \eta^{\mu'}{}_{\nu}=\delta^{\mu'}{}_{\nu}$
(0)
no mixing
unitary

Continuous

Rotation
$G^{\mu'}{}_{\nu} \rightarrow R^{\mu'}{}_{\nu}$
(3)
x:y | x:z | y:z

Motion:Shear
$G^{\mu'}{}_{\nu} \rightarrow S^{\mu'}{}_{\nu}$
(3)
t:x | t:y | t:z

Isotropy
{same all directions}

Translation Transform $\Delta X^{\mu'}$

(1+3=4) 4-Vector

Discrete

4-Zero
$\Delta X^{\mu'} \rightarrow (0,\mathbf{0})$
(0)
no motion

Continuous

Temporal
$\Delta X^{\mu'} \rightarrow (c\Delta t,\mathbf{0})$
(1)
Δt

Spatial
$\Delta X^{\mu'} \rightarrow (0,\mathbf{\Delta x})$
(3)
Δx | Δy | Δz

Homogeneity
{same all points}

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Transforms: Venn Diagram

Poincaré = Lorentz + Translations

(10) (6) (4)

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(6+4=10)

Lorentz Transform $\Lambda^{\mu'}{}_{\nu}$

(3+3=6) 4-Tensor {mixed type-(1,1)}

Discrete	Continuous
Time-reversal $\Lambda^{\mu'}{}_{\nu} \to T^{\mu'}{}_{\nu}$ (0) $t \to -t^*$ time parity anti-unitary	Rotation $\Lambda^{\mu'}{}_{\nu} \to R^{\mu'}{}_{\nu}$ (3) x:y | x:z | y:z
SpatialFlipCombos $\Lambda^{\mu'}{}_{\nu} \to F^{\mu'}{}_{\nu}$ (0) {x|y|z} → -{x|y|z} unitary	Boost $\Lambda^{\mu'}{}_{\nu} \to B^{\mu'}{}_{\nu}$ (3) t:x | t:y | t:z
Parity-Inversion $\Lambda^{\mu'}{}_{\nu} \to P^{\mu'}{}_{\nu}$ (0) **r** → **-r** space parity unitary	
Identity $I_{(4)}$ $\Lambda^{\mu'}{}_{\nu} \to \eta^{\mu'}{}_{\nu}=\delta^{\mu'}{}_{\nu}$ (0) no mixing unitary	
Charge-Conjugation $\Lambda^{\mu'}{}_{\nu} \to C^{\mu'}{}_{\nu}$ (0) **R** → **-R***, q → -q charge parity anti-unitary	
CPT Symmetry {Charge} {Partiy} {Time}	Isotropy [same all directions] {tx,ty,tz;xy,xz,yz}

Translation Transform $\Delta X^{\mu'}$

(1+3=4) 4-Vector

Discrete	Continuous
4-Zero $\Delta X^{\mu'} \to (0,\mathbf{0})$ (0) no motion	Temporal $\Delta X^{\mu'} \to (c\Delta t,\mathbf{0})$ (1) Δt
	Spatial $\Delta X^{\mu'} \to (0,\mathbf{\Delta x})$ (3) $\Delta x \mid \Delta y \mid \Delta z$
	Homogeneity [same all points] {t;x,y,z}

Stress-Energy(Density)-Tensor $T^{\mu\nu}$ (symmetric)
Covariant Decomposition

Tensor Invariants: Symmetry, AntiSymmetry,
Trace→Isotropy, Anisotropy

$T^{\mu\nu}$ = flux of $\hat{e}_\mu$-component of 4-Momentum along $\hat{e}_\nu$

(temporal:mixed:spatial) splitting

1 Temporal:Temporal EnergyDensity $(\rho_{eo})=V_{\mu\nu}T^{\mu\nu}$
3 Temporal:Spatial HeatEnergy Flux $(Q^\mu)=cT_\nu T^{\mu\nu}$
1 Spatial:Spatial Isotropic Pressure $(p_o)=(-1/3)H_{\mu\nu}T^{\mu\nu}$
5 Spatial:Spatial Anisotropic Stress $(\Pi^{\mu\nu})=H^\mu{}_\alpha H^\nu{}_\beta T^{\alpha\beta} + (p_o)H^{\mu\nu}$
10 Total Independent components

Poincaré Transformation Group = 10 Invariances
The group of all isometries of SR:Minkowski Spacetime (6 + 4 = 10)
(preserve quadratic form)
General Linear,Affine Transform $X^{\mu'} = \Lambda^{\mu'}{}_{\nu}X^{\nu} + \Delta X^{\mu'}$ with $\mathrm{Det}[\Lambda^{\mu'}{}_{\nu}] = \pm 1$

4-AngularMomentum $M^{\mu\nu}$ = **X ^ P** = $X^\mu P^\nu - X^\nu P^\mu$
= Generator of Lorentz Transformations (6)
= { $\Lambda^{\mu'}{}_{\nu} \to R^{\mu'}{}_{\nu}$ Rotations (3) + $\Lambda^{\mu'}{}_{\nu} \to B^{\mu'}{}_{\nu}$ Boosts (3) }

4-LinearMomentum P^μ = **P**
= Generator of Translation Transformations (4)
= { $\Delta X^{\mu'} \to (c\Delta t,\mathbf{0})$ Time (1) + $\Delta X^{\mu'} \to (0,\mathbf{\Delta x})$ Space (3) }

[(**R**→ **-R***)] or [(t→ -t*) & (**r**→ **-r**)] imply q→ -q
Feynman-Stueckelberg Interpretation
Amusingly, Inhomogeneous Lorentz adds homogeneity.

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

4-Vector SRQM Interpretation of QM

SRQM Study:
Lie Groups and Generators

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SciRealm@aol.com
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Lie Groups

de Sitter Group SO(1,4)

de Sitter invariant relativity
(?maybe?)

Poincaré Group ISO(1,3)

{ $r << r_{dS}$ = de Sitter Radius}
$r_{dS} = \sqrt{[3/\Lambda]} = L_H/\sqrt{[\Omega_\Lambda]}$

SR & GR Physics
(** *currently thought correct* **)

$\Lambda^{\mu'}{}_{\nu} \rightarrow B^{\mu'}{}_{\nu}$ = Lorentz Boost

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \gamma & -\beta\gamma & 0 & 0 \\ x & -\beta\gamma & \gamma & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array}$$

Galilei Group

{ $|\mathbf{v}| << c$ }
Classical Physics

$G^{\mu'}{}_{\nu} \rightarrow S^{\mu'}{}_{\nu}$ = Motion:Shear

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & 0 \\ x & -\beta & 1 & 0 & 0 \\ y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array}$$

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{array}{c} \\ t \\ x \\ y \\ z \end{array}\!\!\begin{array}{c} \begin{array}{cccc} t & x & y & z \end{array} \\ \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} + \begin{bmatrix} c\Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

SRQM: Lorentz Boost

$ct' = (\gamma)\, ct - (\beta)(\gamma)\, x$ $\qquad y' = y$
$x' = -(\beta)(\gamma)\, ct + (\gamma)\, x$ $\qquad z' = z$

$t' = \gamma t - \beta\gamma x/c$
$x' = \gamma x - \beta\gamma ct$

Classical: Galilean Motion Shear
take limit of $|\mathbf{v}|<<c$, { $\gamma\rightarrow 1$, $c\beta\rightarrow v$, $v/c=\beta\rightarrow 0$ }

$ct' = (1)\, ct - (\beta)(1)\, x$ $\qquad y' = y$
$x' = -(\beta)(1)\, ct + (1)\, x$ $\qquad z' = z$

$ct' = (1)\, ct - (0)(1)\, x$
$x' = -(v)\, t + (1)\, x$

$ct' = ct$
$x' = x - vt$

$t' = t$
$x' = x - vt$

	M^{01}	M^{02}	M^{03}
M^{10}		M^{12}	M^{13}
M^{20}	M^{21}		M^{23}
M^{30}	M^{31}	M^{32}	

P^0
P^1
P^2
P^3

4-AngularMomentum $M^{\mu\nu} = X^\mu \wedge P^\nu = X^\mu P^\nu - X^\nu P^\mu$
= Generator of Lorentz Transformations (6)
= { $\Lambda^{\mu'}{}_{\nu}\rightarrow R^{\mu'}{}_{\nu}$ Rotations (3) + $\Lambda^{\mu'}{}_{\nu}\rightarrow B^{\mu'}{}_{\nu}$ Boosts (3) }

4-LinearMomentum P^μ
= Generator of Translation Transformations (4)
= { $\Delta X^{\mu'}\rightarrow(c\Delta t,\mathbf{0})$ Time (1) + $\Delta X^{\mu'}\rightarrow(0,\mathbf{\Delta x})$ Space (3) }

$\mathrm{Det}[\Lambda^{\mu'}{}_{\nu}] = +1$ for Proper Lorentz Transforms
$\mathrm{Det}[\Lambda^{\mu'}{}_{\nu}] = -1$ for Improper Lorentz Transforms

Lorentz Matrices can be generated by a matrix M with Tr[M]=0 which gives:
{ $\Lambda = e \wedge M = e \wedge (+\boldsymbol{\theta}\cdot\mathbf{J} - \boldsymbol{\zeta}\cdot\mathbf{K})$ }
{ $\Lambda^T = (e \wedge M)^T = e \wedge M^T$ }
{ $\Lambda^{-1} = (e \wedge M)^{-1} = e \wedge -M$ }

$M = +\boldsymbol{\theta}\cdot\mathbf{J} - \boldsymbol{\zeta}\cdot\mathbf{K}$
$B[\boldsymbol{\zeta}] = e\wedge(-\boldsymbol{\zeta}\cdot\mathbf{K})$
$R[\boldsymbol{\theta}] = e\wedge(+\boldsymbol{\theta}\cdot\mathbf{J})$
$\Lambda = e \wedge M = e \wedge (+\boldsymbol{\theta}\cdot\mathbf{J} - \boldsymbol{\zeta}\cdot\mathbf{K})$

SR:Lorentz Transform

$\partial_\nu[R^{\mu'}] = \partial R^{\mu'}/\partial R^\nu = \Lambda^{\mu'}{}_{\nu}$
$\Lambda^\mu{}_\nu = (\Lambda^{-1})_\nu{}^\mu : \Lambda^\mu{}_\alpha(\Lambda^{-1})^\alpha{}_\nu = \eta^\mu{}_\nu = \delta^\mu{}_\nu$
$\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta = \eta_{\alpha\beta}$
$\mathrm{Det}[\Lambda^\mu{}_\nu]=\pm 1$ $\qquad \Lambda_{\mu\nu}\Lambda^{\mu\nu}=4$

Rotations $J_i = -\varepsilon_{imn}M^{mn}/2$, Boosts $K_i = M_{i0}$

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

Review of SR Transforms

10 Poincaré Symmetries, 10 Conservation Laws 10 Generators : Noether's Theorem

4-Vector SRQM Interpretation of QM

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4-Displacement
$\mathbf{\Delta X}=(c\Delta t,\mathbf{\Delta x})$

ΔTime Transform $\Delta X^{\mu'}\rightarrow(c\Delta t,\mathbf{0})$
Generated by energy $E = cp^0$

Translation Transform
Generated $\Delta X^{\mu'}(t,\mathbf{x}) = \exp[\mathbf{X\cdot P}/\hbar]^{\mu'}$

ΔSpace Transform $\Delta X^{\mu'}\rightarrow(0,\mathbf{\Delta x})$
Generated by 3-momentum $\mathbf{p} = p^j$

$\partial\cdot\mathbf{X}=4$ SpaceTime Dimension

Minkowski $\partial[\mathbf{X}]=\partial^\mu[X^\nu]=\eta^{\mu\nu}$ Metric

Conservation of scalar Energy (temporal)

Conservation of 4-Momentum
(1 + 3) = (4) Laws

Conservation of linear 3-momentum (spatial)

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{X}/d\tau$

m_o
E_o/c^2

4-Momentum
$P^\mu = \mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

* Generator P^μ

4-AngMomentum Tensor
$M^{\mu\nu} = X^\mu P^\nu - X^\nu P^\mu = \mathbf{X} \wedge \mathbf{P}$
→

$$\begin{bmatrix} 0 & -cn^x & -cn^y & -cn^z \\ +cn^x & 0 & +l^z & -l^y \\ +cn^y & -l^z & 0 & +l^x \\ +cn^z & +l^y & -l^x & 0 \end{bmatrix}$$

=

$$\begin{bmatrix} 0 & , -cn^j \\ +cn^i, & \varepsilon^{ij}{}_k l^k \end{bmatrix}$$

Angular $M^{\mu\nu}$ + Linear P^μ
(3 + 3) + (1 + 3)
= (6) + (4)
= 10 Symmetries = 10 Generators = 10 Conservation Laws : Noether's Theorem
= 10 Independent Components of Symmetric 4D (2,0)-Tensor { Relativistic Fluid Stress-EnergyDensity = [1 temporal + 3 mixed + (1+5) spatial] }

* Generator $M^{\mu\nu}$

4-Gradient
$\partial=(\partial_t/c,-\nabla)$

Lagrange "Shift Operator" version of Taylor's Theorem: $e^{a(d/dx)}f(x)=f(x+a)$
Bloch Theorem:Translation Operator: $e^{i(\mathbf{K}\cdot\mathbf{R})}\psi(\mathbf{X}) = \psi(\mathbf{X}+\mathbf{R})$, with $\mathbf{K}$ as reciprocal lattice

Lorentz General Time-Space Boost Transform
$\Lambda^{\mu'}{}_\nu\rightarrow B^{\mu'}{}_\nu =$

$$\begin{matrix} \gamma & -\gamma\beta_j \\ -\gamma\beta^i & (\gamma-1)\beta^i\beta_j/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\delta^i_j \end{matrix}$$

Generated by relativistic massmoment 3-vector $c\mathbf{n}$

Conservation of relativistic 3-mass-moment (temporal-spatial)

Lorentz Transform $\partial_\nu[X^\mu] = \partial X^\mu/\partial X^\nu = \Lambda^{\mu'}{}_\nu$
Generated $\Lambda^{\mu'}{}_\nu(\boldsymbol{\zeta},\boldsymbol{\theta}) = \exp[1/2\ \omega_{\alpha\beta}M^{\alpha\beta}]^{\mu'}{}_\nu = \exp[\boldsymbol{\zeta}\cdot\mathbf{K} + \boldsymbol{\theta}\cdot\mathbf{J}]^{\mu'}{}_\nu$

Lorentz General Space-Space Rotation Transform
$\Lambda^{\mu'}{}_\nu\rightarrow R^{\mu'}{}_\nu =$

$$\begin{matrix} 1 & 0_j \\ 0^i & (\delta^i_j-n^in_j)\cos(\theta)-(\varepsilon^i{}_{jk}n^k)\sin(\theta)+n^in_j \end{matrix}$$

Generated by angular-momentum-3-vector $\mathbf{l} = \mathbf{r} \wedge \mathbf{p}$

Conservation of 4-AngularMomentum
(3 + 3) = (6) Laws

Conservation of angular 3-momentum (spatial-spatial)

Poincaré Transformation Group aka. Inhomogeneous Lorentz Transformation
The group of all isometries of SR:Minkowski Spacetime (6 + 4 = 10)
(preserve quadratic form)
General Linear,Affine Transform $X^{\mu'} = \Lambda^{\mu'}{}_\nu X^\nu + \Delta X^{\mu'}$ with $\mathrm{Det}[\Lambda^{\mu'}{}_\nu] = \pm 1$

4-AngularMomentum $M^{\mu\nu} = \mathbf{X} \wedge \mathbf{P} = X^\mu P^\nu - X^\nu P^\mu$
= Generator of Lorentz Transformations (6)
= { $\Lambda^{\mu'}{}_\nu\rightarrow R^{\mu'}{}_\nu$ Rotations (3) + $\Lambda^{\mu'}{}_\nu\rightarrow B^{\mu'}{}_\nu$ Boosts (3) }

0	$-c\mathbf{n}$
$c\mathbf{n}^T$	$\mathbf{l} = \mathbf{x} \wedge \mathbf{p}$

4-LinearMomentum $P^\mu = \mathbf{P}$
= Generator of Translation Transformations (4)
= { $\Delta X^{\mu'}\rightarrow(c\Delta t,\mathbf{0})$ Time (1) + $\Delta X^{\mu'}\rightarrow(0,\mathbf{\Delta x})$ Space (3) }

$E/c = p^0$
$\mathbf{p} = p^j$

Jacobi's Formula for Complex Square Matrix A: $\mathrm{Det}(\mathrm{Exp}[A])=\mathrm{Exp}(\mathrm{Tr}[A])$
$\mathrm{Det}(A)_{4D} = [\ (\mathrm{tr}\,A)^4 - 6\,\mathrm{tr}(A^2)(\mathrm{tr}\,A)^2 + 3(\mathrm{tr}(A^2))^2 + 8\,\mathrm{tr}(A^3)\,\mathrm{tr}\,A - 6\,\mathrm{tr}(A^4)\ ] / 4!$

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

Review of SR Transforms
Poincaré Algebra & Generators
Casimir Invariants

4-Vector SRQM Interpretation of QM

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The (10) one-parameter groups can be expressed directly as exponentials of the generators:

$U[I, (a^0,\mathbf{0})] = e^{\wedge}(ia^0\cdot H) = e^{\wedge}(ia^0\cdot p^0)$: (1) Hamiltonian (Energy) = Temporal Momentum H

$U[I, (0,\lambda\hat{\mathbf{a}})] = e^{\wedge}(-i\lambda\hat{\mathbf{a}}\cdot\mathbf{p})$: (3) Linear Momentum **p**

$U[\Lambda(i\lambda\hat{\boldsymbol{\theta}}/2), 0] = e^{\wedge}(i\lambda\hat{\boldsymbol{\theta}}\cdot\mathbf{j})$: (3) Angular Momentum **j = l**

$U[\Lambda(\lambda\hat{\boldsymbol{\varphi}}/2), 0] = e^{\wedge}(i\lambda\hat{\boldsymbol{\varphi}}\cdot\mathbf{k})$: (3) Dynamic Mass Moment **k = n**

The Poincaré Algebra is the Lie Algebra of the Poincaré Group:

Total of {1+3+3+3 = (1+3)+(3+3) = 4+6 = 10} Invariances from Poincaré Symmetry

Poincaré Algebra is the Lie Algebra of the Poincaré Group.

	$M^{01}= -cn^1$	$M^{02}= -cn^2$	$M^{03}= -cn^3$
$M^{10}= cn^1$		$M^{12}= l^3$	$M^{13}= -l^2$
$M^{20}= cn^2$	$M^{21}= -l^3$		$M^{23}= l^1$
$M^{30}= cn^3$	$M^{31}= l^2$	$M^{32}= -l^1$	

P^0
P^1
P^2
P^3

Covariant form:

These are the commutators of the the Poincaré Algebra :

$[X^\mu, X^\nu] = 0^{\mu\nu}$

$[P^\mu, P^\nu] = -i\hbar q(F^{\mu\nu})$ if interacting with EM field; otherwise $= 0^{\mu\nu}$ for free particles

$M^{\mu\nu} = (X^\mu P^\nu - X^\nu P^\mu) = i\hbar(X^\mu\partial^\nu - X^\nu\partial^\mu)$

$[M^{\mu\nu}, P^\rho] = i\hbar(\eta^{\rho\nu}P^\mu - \eta^{\rho\mu}P^\nu)$

$[M^{\mu\nu}, M^{\rho\sigma}] = i\hbar(\eta^{\nu\rho}M^{\mu\sigma} + \eta^{\mu\sigma}M^{\nu\rho} + \eta^{\sigma\nu}M^{\rho\mu} + \eta^{\rho\mu}M^{\sigma\nu})$

$M^{\mu\nu} = \mathbf{X} \wedge \mathbf{P} = X^\mu P^\nu - X^\nu P^\mu$

$P^\mu = \mathbf{P}$

0	-cn
cn^T	**l = x ^ p**

$H/c = E_T/c = p^0$
p = p^j

M = Generator of Lorentz Transformations (6) = { Rotations (3) + Boosts (3) }

P = Generator of Translation Transformations (4) = { Time-Move (1) + Space-Moves (3) }

Component form: Rotations $J_i = -\varepsilon_{imn}M^{mn}/2$, Boosts $K_i = M_{i0}$

$[J_m,P_n] = i\varepsilon_{mnk}P^k$

$[J_m,P_0] = 0$

$[K_j,P_k] = i\eta_{jk}P^0$

$[K_j,P_0] = -iP_j$

$[J_m,J_n] = i\varepsilon_{mnk}J^k$

$[J_m,K_n] = i\varepsilon_{mnk}K^k$

$[K_m,K_n] = -i\varepsilon_{mnk}J^k$, a Wigner Rotation resulting from consecutive boosts

$[J_m + iK_m,J_n - iK_n] = 0$

Rotations $J_i = -\varepsilon_{imn}M^{mn}/2$, Boosts $K_i = M_{i0}$

The set of all Lorentz Generators V = {$\boldsymbol{\zeta}\cdot\mathbf{K} + \boldsymbol{\theta}\cdot\mathbf{J}$} forms a vector space over the real numbers. The generators {J_x , J_y , J_z , K_x , K_y , K_z} form a basis set of V. The components of the axis-angle vector and rapidity vector {θ_x , θ_y , θ_z , ζ_x , ζ_y , ζ_z} are the coordinates of a Lorentz generator wrt. this basis.

Very importantly, the Poincaré group has Casimir Invariant Eigenvalues = { Mass m, Spin j }, hence Mass *and* Spin are purely SR phenomena, no QM axioms required!

This Representation of the Poincaré Group or Representation of the Lorentz Group is known as Wigner's Classification in Representation Theory of Particle Physics

Poincaré Algebra has 2 Casimir Invariants = Operators that commute with all of the Poincaré Generators

These are { $P^2 = P^\mu P_\mu = (m_o c)^2$, $W^2 = W^\mu W_\mu = -(m_o c)^2 j(j + 1)$ }, with $W^\mu = (-1/2)\varepsilon^{\mu\nu\rho\sigma}J_{\nu\rho}P_\sigma$ as the Pauli-Lubanski Pseudovector

$[P^2,P^0] = [P^2,P^i] = [P^2,J^i] = [P^2,K^i] = 0$: Hence the 4-Momentum Magnitude squared commutes with all Poincaré Generators

$[W^2,P^0] = [W^2,P^i] = [W^2,J^i] = [W^2,K^i] = 0$: Hence the 4-SpinMomentum Magnitude squared commutes with all Poincaré Generators

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$

$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$

= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4

A Tensor Study of Physical 4-Vectors

SRQM Study:
10 Poincaré Symmetry Invariances
Noether's Theorem: 10 SR Conservation Laws

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d'Alembertian Invariant Wave Equation: $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = (\partial_\tau/c)^2$

4

4-Gradient
$\partial=\partial^\mu=(\partial_t c,-\nabla)=\partial/\partial R_\mu$
$=(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$

Math/Phys Wave Solution

4-WaveVector
$\mathbf{K}=K^\mu=(\omega/c=2\pi\nu/c,\mathbf{k})=-\partial[\Phi]$
$=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})=(1/c\mathrm{T},\hat{\mathbf{n}}/\lambda)$

Time Translation:
Let $\mathbf{X}_T = (ct+c\Delta t,\mathbf{x})$, then $\partial[\mathbf{X}_T] = (\partial_t/c,-\nabla)(ct+c\Delta t,\mathbf{x}) = \mathrm{Diag}[1,-1] = \partial[\mathbf{X}] = \boldsymbol{\eta}^{\mu\nu}$
so $\partial[\mathbf{X}_T] = \partial[\mathbf{X}]$ and $\partial[\mathbf{K}] = [[\mathbf{0}]]$
$(\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}_T] = \partial\cdot(\partial[\mathbf{K}\cdot\mathbf{X}_T]) = \partial[\mathbf{K}]\cdot\mathbf{X}_T+\mathbf{K}\cdot\partial[\mathbf{X}_T] = 0+\mathbf{K}\cdot\partial[\mathbf{X}] = \partial[\mathbf{K}]\cdot\mathbf{X}+\mathbf{K}\cdot\partial[\mathbf{X}] = \partial\cdot(\partial[\mathbf{K}\cdot\mathbf{X}]) = (\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}]$:

Time Translation Invariance (1)
Conservation of Energy = (Temporal) 1-energy (E)
Temporal part of $P^\mu = (E/c,\mathbf{p})$

$H/c = E_T/c = p^0$	
$\mathbf{p} = p^j$	

Space Translation:
Let $\mathbf{X}_S = (ct,\mathbf{x}+\Delta\mathbf{x})$, then $\partial[\mathbf{X}_S] = (\partial_t/c,-\nabla)(ct,\mathbf{x}+\Delta\mathbf{x}) = \mathrm{Diag}[1,-1] = \partial[\mathbf{X}] = \boldsymbol{\eta}^{\mu\nu}$
so $\partial[\mathbf{X}_S] = \partial[\mathbf{X}]$ and $\partial[\mathbf{K}] = [[\mathbf{0}]]$
$(\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}_S] = \partial\cdot(\partial[\mathbf{K}\cdot\mathbf{X}_S]) = \partial[\mathbf{K}]\cdot\mathbf{X}_S+\mathbf{K}\cdot\partial[\mathbf{X}_S] = 0+\mathbf{K}\cdot\partial[\mathbf{X}] = \partial[\mathbf{K}]\cdot\mathbf{X}+\mathbf{K}\cdot\partial[\mathbf{X}] = \partial\cdot(\partial[\mathbf{K}\cdot\mathbf{X}]) = (\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}]$:

Space Translation Invariances (3)
Conservation of Linear (Spatial) 3-momentum (p)
Spatial part of $P^\mu = (E/c,\mathbf{p})$

Lorentz Space-Space Rotation:
Let $\mathbf{X}_R = (ct,R[\mathbf{x}])$, then $\partial[\mathbf{X}_R] = (\partial_t/c,-\nabla)(ct,R[\mathbf{x}]) = \mathrm{Diag}[1,-1] = \partial[\mathbf{X}] = \boldsymbol{\eta}^{\mu\nu}$
so $\partial[\mathbf{X}_R] = \partial[\mathbf{X}]$ and $\partial[\mathbf{K}] = [[\mathbf{0}]]$
$(\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}_R] = \partial\cdot(\partial[\mathbf{K}\cdot\mathbf{X}_R]) = \partial[\mathbf{K}]\cdot\mathbf{X}_R+\mathbf{K}\cdot\partial[\mathbf{X}_R] = 0+\mathbf{K}\cdot\partial[\mathbf{X}] = \partial[\mathbf{K}]\cdot\mathbf{X}+\mathbf{K}\cdot\partial[\mathbf{X}] = \partial\cdot(\partial[\mathbf{K}\cdot\mathbf{X}]) = (\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}]$:

Lorentz Space-Space Rotation Invariances (3)
Conservation of Angular (Spatial) 3-angmomentum (l)
Spatial-Spatial part of $M^{\mu\nu} = \mathbf{X}\wedge\mathbf{P}$

0	-cn
cn^T	$\mathbf{l} = \mathbf{x} \wedge \mathbf{p}$

Lorentz Time-Space Boost:
Let $\mathbf{X}_B = \gamma(ct-\boldsymbol{\beta}\cdot\mathbf{x},-\boldsymbol{\beta}ct+\mathbf{x})$, then $\partial[\mathbf{X}_B] = (\partial_t/c,-\nabla)\gamma(ct-\boldsymbol{\beta}\cdot\mathbf{x},-\boldsymbol{\beta}ct+\mathbf{x}) = [[\gamma,-\gamma\boldsymbol{\beta}],[-\gamma\boldsymbol{\beta},\gamma]] = \boldsymbol{\Lambda}^{\mu\nu}$
$\partial[\mathbf{K}\cdot\mathbf{X}_B] = \partial[\mathbf{K}]\cdot\mathbf{X}_B+\mathbf{K}\cdot\partial[\mathbf{X}_B] = \boldsymbol{\Lambda}^{\mu\nu}\mathbf{K} = \mathbf{K}_B$ = a Lorentz Boosted $\mathbf{K}$, as expected
$\partial\cdot\mathbf{K}_B = \partial\cdot\boldsymbol{\Lambda}^{\mu\nu}\mathbf{K} = \boldsymbol{\Lambda}_{\mu\nu}(\partial\cdot\mathbf{K}) = \boldsymbol{\Lambda}^{\mu\nu}(0) = 0 = \partial\cdot\mathbf{K}$ = Divergence of $\mathbf{K}$ = 0, as expected
$(\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}_B] = \partial\cdot(\partial[\mathbf{K}\cdot\mathbf{X}_B]) = \partial\cdot\mathbf{K}_B = \partial\cdot\mathbf{K} = \partial\cdot(\partial[\mathbf{K}\cdot\mathbf{X}]) = (\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}]$:

Lorentz Time-Space Boost Invariances (3)
Conservation of Relativistic 3-mass-moment (n)
Temporal-Spatial part of $M^{\mu\nu} = \mathbf{X}\wedge\mathbf{P}$
see Wikipedia: Relativistic Angular Momentum

SR Waves:
Let $\Psi = ae^{-i(\mathbf{K}\cdot\mathbf{X})}$, $\Psi_T = ae^{-i(\mathbf{K}\cdot\mathbf{X}_T)}$, $\Psi_S = ae^{-i(\mathbf{K}\cdot\mathbf{X}_S)}$, $\Psi_R = ae^{-i(\mathbf{K}\cdot\mathbf{X}_R)}$, $\Psi_B = ae^{-i(\mathbf{K}\cdot\mathbf{X}_B)}$
$(\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}_T] = (\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}_S] = (\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}_R] = (\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}_B] = (\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}]$: Wave Equation Invariant under all Poincaré transforms
Total of (1+3+3+3 = 10) Invariances from Poincaré Symmetry

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

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4-AngularMomentum $L^{\mu\nu}$
$= (1/c)\int_V (x^\mu T^{\nu 0} - x^\nu T^{\mu 0})dV$

$\mathbf{h} = (L^{23},L^{31},L^{12}) = \int_V (\mathbf{r} \times \mathbf{g})dV$

4-AngularMomentum $\mathbf{M} = M^{\mu\nu}$
$M^{\alpha\beta}[\bar{X}] = \oint_{\partial\Omega} M^{\alpha\beta\gamma}\, d\Sigma_\gamma$
with $M^{\alpha\beta\gamma} = (X^\alpha - \bar{X}^\alpha)T^{\beta\gamma} - (X^\beta - \bar{X}^\beta)T^{\alpha\gamma}$

	$M^{01} = -cn^1$	$M^{02} = -cn^2$	$M^{03} = -cn^3$
$M^{10} = cn^1$		$M^{12} = l^3$	$M^{13} = -l^2$
$M^{20} = cn^2$	$M^{21} = -l^3$		$M^{23} = l^1$
$M^{30} = cn^3$	$M^{31} = l^2$	$M^{32} = -l^1$	

6 [DoF]

0	$-c\mathbf{n}$
$c\mathbf{n}^T$	$\mathbf{l} = \mathbf{x} \wedge \mathbf{p}$

Lorentz General Time-Space Boost Transform
$\Lambda^{\mu'}{}_\nu \rightarrow B^{\mu'}{}_\nu =$

γ	$-\gamma\beta_j$
$-\gamma\beta^i$	$(\gamma-1)\beta^i\beta_j/(\boldsymbol{\beta}\cdot\boldsymbol{\beta}) + \delta^i_j$

Generated by relativistic massmoment 3-vector $c\mathbf{n}$

6 [DoF]

Lorentz Transform $\partial_\nu[X^{\mu'}] = \partial X^{\mu'}/\partial X^\nu = \Lambda^{\mu'}{}_\nu$
Generated $\Lambda^{\mu'}{}_\nu(\boldsymbol{\zeta},\boldsymbol{\theta}) = \exp[1/2\, \omega_{\alpha\beta}M^{\alpha\beta}]^{\mu'}{}_\nu = \exp[\boldsymbol{\zeta}\cdot\mathbf{K} + \boldsymbol{\theta}\cdot\mathbf{J}]^{\mu'}{}_\nu$

Lorentz Identity Transform
$\Lambda^{\mu'}{}_\nu \rightarrow$
$\eta^{\mu'}{}_\nu = \delta^{\mu'}{}_\nu = I_{(4)}$

1	0_j
0^i	δ^i_j

Lorentz General Space-Space Rotation Transform
$\Lambda^{\mu'}{}_\nu \rightarrow R^{\mu'}{}_\nu =$

1	0_j
0^i	$(\delta^i_j - n^i n_j)\cos(\theta) - (\varepsilon^i_{jk}n^k)\sin(\theta) + n^i n_j$

Generated by angular-momentum-3-vector $\mathbf{l} = \mathbf{r} \wedge \mathbf{p}$

Isotropy
same all directions
{tx,ty,tz;xy,xz,yz}

Full Relativistic Fluid Stress-Energy(Density)
$T^{\mu\nu} \rightarrow$
$(\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} + (\bar{T}^\mu H^\nu{}_\sigma Q^\sigma + Q^\sigma H^\mu{}_\sigma \bar{T}^\nu)/c + \Pi^{\mu\nu}$

[J/m³] [N/m²] [Pa]

10 [DoF]

$\rightarrow$ {MCRF}

$Q^\mu \bar{T}_\mu = 0$

$\Pi^{\mu\nu}\bar{T}_\mu = 0$

$\rho_e = \rho_m c^2$ Q^{0j}/c

Q^{i0}/c $p\delta^{ij} + \Pi^{ij}$

4-Tensor
Symmetric
10 independent components

4-LinearMomentum $\mathbf{P} = P^\mu$

4 [DoF]

P^0
P^1
P^2
P^3

$H/c = E_T/c = p^0$
$\mathbf{p} = p^j$

4-AngularMomentum $M^{\mu\nu} = X^\mu \wedge P^\nu = X^\mu P^\nu - X^\nu P^\mu$
= Generator of Lorentz Transformations (6)
$= \{ \Lambda^{\mu'}{}_\nu \rightarrow R^{\mu'}{}_\nu$ Rotations (3) $+ \Lambda^{\mu'}{}_\nu \rightarrow B^{\mu'}{}_\nu$ Boosts (3) $\}$

4-LinearMomentum P^μ
= Generator of Translation Transformations (4)
$= \{ \Delta X^{\mu'} \rightarrow (c\Delta t, \mathbf{0})$ Time (1) $+ \Delta X^{\mu'} \rightarrow (0, \Delta\mathbf{x})$ Space (3)

Poincaré Transform
aka. Inhomogenous Lorentz
$X^{\mu'} = \Lambda^{\mu'}{}_\nu X^\nu + \Delta X^{\mu'}$
6 [DoF] 4 [DoF]
10 total parameters

Homogeneity
same at all points
{t;x,y,z}

4 [DoF]
4-Displacement
$\Delta\mathbf{X} = (c\Delta t, \Delta\mathbf{x})$

ΔTime Transform $\Delta X^{\mu'} \rightarrow (c\Delta t, \mathbf{0})$
Generated by energy $E = cp^0$

Translation Transform
Generated $\Delta X^{\mu'}(t,\mathbf{x}) = \exp[\mathbf{X}\cdot\mathbf{P}/\hbar]^{\mu'}$

ΔSpace Transform $\Delta X^{\mu'} \rightarrow (0, \Delta\mathbf{x})$
Generated by 3-momentum $\mathbf{p} = p^j$

4-Zero Z $=(0,0)$

4-LinearMomentum P^μ
$= (1/c)\int_V (T^{\mu 0})dV$
$= (E/c, \mathbf{p}) = (1/c)\int_V (\rho, c\mathbf{g})dV$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

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SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: 4-Vector Operations

Lorentz Scalar Product $\mathbf{A}\cdot\mathbf{B} = A_\mu B^\mu$

Exterior Product $\mathbf{A}\wedge\mathbf{B} = A^\mu B^\nu - A^\nu B^\mu$

4-Vector SRQM Interpretation of QM

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4-Gradient
$\partial=(\partial_t/c,-\nabla)=\partial/\partial R_\mu$

$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$
d'Alembertian

Minkowski
$\partial[\mathbf{R}]=\partial^\mu[R^\nu]=\eta^{\mu\nu}$
Metric

Lorentz
$\partial_\nu[R^{\mu'}]=\Lambda^{\mu'}{}_\nu$
Transform

SpaceTime
$\partial\cdot\mathbf{R}=\partial_\mu R^\mu=4$
Dimension

4-Position
$\mathbf{R}=(ct,\mathbf{r})\in$<Event>

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

$\partial\cdot\mathbf{A}=0$
Lorenz Gauge

$\mathbf{R}\cdot\mathbf{P} = -S_{action,free}$
Action Scalar

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})=d\mathbf{R}/d\tau$

$F^{\mu\nu}=$
$\partial\wedge\mathbf{A}=\partial^\mu A^\nu - \partial^\nu A^\mu$
EM Faraday 4-Tensor

$M^{\mu\nu}=$
$\mathbf{R}\wedge\mathbf{P}=R^\mu P^\nu - R^\nu P^\mu$
4-AngularMomentum 4-Tensor

EM φ_o/c^2

ρ_o

m_o
E_o/c^2
Energy:Mass
$E = mc^2$

4-EMVectorPotential
$\mathbf{A}=(\varphi/c,\mathbf{a})$

$(\partial\cdot\partial)\mathbf{A} - \partial(\partial\cdot\mathbf{A})=\mu_o\mathbf{J}$
Maxwell EM Wave Eqn

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

$\varepsilon_o c^2$
$1/\mu_o$

4-ChargeFlux
4-CurrentDensity
$\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

Electric:Magnetic
$1/(\varepsilon_o\mu_o) = c^2$

There are at least three 4-Vector relations which use the Exterior (Wedge=^) Product.

$\partial\wedge\mathbf{A} = \partial^\mu \wedge A^\nu = \partial^\mu A^\nu - \partial^\nu A^\mu = F^{\mu\nu}$: the Faraday EM 4-Tensor
$\mathbf{R}\wedge\mathbf{P} = R^\mu \wedge P^\nu = R^\mu P^\nu - R^\nu P^\mu = M^{\mu\nu}$: the 4-Angular-Momentum Tensor
$\mathbf{R}\wedge\mathbf{F} = R^\mu \wedge F^\nu = R^\mu F^\nu - R^\nu F^\mu = T^{\mu\nu}$: the 4-(Angular-)Torque Tensor

This gives the components of each remarkably similar properties.
Likewise, each of these has a physical (Dot=·) Product relation as well.

$\partial\cdot\mathbf{A} = \partial_\mu A^\mu = 0$: the Lorenz Gauge, a conservation of 4-EMVectorPotential
$\mathbf{R}\cdot\mathbf{P} = R_\mu P^\mu = -S_{action,free}$: the Action Scalar
$\mathbf{R}\cdot\mathbf{F} = R_\mu F^\mu = (ct,\mathbf{r})\cdot\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}}) = \gamma(\dot{E}t-\mathbf{r}\cdot\mathbf{f}) = W_o$: Work W_o = power$_{relativistic}$*time-**distance·force**$_{relativistic}$

$$d(\mathbf{R}\cdot\mathbf{P})/d\tau = d\mathbf{R}/d\tau\cdot\mathbf{P}+\mathbf{R}\cdot d\mathbf{P}/d\tau = \mathbf{U}\cdot\mathbf{P}+\mathbf{R}\cdot\mathbf{F} = d(-S_{action,free})/d\tau = \overbrace{H_o}^{E_o + W_o} = -L_o$$

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$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study:
4-Momentum → 4-Force
4-AngularMomentum → 4-Torque

4-Vector SRQM Interpretation of QM

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Linear:
4-Force **F** is the ProperTime Derivative ($\mathbf{F}\cdot\partial_P$) = $d/d\tau$ of 4-Momentum **P**

Angular:
4-Torque $T^{\mu\nu}$ = **X^F** is the ProperTime Derivative ($\mathbf{F}\cdot\partial_P$) = $d/d\tau$ of 4-AngularMomentum $M^{\mu\nu}$ = **X^P**

$d/d\tau[\, M^{\mu\nu} \,] = d/d\tau[\, \mathbf{X^P} \,] = (\mathbf{F}\cdot\partial_P)[\, \mathbf{X^P} \,]$
$= d/d\tau[\, X^\mu P^\nu - X^\nu P^\mu \,]$
$= [\, U^\mu P^\nu + X^\mu F^\nu - U^\nu P^\mu - X^\nu F^\mu \,]$
$= [\, U^\mu m_o U^\nu + X^\mu F^\nu - U^\nu m_o U^\mu - X^\nu F^\mu \,]$
$= [\, U^\mu m_o U^\nu - U^\nu m_o U^\mu + X^\mu F^\nu - X^\nu F^\mu \,]$
$= [\, m_o(U^\mu U^\nu - U^\nu U^\mu) + X^\mu F^\nu - X^\nu F^\mu \,]$
$= [\, m_o(0^{\mu\nu}) + X^\mu F^\nu - X^\nu F^\mu \,]$
$= [\, X^\mu F^\nu - X^\nu F^\mu \,]$

$d/d\tau[\, M^{\mu\nu} \,] = T^{\mu\nu} = [\, X^\mu F^\nu - X^\nu F^\mu \,] = \mathbf{X^F}$

SpaceTime
$\partial\cdot\mathbf{X} = \partial_\mu X^\mu = 4$
Dimension

Minkowski
$\partial[\mathbf{X}] = \partial^\mu[X^\nu] = \eta^{\mu\nu}$
Metric

Lorentz
$\partial_\nu[X^{\mu'}] = \Lambda^{\mu'}{}_\nu$
Transform

4-Gradient
$\partial = (\partial_t/c, -\nabla)$

4-Displacement
$\Delta\mathbf{X} = (c\Delta t, \Delta\mathbf{x})$

4-Position
$\mathbf{X} = (ct, \mathbf{x})$

$\mathbf{X}\cdot\mathbf{P} = -S_{action,free}$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

4-Velocity
$\mathbf{U} = \gamma(c, \mathbf{u}) = d\mathbf{X}/d\tau$

m_o
E_o/c^2

4-Momentum
$P^\mu = \mathbf{P} = (mc, \mathbf{p}) = (E/c, \mathbf{p})$

4-AngMomentum Tensor
$M^{\mu\nu} = X^\mu P^\nu - X^\nu P^\mu = \mathbf{X^P}$
→

$$\begin{bmatrix} 0 & -cn^x & -cn^y & -cn^z \\ +cn^x & 0 & +l^z & -l^y \\ +cn^y & -l^z & 0 & +l^x \\ +cn^z & +l^y & -l^x & 0 \end{bmatrix}$$

=

$$\begin{bmatrix} 0 & , -cn^j \\ +cn^i, & \varepsilon^{ij}{}_k l^k \end{bmatrix}$$

$(\mathbf{U}\cdot\partial_R)\,[..]$
$(\mathbf{F}\cdot\partial_P)\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

4-Torque Tensor
$T^{\mu\nu} = X^\mu F^\nu - X^\nu F^\mu = \mathbf{X^F}$
=
$dM^{\mu\nu}/d\tau$

=

$(\mathbf{U}\cdot\partial_R)\,[..]$
$(\mathbf{F}\cdot\partial_P)\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

4-Force
$\mathbf{F} = \gamma(\dot{E}/c, \mathbf{f} = \dot{\mathbf{p}})$
$= d\mathbf{P}/d\tau$

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$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SR 4-Vectors & 4-Tensors
Lorentz Scalar Product & Tensor Trace Invariants: Similarities

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

All {4-Vectors:4-Tensors} have an associated {Lorentz Scalar Product:Trace}

Each 4-Vector has a "magnitude" given by taking the Lorentz Scalar Product of itself.
$\mathbf{V}\cdot\mathbf{V} = V^\mu \eta_{\mu\nu} V^\nu = V^\mu V_\mu = V_\nu V^\nu = (v_0 v^0 + v_1 v^1 + v_2 v^2 + v_3 v^3) = (v^0 v^0 - \mathbf{v}\cdot\mathbf{v}) = (v^0{}_o)^2$
The magnitude^2 of **V** is **V·V**, which can be (+/0/-)

Each 4-Tensor has a "magnitude" given by taking the Tensor Trace of itself.
$\text{Trace}[T^{\mu\nu}] = \text{Tr}[T^{\mu\nu}] = \eta_{\mu\nu} T^{\mu\nu} = T^\mu{}_\mu = T_\nu{}^\nu = (T_0{}^0 + T_1{}^1 + T_2{}^2 + T_3{}^3) = (T^{00} - T^{11} - T^{22} - T^{33}) = T$
Note that the Trace runs down the diagonal of the 4-Tensor.

Notice the similarities. In both cases there is a tensor contraction with the Minkowski Metric Tensor $\eta_{\mu\nu} \rightarrow \text{Diag}[+1,-1,-1,-1]_{\{\text{Cartesian basis}\}}$ with result (+/0/-)

ex. $\mathbf{P}\cdot\mathbf{P} = (E/c)^2 - \mathbf{p}\cdot\mathbf{p} = (E_o/c)^2 = (m_o c)^2$
which says that the "magnitude" of the 4-Momentum is the RestEnergy/c = RestMass*c

ex. $\text{Trace}[\eta^{\mu\nu}] = (\eta^{00} - \eta^{11} - \eta^{22} - \eta^{33}) = 1 -(-1) -(-1) -(-1) = 1+1+1+1 = 4$
which says that the "magnitude" of the Minkowski Metric = SpaceTime Dimension = 4

Lorentz Scalar Invariant

$\mathbf{V}\cdot\mathbf{V} = V^\mu V_\mu = (v^0 v^0 - \mathbf{v}\cdot\mathbf{v}) = (v^0{}_o)^2$

4-Vector
$\mathbf{V} = V^\mu = (v^0, \mathbf{v})$

Trace Tensor Invariant

$\text{Tr}[T^{\mu\nu}] = T^\mu{}_\mu = (T^{00} - T^{11} - T^{22} - T^{33}) = T$

4-Tensor
$T^{\mu\nu} = [T^{00}, T^{01}, T^{02}, T^{03}]$
$[T^{10}, T^{11}, T^{12}, T^{13}]$
$[T^{20}, T^{21}, T^{22}, T^{23}]$
$[T^{30}, T^{31}, T^{32}, T^{33}]$

$\mathbf{P}\cdot\mathbf{P} = (m_o c)^2 = (E_o/c)^2$

4-Momentum
$\mathbf{P} = (mc, \mathbf{p}) = (E/c, \mathbf{p})$

$\text{Tr}[\eta^{\mu\nu}] = 4$

Minkowski Metric
$\partial[\mathbf{R}] = \eta^{\mu\nu} \rightarrow \text{Diag}[1,-1,-1,-1]$

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SR 4-Vectors & 4-Tensors
More 4-Vector-based Invariants
Phase Space Integration

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John B. Wilson
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Some other SR Invariants include:

Lorentz Scalar Invariant

$\mathbf{V}\cdot\mathbf{V}=V^\mu V_\mu=(v^0v^0 - \mathbf{v}\cdot\mathbf{v})=(v^0_o)^2$

4-Vector
$\mathbf{V} = V^\mu = (v^0,\mathbf{v})$

$d\mathbf{v}/v^0 = d^3v/v^0$ if $\mathbf{v}\cdot\mathbf{v}$=(constant)

Phase Space Invariant

$\mathbf{P}\cdot\mathbf{P}=(m_oc)^2=(E_o/c)^2$

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

$d^3\mathbf{p}/E$

Particle #
$N = (-V_o/c)\int\mathbf{dT}\cdot\mathbf{N}$
$= \int n d^3\mathbf{x} = \int\gamma n_o d^3\mathbf{x}$
$\rightarrow n_oV_o$

q

EM Charge
$Q = (-V_o/c)\int\mathbf{dT}\cdot\mathbf{J}$
$= \int\rho d^3\mathbf{x} = \int\gamma\rho_o d^3\mathbf{x}$
$\rightarrow \rho_oV_o$

n_o

ρ_o

γdV
$= \gamma\, dx\cdot dy\cdot dz$
$= \gamma(dr)\cdot(dA)$
$= \gamma d^3\mathbf{x}$

$\int[..]$

Rest Volume $V_o = \int\gamma dV = \int\gamma d^3\mathbf{x}$
$= -cN/\int\mathbf{dT}\cdot\mathbf{N} = -cQ/\int\mathbf{dT}\cdot\mathbf{J}$

$c\,d\tau$

$d^4\mathbf{X}$
$= cdt\cdot dx\cdot dy\cdot dz$
$= c\gamma d\tau\cdot dx\cdot dy\cdot dz$
$= cd\tau\cdot\gamma d^3\mathbf{x}$
$= cdt\cdot d^3\mathbf{x}$

$d^4\mathbf{P}$
$= (dE/c)\, dp^x\, dp^y\, dp^z$
$= (dE/c)\, d^3\mathbf{p}$

$d^4\mathbf{K}$
$= (d\omega/c)\, dk^x\, dk^y\, dk^z$
$= (d\omega/c)\, d^3\mathbf{k}$

$d^3\mathbf{p}\, d^3\mathbf{x}$
$= dp^x\, dp^y\, dp^z\, dx\, dy\, dz$

$d^3\mathbf{k}\, d^3\mathbf{x}$
$= dk^x\, dk^y\, dk^z\, dx\, dy\, dz$

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SR 4-Vectors & 4-Tensors

More 4-Vector-based Invariants

Phase Space Integration

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Some 4-Vectors have an alternate form of Tensor Invariant: ($d\mathbf{v}'/v^{0'} = d\mathbf{v}/v^0$) or ($d^3v'/v^{0'} = d^3v/v^0$) in addition to the standard Lorentz Invariant $\mathbf{V}\cdot\mathbf{V} = V^\mu V_\mu = (v^0v^0 - \mathbf{v}\cdot\mathbf{v}) = (v^0_o)^2$

Lorentz Scalar Invariant

$\mathbf{V}\cdot\mathbf{V}=V^\mu V_\mu=(v^0v^0 - \mathbf{v}\cdot\mathbf{v})=(v^0_o)^2$

4-Vector
$\mathbf{V} = V^\mu = (v^0,\mathbf{v})$

$d\mathbf{v}/v^0 \longrightarrow d^3v/v^0$ if $\mathbf{v}\cdot\mathbf{v}$=(constant)

Phase Space Invariant

<u>If $\mathbf{V}\cdot\mathbf{V}$ = (constant):</u>, with $\mathbf{V} = (v^0,\mathbf{v})$
then $d(\mathbf{V}\cdot\mathbf{V}) = 2*(\mathbf{V}\cdot d\mathbf{V}) = d(\text{constant}) = 0$
hence $(\mathbf{V}\cdot d\mathbf{V}) = 0 = v^0dv^0 - \mathbf{v}\cdot d\mathbf{v}$
$dv^0 = \mathbf{v}\cdot d\mathbf{v}/v^0$

<u>Generally:</u> with $\Lambda = \Lambda^{\mu'}{}_\nu$ = Lorentz Boost Transform in the $\boldsymbol{\beta}$-direction
$\mathbf{V}' = \Lambda\mathbf{V}$: from which the temporal component $v^{0'} = (\gamma v^0 - \gamma\boldsymbol{\beta}\cdot\mathbf{v})$
$d\mathbf{V}' = \Lambda d\mathbf{V}$: from which the spatial component $d\mathbf{v}' = (\gamma d\mathbf{v} - \gamma\boldsymbol{\beta}dv^0)$

<u>Combining:</u>
$d\mathbf{v}' = (\gamma d\mathbf{v} - \gamma\boldsymbol{\beta}(\mathbf{v}\cdot d\mathbf{v}/v^0))$
$d\mathbf{v}' = (1/v^0)*(\gamma v^0 d\mathbf{v} - \gamma\boldsymbol{\beta}(\mathbf{v}\cdot d\mathbf{v}))$
$d\mathbf{v}' = (1/v^0)*(\gamma v^0 - \gamma\boldsymbol{\beta}\cdot\mathbf{v})d\mathbf{v}$
$d\mathbf{v}' = (\gamma v^0 - \gamma\boldsymbol{\beta}\cdot\mathbf{v})*(1/v^0)*d\mathbf{v}$
$d\mathbf{v}' = (v^{0'}/v^0)d\mathbf{v}$
$d\mathbf{v}'/v^{0'} = d\mathbf{v}/v^0$ = Invariant of $\mathbf{V} = (v^0,\mathbf{v})$ for $\mathbf{V}\cdot\mathbf{V}$ = (constant)

An alternate approach is:
$\int d^4p\ \delta[p^2-(m_oc)^2]$
$= \int d^4p\ (1/2|m_oc|)\ (\delta[p+m_oc] + \delta[p-m_oc])$
$= cd^3p/2E$
= Invariant

$\mathbf{P}\cdot\mathbf{P}=(m_oc)^2=(E_o/c)^2$

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

d^3p/E

<u>So, for example:</u>
$\mathbf{P}\cdot\mathbf{P} = (m_oc)^2$ = (constant, assuming rest mass doesn't change)

Thus, $d\mathbf{p}'/(E'/c) = d\mathbf{p}/(E/c)$ = Invariant
Or: $d\mathbf{p}'/E' = d\mathbf{p}/E \rightarrow d^3p/E = dp^xdp^ydp^z/E$ = Invariant, usually seen as $\int$ F(various invariants)*d^3p/E = Invariant

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(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0_o)^2$
= Lorentz Scalar Invariant

SR 4-Vectors & 4-Tensors

More 4-Vector-based Invariants

Phase Space Integration

SciRealm.org
John B. Wilson
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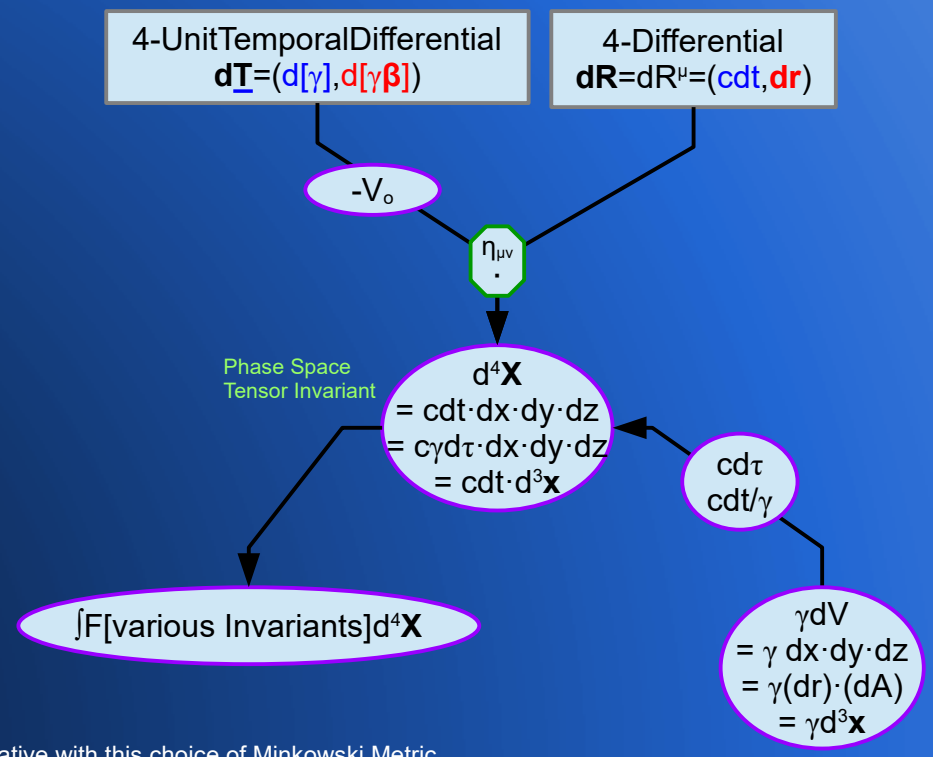
http://scirealm.org/SRQM.pdf

Invariant $d^4\mathbf{X} = -(V_o)\mathbf{dT}\cdot\mathbf{dX} = -(dV_o)\mathbf{T}\cdot\mathbf{dX} = cdt\, d^3\mathbf{x} = cdt\cdot dx\cdot dy\cdot dz$
The 4D Position coords that are integrated to give a 4D volume: SI units [m^4]

4-Differential $\mathbf{dX} = (cdt,\mathbf{dx})$; $\mathbf{dR} = (cdt,\mathbf{dr})$;
4-UnitTemporal $\mathbf{T} = \gamma(1,\boldsymbol{\beta}) = (\gamma,\gamma\boldsymbol{\beta})$
4-UnitTemporalDifferential $\mathbf{dT} = d[(\gamma,\gamma\boldsymbol{\beta})] = (d[\gamma],d[\gamma\boldsymbol{\beta}])$

$V = \int dV = \int dx \int dy \int dz = \iiint dx\, dy\, dz = \int d^3\mathbf{x}$
$V = V_o/\gamma$ = 3D Spatial Volume: SI units [m^3]
$dV = d^3\mathbf{x}$ = 3D Spatial Volume Element
$\gamma = V_o/V$
$d\gamma = -(V_o/V^2)dV$

$-(V_o)\mathbf{dT}\cdot\mathbf{dX}$ = Invariant, because (Rest Scalar * Lorentz Scalar Product) = Invariant
$= -(V_o)(d[\gamma],d[\gamma\boldsymbol{\beta}])\cdot(cdt,\mathbf{dx})$
$= -(V_o)(d[\gamma]cdt - d[\gamma\boldsymbol{\beta}]\cdot\mathbf{dx})$
$= -(V_o)(-(V_o/V^2)dVcdt - d[\gamma\boldsymbol{\beta}]\cdot\mathbf{dx})$
$= -(V_o)(-(V_o/V_o^2)dVcdt - d[(1)(\mathbf{0})]\cdot\mathbf{dx})$ by taking the usual rest-case
$= -(V_o)(-(V_o/V_o^2)dVcdt)$
$= -(V_o)(-(1/V_o)dVcdt)$
$= dVcdt$
$= cdt\, dV$
$= cdt\cdot dx\cdot dy\cdot dz$
$= cdt\, d^3\mathbf{x}$
$= d^4\mathbf{X}$ = Invariant
And, this makes sense.
T is a temporal 4-Vector with fixed magnitude: $\mathbf{T}\cdot\mathbf{T} = 1$. $d(\mathbf{T}\cdot\mathbf{T}) = d(1) = 0 = 2(d\mathbf{T}\cdot\mathbf{T})$
Since $(d\mathbf{T}\cdot\mathbf{T})=0$, **dT** must orthogonal to **T** and thus must be a spatial 4-Vector
If **dX** is also spatial, then the Lorentz scalar product { $(\mathbf{dT}\cdot\mathbf{dX})$ = -magnitude } will be negative with this choice of Minkowski Metric.
Thus, multiplying by $-(V_o)$ gives a positive volume element{ $cdt\, dx\, dy\, dz = d^4\mathbf{X}$}
It is sort of quirky though, that the temporal (cdt) comes from the **dX** part, and the spatial ($d^3\mathbf{x}$) comes from the **dT** part.



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= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SR 4-Vectors & 4-Tensors

More 4-Vector-based Invariants

Phase Space Integration

4-Vector SRQM Interpretation of QM

SciRealm.org
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$\rho\, d^3\mathbf{x} = \rho'\, d^3\mathbf{x}' = (-V_o/c)\mathbf{dT}\cdot\mathbf{J}$ = Lorentz Scalar Invariant
$n\, d^3\mathbf{x} = n'\, d^3\mathbf{x}' = (-V_o/c)\mathbf{dT}\cdot\mathbf{N}$ = Lorentz Scalar Invariant

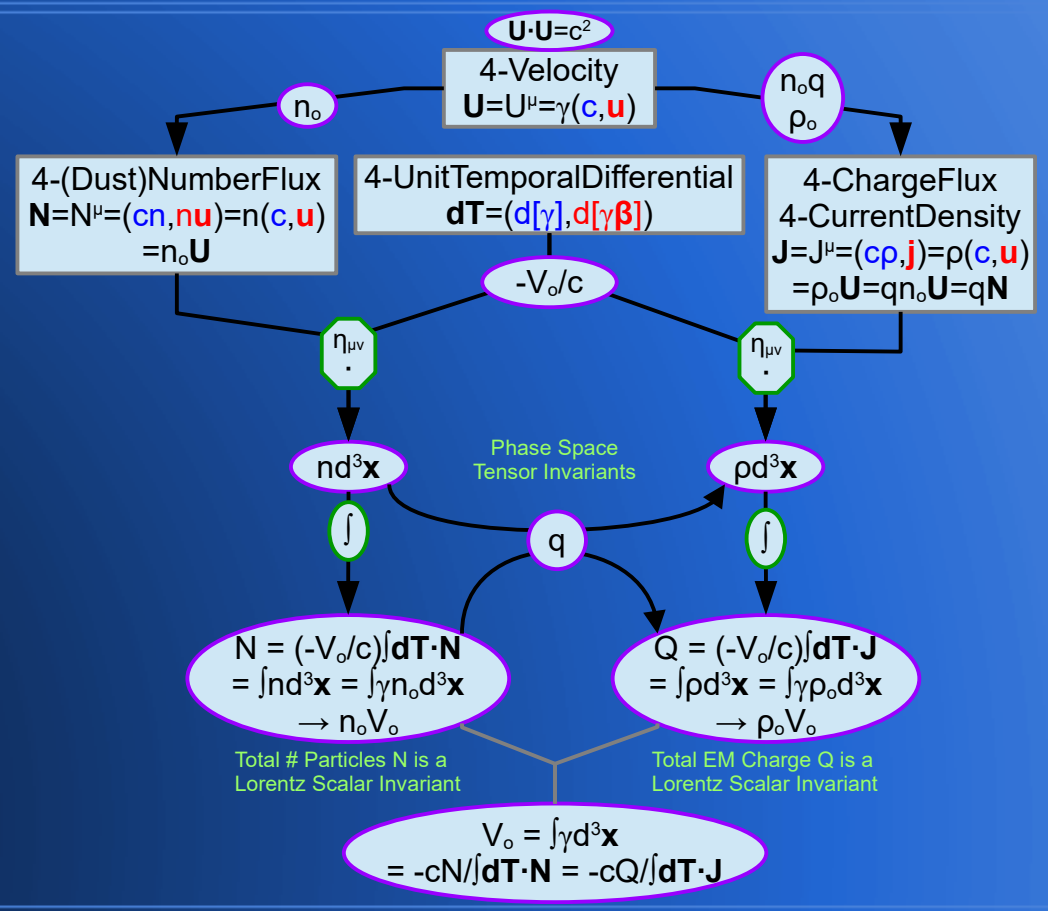
4-CurrentDensity $\mathbf{J} = (\rho c, \mathbf{j}) = \rho_o\mathbf{U}$
4-NumberFlux $\mathbf{N} = (nc, \mathbf{n}) = n_o\mathbf{U}$
4-UnitTemporal $\mathbf{T} = \gamma(1,\boldsymbol{\beta}) = (\gamma,\gamma\boldsymbol{\beta})$
4-UnitTemporalDifferential $\mathbf{dT} = d[(\gamma,\gamma\boldsymbol{\beta})] = (d[\gamma],d[\gamma\boldsymbol{\beta}])$

$V = V_o/\gamma$
$d\gamma = -(V_o/V^2)dV$

$(-V_o/c)\mathbf{dT}\cdot\mathbf{J}$ = Invariant, because (Rest Scalar * Lorentz Scalar Product) = Invariant
$= (-V_o/c)(d[\gamma],d[\gamma\boldsymbol{\beta}])\cdot(\rho c,\mathbf{j})$
$= (-V_o/c)(d[\gamma]\rho c - d[\gamma\boldsymbol{\beta}]\cdot\mathbf{j})$
$= (-V_o/c)(-(V_o/V^2)(dV)(\rho c) - d[\gamma\boldsymbol{\beta}]\cdot\mathbf{j})$
$= (-V_o/c)(-(V_o/V_o^2)(dV)(\rho c) - d[(1)\mathbf{0}]\cdot\mathbf{j})$
$= (-V_o/c)(-(V_o/V_o^2)(dV)(\rho c))$
$= (dV/c)(\rho c)$
$= (\rho c)(dV/c)$
$= (\rho)(dV)$
$= \rho\, d^3\mathbf{x}$

Total Charge $Q = \int\gamma\rho_o\, d^3\mathbf{x} = \int\rho\, d^3\mathbf{x}$ = Lorentz Scalar Invariant
Total Particle # $N = \int\gamma n_o\, d^3\mathbf{x} = \int n\, d^3\mathbf{x}$ = Lorentz Scalar Invariant
Total RestVolume $V_o = \int\gamma(1)d^3\mathbf{x}$ = Lorentz Scalar Invariant

This also gives an alternate way to define the RestVolume Invariant V_o.
$(-V_o/c)\mathbf{dT}\cdot\mathbf{N} = nd^3\mathbf{x}$
$N = \int nd^3\mathbf{x} = \int(-V_o/c)\mathbf{dT}\cdot\mathbf{N}$
$cN/V_o = -\int\mathbf{dT}\cdot\mathbf{N}$
$V_o = -cN/\int\mathbf{dT}\cdot\mathbf{N}$



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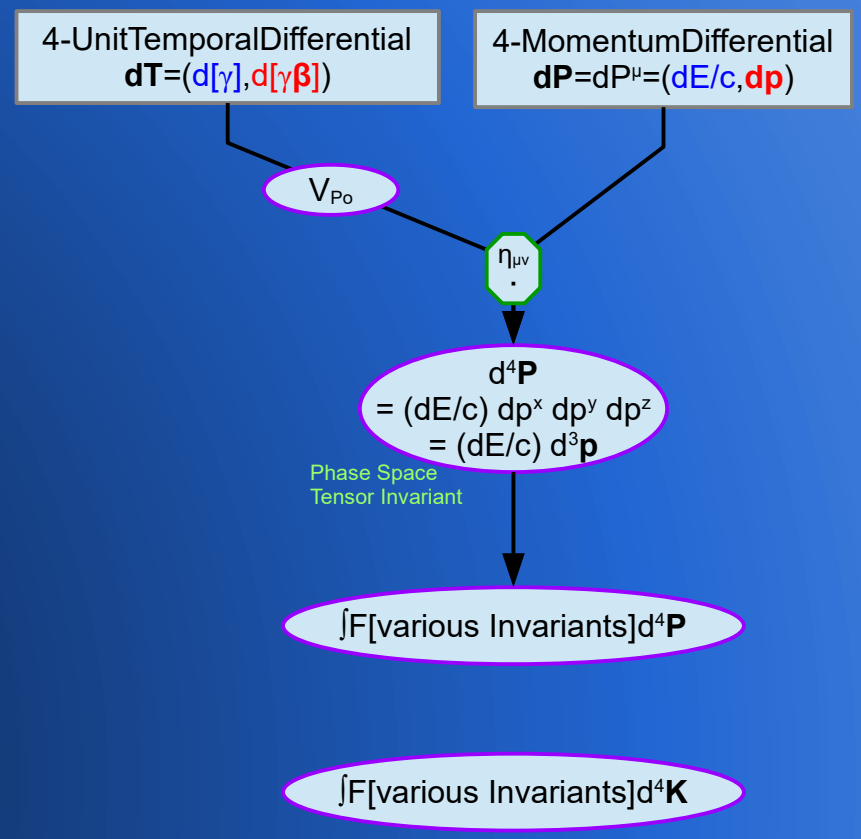
$d^4\mathbf{P} = (V_{Po})\mathbf{dT}\cdot\mathbf{dP} = (dE/c)\, d^3\mathbf{p} = (dE/c)\, dp^x\, dp^y\, dp^z$
$d^4\mathbf{K} = (V_{Ko})\mathbf{dT}\cdot\mathbf{dK} = (d\omega/c)\, d^3\mathbf{k} = (d\omega/c)\, dk^x\, dk^y\, dk^z$
The 4D Momentum coords that are integrated to give a 4D Momentum Volume: SI Units $[(kg\cdot m/s)^4]$
The 4D WaveVector coords that are integrated to give a 4D WaveVector Volume: SI Units $[(1/m)^4]$

4-DifferentialMomentum $\mathbf{dP} = (dE/c, \mathbf{dp})$
4-DifferentialWaveVector $\mathbf{dK} = (d\omega/c, \mathbf{dk})$
4-UnitTemporal $\mathbf{T} = \gamma(1, \boldsymbol{\beta}) = (\gamma, \gamma\boldsymbol{\beta})$
4-UnitTemporalDifferential $\mathbf{dT} = d[(\gamma, \gamma\boldsymbol{\beta})] = (d[\gamma], d[\gamma\boldsymbol{\beta}])$

$V_P = \int dV_P = \int dp^x \int dp^y \int dp^z = \iiint dp^x\, dp^y\, dp^z = \int d^3\mathbf{p}$
$V_P = \gamma(V_{Po})$ = 3D Volume in Momentum Space: SI Units $[(kg\cdot m/s)^3]$
$dV_P = d\gamma(V_{Po})$ = 3D Volume Element in Momentum Space
$\gamma = (V_P)/(V_{Po})$
$d\gamma = (dV_P)/(V_{Po})$

$(V_{Po})\mathbf{dT}\cdot\mathbf{dP}$ = Invariant, because Rest Scalar * Lorentz Scalar Product
$= (V_{Po}))(d[\gamma], d[\gamma\boldsymbol{\beta}])\cdot(dE/c, \mathbf{dp})$
$= (V_{Po})(d[\gamma]dE/c - d[\gamma\boldsymbol{\beta}]\cdot\mathbf{dp})$
$= (V_{Po})((dV_P/V_{Po})dE/c - d[\gamma\boldsymbol{\beta}]\cdot\mathbf{dp})$
$= (V_{Po}))((dV_P/V_{Po})dE/c - d[(1)(\mathbf{0})]\cdot\mathbf{dp})$ by taking the usual rest-case
$= (V_{Po}))((dV_P/V_{Po})dE/c)$
$= (dV_P)\,(dE/c)$
$= d^3\mathbf{p}\,(dE/c)$
$= (dE/c)\, d^3\mathbf{p}$
$= (dE/c)\, dp^x\, dp^y\, dp^z$
$= d^4\mathbf{P}$ = Invariant

Likewise, $d^4\mathbf{K}$ = Invariant



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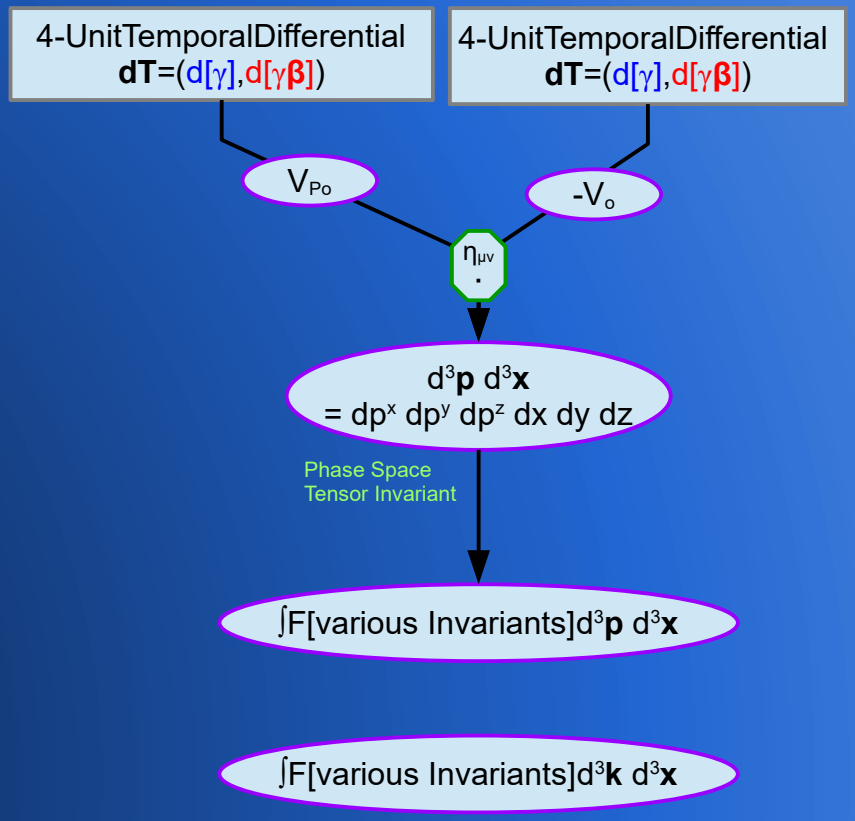
$d^3\mathbf{p}\ d^3\mathbf{x} = (V_{Po})\mathbf{dT}\cdot(-V_o)\mathbf{dT} = (-V_o)(V_{Po})\mathbf{dT}\cdot\mathbf{dT}$
$d^3\mathbf{k}\ d^3\mathbf{x} = (V_{Ko})\mathbf{dT}\cdot(-V_o)\mathbf{dT} = (-V_o)(V_{Ko})\mathbf{dT}\cdot\mathbf{dT}$

4-UnitTemporal $\mathbf{T} = \gamma(1,\boldsymbol{\beta}) = (\gamma,\gamma\boldsymbol{\beta})$
4-UnitTemporalDifferential $\mathbf{dT} = d[(\gamma,\gamma\boldsymbol{\beta})] = (d[\gamma],d[\gamma\boldsymbol{\beta}])$

$(V_{po})\mathbf{dT}\cdot(-V_o)\mathbf{dT}$ = Invariant
$= (V_{Po})(d[\gamma],d[\gamma\boldsymbol{\beta}])\cdot(-V_o)(d[\gamma],d[\gamma\boldsymbol{\beta}])$
$= (V_{Po})(-V_o)(d[\gamma]d[\gamma] - d[\gamma\boldsymbol{\beta}]\cdot d[\gamma\boldsymbol{\beta}])$
$= (V_{Po})(-V_o)(-(V_o/V^2)dV(dV_P/(V_{Po})) - d[\gamma\boldsymbol{\beta}]\cdot d[\gamma\boldsymbol{\beta}])$
$= (V_{Po})(-V_o)(-(V_o/V_o^2)dV(dV_P/(V_{Po})) - d[(1)\mathbf{0}]\cdot d[(1)\mathbf{0}])$
$= (V_{Po})(-V_o)(-(V_o/V_o^2)dV(dV_P/(V_{Po}))$
$= (V_{Po})dV(dV_P/(V_{Po}))$
$= dV\ dV_P$
$= dV_P\ dV$
$= d^3\mathbf{p}\ d^3\mathbf{x}$ = Invariant

Likewise, $d^3\mathbf{k}\ d^3\mathbf{x}$ = Invariant



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= Lorentz Scalar Invariant

SRQM Study: SR 4-Tensor Properties

General → Symmetric & Anti-Symmetric

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John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Any SR Tensor $T^{\mu\nu} = (S^{\mu\nu} + A^{\mu\nu})$ can be decomposed into parts:

Symmetric $S^{\mu\nu} = T^{(\mu\nu)} = (T^{\mu\nu}+T^{\nu\mu})/2$ with $S^{\mu\nu} = +S^{\nu\mu}$

Anti-Symmetric $A^{\mu\nu} = T^{[\mu\nu]} = (T^{\mu\nu}-T^{\nu\mu})/2$ with $A^{\mu\nu} = -A^{\nu\mu}$

$$S^{\mu\nu} + A^{\mu\nu} = (T^{\mu\nu}+T^{\nu\mu})/2+(T^{\mu\nu}-T^{\nu\mu})/2 = T^{\mu\nu}/2 + T^{\mu\nu}/2 + T^{\nu\mu}/2 - T^{\nu\mu}/2 = T^{\mu\nu} + 0 = T^{\mu\nu}$$

Independent components: $\{ 4^2 = 16 = 10 + 6 \}$

Max 16 possible

General 4-Tensor

$$T^{\mu\nu} = \begin{bmatrix} T^{00}, T^{01}, T^{02}, T^{03} \\ T^{10}, T^{11}, T^{12}, T^{13} \\ T^{20}, T^{21}, T^{22}, T^{23} \\ T^{30}, T^{31}, T^{32}, T^{33} \end{bmatrix}$$

=

Max 10 possible

Symmetric 4-Tensor

$$S^{\mu\nu} = \begin{bmatrix} S^{00}, S^{01}, S^{02}, S^{03} \\ S^{10}, S^{11}, S^{12}, S^{13} \\ S^{20}, S^{21}, S^{22}, S^{23} \\ S^{30}, S^{31}, S^{32}, S^{33} \end{bmatrix} = \begin{bmatrix} S^{00}, & S^{01}, & S^{02}, & S^{03} \\ +S^{01}, & S^{11}, & S^{12}, & S^{13} \\ +S^{02}, & +S^{12}, & S^{22}, & S^{23} \\ +S^{03}, & +S^{13}, & +S^{23}, & S^{33} \end{bmatrix}$$

$Tr[S^{\mu\nu}]=S^{\mu}{}_{\mu}$

+

Max 6 possible

Anti-Symmetric 4-Tensor

$$A^{\mu\nu} = \begin{bmatrix} A^{00}, A^{01}, A^{02}, A^{03} \\ A^{10}, A^{11}, A^{12}, A^{13} \\ A^{20}, A^{21}, A^{22}, A^{23} \\ A^{30}, A^{31}, A^{32}, A^{33} \end{bmatrix} = \begin{bmatrix} 0, & A^{01}, & A^{02}, & A^{03} \\ -A^{01}, & 0, & A^{12}, & A^{13} \\ -A^{02}, & -A^{12}, & 0, & A^{23} \\ -A^{03}, & -A^{13}, & -A^{23}, & 0 \end{bmatrix}$$

$Tr[A^{\mu\nu}]=0$

aka Skew-Symmetric

Importantly, the Contraction of any Symmetric tensor with any Anti-Symmetric tensor on the same pair of indices is always 0.

Note These don't have to be composed from a single general tensor.

$S^{\mu\nu} A_{\mu\nu} = 0$

Proof:
$S^{\mu\nu} A_{\mu\nu}$
$= S^{\nu\mu} A_{\nu\mu}$: because we can switch dummy indices
$= (+S^{\mu\nu})A_{\nu\mu}$: because of symmetry
$= S^{\mu\nu}(-A_{\mu\nu})$: because of anti-symmetry
$= -S^{\mu\nu} A_{\mu\nu}$
$= 0$: because the only solution of $\{c = -c\}$ is 0

Physically, the anti-symmetric part contains rotational information and the symmetric part contains information about isotropic scaling and anisotropic shear.

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Maximum Degrees of Freedom (DoF)
= # of possible independent components
= (Tensor dimension)^(Tensor rank)

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM Study: SR 4-Tensor Properties

Symmetric → Isotropic & Anisotropic

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Any Symmetric SR Tensor $S^{\mu\nu} = (T_{iso}{}^{\mu\nu} + T_{aniso}{}^{\mu\nu})$ can be decomposed into parts:

Isotropic $T_{iso}{}^{\mu\nu} = (1/4)\mathrm{Trace}[S^{\mu\nu}]\,\eta^{\mu\nu} = (T)\,\eta^{\mu\nu}$

Anisotropic $T_{aniso}{}^{\mu\nu} = S^{\mu\nu} - T_{iso}{}^{\mu\nu}$

The Anisotropic part is Traceless by construction, and the Isotropic part has the same Trace as the original Symmetric Tensor. The Minkowski Metric is a symmetric, isotropic 4-tensor with T=1.

Independent components:

Max 10 possible

Symmetric 4-Tensor

$$S^{\mu\nu} = \begin{bmatrix} S^{00}, & S^{01}, & S^{02}, & S^{03} \\ S^{10}, & S^{11}, & S^{12}, & S^{13} \\ S^{20}, & S^{21}, & S^{22}, & S^{23} \\ S^{30}, & S^{31}, & S^{32}, & S^{33} \end{bmatrix} = \begin{bmatrix} S^{00}, & S^{01}, & S^{02}, & S^{03} \\ +S^{01}, & S^{11}, & S^{12}, & S^{13} \\ +S^{02}, & +S^{12}, & S^{22}, & S^{23} \\ +S^{03}, & +S^{13}, & +S^{23}, & S^{33} \end{bmatrix}$$

$\mathrm{Tr}[S^{\mu\nu}] = S^{\mu}{}_{\mu} = 4T$

=

Max 1 possible

Symmetric Isotropic 4-Tensor

$$T_{iso}{}^{\mu\nu} = \begin{bmatrix} T, & 0, & 0, & 0 \\ 0, & -T, & 0, & 0 \\ 0, & 0, & -T, & 0 \\ 0, & 0, & 0, & -T \end{bmatrix}$$

with $T = (1/4)\mathrm{Trace}[S^{\mu\nu}]$

$\mathrm{Tr}[T_{iso}{}^{\mu\nu}] = 4T$

+

Max 9 possible

Symmetric Anisotropic 4-Tensor

$$T_{aniso}{}^{\mu\nu} = \begin{bmatrix} S^{00}-T, & S^{01}, & S^{02}, & S^{03} \\ S^{10}, & S^{11}+T, & S^{12}, & S^{13} \\ S^{20}, & S^{21}, & S^{22}+T, & S^{23} \\ S^{30}, & S^{31}, & S^{32}, & S^{33}+T \end{bmatrix} = \begin{bmatrix} S^{00}-T, & S^{01}, & S^{02}, & S^{03} \\ +S^{01}, & S^{11}+T, & S^{12}, & S^{13} \\ +S^{02}, & +S^{12}, & S^{22}+T, & S^{23} \\ +S^{03}, & +S^{13}, & +S^{23}, & S^{33}+T \end{bmatrix}$$

aka Deviatoric

$\mathrm{Tr}[T_{aniso}{}^{\mu\nu}] = 0$

Importantly, the Contraction of any Symmetric tensor with any Anti-Symmetric tensor on the same index is always 0.

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$S^{\mu\nu} A_{\mu\nu} = 0$

Proof:

$S^{\mu\nu} A_{\mu\nu}$

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$= -S^{\mu\nu} A_{\mu\nu}$

= 0: because the only solution of {c = -c} is 0

Physically, the isotropic part represents a direction independent transformation (e.g., a uniform scaling or uniform pressure); the deviatoric part represents the distortion

An Isotropic Tensor has the same components in all possible coordinate-frames.

Rank 0: All Scalars are isotropic

Rank 1: There are no non-zero isotropic vectors

Rank 2: Most general isotropic 2nd rank tensor must equal to $\lambda\delta^{\mu}{}_{\nu} = \lambda\eta^{\mu}{}_{\nu}$ for some scalar λ.

Rank 3: Most general isotropic 3rd rank tensor must equal to $\lambda\varepsilon^{ijk}$ for some scalar λ.

Rank 4: Most general isotropic 4th rank tensor must equal to $a\delta^{\mu\nu}\delta^{\alpha\beta} + b\delta^{\mu\alpha}\delta^{\nu\beta} + c\delta^{\mu\beta}\delta^{\nu\alpha}$ for scalars {a,b,c}.

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= Lorentz Scalar Invariant

SRQM Study: SR 4-Tensors

4-Tensor Decomposition based on Tensor Invariants

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General (rank=2) 4-Tensor $T^{\mu\nu}$
$= T_{symm}^{\mu\nu} + T_{anti\text{-}symm}^{\mu\nu}$

Symmetric 4-Tensor $T_{symm}^{\mu\nu}$
$= (T^{\mu\nu} + T^{\nu\mu})/2$
$= T_{iso}^{\mu\nu} + T_{aniso}^{\mu\nu}$

Isotropic Symm 4-Tensor
$T_{iso}^{\mu\nu} = (Tr[T_{symm}^{\mu\nu}]/4)\, \eta^{\mu\nu}$
$Tr[T_{iso}^{\mu\nu}] = Tr[T_{symm}^{\mu\nu}]$
uniform scaling info
max DoF = 1

+

Anisotropic Symm 4-Tensor
$T_{aniso}^{\mu\nu} = T_{symm}^{\mu\nu} - T_{iso}^{\mu\nu}$
$Tr[T_{aniso}^{\mu\nu}] = 0$
shearing:distortion info
max DoF = 9

$T_{symm}^{\mu\nu} = +T_{symm}^{\nu\mu}$
max DoF = 10

+

Anti-Symmetric 4-Tensor $T_{anti\text{-}symm}^{\mu\nu}$
$= (T^{\mu\nu} - T^{\nu\mu})/2$

$Tr[T_{anti\text{-}symm}^{\mu\nu}] = 0$

$T_{anti\text{-}symm}^{\mu\nu} = -T_{anti\text{-}symm}^{\nu\mu}$
rotational info
max DoF = 6

max DoF = (dim)^(rank) = 4^2 = 4x4 =16 = (10+6)

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Maximum Degrees of Freedom (DoF)
= # of possible independent components
= (Tensor dimension)^(Tensor rank)

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4

A Tensor Study of Physical 4-Vectors

SRQM Study: SR 4-Tensors

4-Tensor Temporal:Mixed:Spatial (1+3) Decomposition

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

General (rank=2) 4-Tensor $T^{\mu\nu}$
$= T_{symm}{}^{\mu\nu} + T_{anti\text{-}symm}{}^{\mu\nu}$

Symmetric 4-Tensor $T_{symm}{}^{\mu\nu}$

$= (T^{\mu\nu} + T^{\nu\mu})/2 = T_{symtemp}{}^{\mu\nu} + T_{symmixed}{}^{\mu\nu} + T_{symspat}{}^{\mu\nu}$

Temporal Symm 4-Tensor

$T_{symtemp}{}^{\mu\nu} = V^{\mu}{}_{\alpha}V^{\nu}{}_{\beta}(T_{symm}{}^{\alpha\beta})$

effective scalar
temporal info
ex. energy_density (ρ_e)
max DoF = 1

+

Mixed Symm 4-Tensor

$T_{symmixed}{}^{\mu\nu} = V^{\mu}{}_{\alpha}H^{\nu}{}_{\beta}(T_{symm}{}^{\alpha\beta}) + H^{\mu}{}_{\alpha}V^{\nu}{}_{\beta}(T_{symm}{}^{\alpha\beta})$

effective 3-vector
mixed time:space info
ex.. heat_flux (Q^{μ})
max DoF = 3

+

Spatial Symm 4-Tensor

$T_{symspat}{}^{\mu\nu} = H^{\mu}{}_{\alpha}H^{\nu}{}_{\beta}(T_{symm}{}^{\alpha\beta}) = T_{iso}{}^{\mu\nu} + T_{aniso}{}^{\mu\nu}$

Isotropic Spatial Symm 4-Tensor

$T_{iso}{}^{\mu\nu} = (Tr[T_{symspat}{}^{\mu\nu}]/3)\, H^{\mu\nu}$

$Tr[T_{iso}{}^{\mu\nu}] = Tr[T_{symm}{}^{\mu\nu}]$

effective scalar
uniform scaling info
ex. pressure (p)
max DoF = 1

+

Anisotropic Spatial Symm 4-Tensor

$T_{aniso}{}^{\mu\nu} = T_{symspat}{}^{\mu\nu} - T_{iso}{}^{\mu\nu}$

$Tr[T_{aniso}{}^{\mu\nu}] = 0$

effective 3-tensor
shearing:distortion info
ex. ViscousShear ($\Pi^{\mu\nu}$)
max DoF = 5

effective symm 3-tensor
spatial info
max DoF = 6

$T_{symm}{}^{\mu\nu} = +T_{symm}{}^{\nu\mu}$
max DoF = 10

+

Anti-Symmetric 4-Tensor $T_{anti\text{-}symm}{}^{\mu\nu}$

$= (T^{\mu\nu} - T^{\nu\mu})/2$

$Tr[T_{anti\text{-}symm}{}^{\mu\nu}] = 0$

$T_{anti\text{-}symm}{}^{\mu\nu} = -T_{anti\text{-}symm}{}^{\nu\mu}$
rotational info
max DoF = 6

ex.
Relativistic Fluid
Symmetric 4-Tensor

$\rho_e = \rho_m c^2$	Q^{0j}/c
Q^{i0}/c	$p\delta^{ij} + \Pi^{ij}$

max DoF = (dim)^(rank) = 4^2 = 4x4 =16 = (10+6)

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(0,0)-Tensor S or S_o
Lorentz Scalar

Maximum Degrees of Freedom (DoF)
= # of possible independent components
= (Tensor dimension)^(Tensor rank)

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
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(0,0)-Tensor = Lorentz Scalar S: Has either (0) or (1) Tensor Invariant, depending on exact meaning

(S) itself is Invariant

S

(1,0)-Tensor = 4-Vector V^μ: Has (1) Tensor Invariant = The Lorentz Scalar Product

$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = \eta_{\mu\nu}V^\mu V^\nu = Tr[V^\mu V^\nu] = V_\nu V^\nu = (v_0v^0 + v_1v^1 + v_2v^2 + v_3v^3) = (v^0v^0 - \mathbf{v}\cdot\mathbf{v}) = (v^0_o)^2$

$\mathbf{V}=V^\mu=(v^\mu)=(v^0,v^1,v^2,v^3)$

$\mathbf{V}\cdot\mathbf{V} = (v^0v^0 - \mathbf{v}\cdot\mathbf{v}) = (v^0_o)^2$

(2,0)-Tensor = 4-Tensor $T^{\mu\nu}$: Has (4+) Tensor Invariants (though not all independent)

a) $T^\alpha{}_\alpha$ = Trace = Sum of EigenValues for (1,1)-Tensors (mixed)

b) $T^\alpha{}_{[\alpha}T^\beta{}_{\beta]}$ = Asymm Bi-Product → Inner Product

c) $T^\alpha{}_{[\alpha}T^\beta{}_\beta T^\gamma{}_{\gamma]}$ = Asymm Tri-Product → ?Name?

d) $T^\alpha{}_{[\alpha}T^\beta{}_\beta T^\gamma{}_\gamma T^\delta{}_{\delta]}$ = Asymm Quad-Product → 4D Determinant = Product of EigenValues for (1,1)-Tensors

eg. $T^\alpha{}_{[\alpha}T^\beta{}_{\beta]} = T^\alpha{}_\alpha T^\beta{}_\beta - T^\alpha{}_\beta T^\beta{}_\alpha = (T^\gamma{}_\gamma)^2 - T^\alpha{}_\beta T^\beta{}_\alpha\{1\} = (T^\gamma{}_\gamma)^2 - T^\alpha{}_\beta T^\beta{}_\alpha\{(¼)\eta_{\gamma\delta}\eta^{\gamma\delta}\}$

and, bending tensor rules slightly: $= (T^\gamma{}_\gamma)^2 - T^\alpha{}_\beta T^\beta{}_\alpha\{(¼)\eta_{\beta\delta}\eta^{\beta\delta}\} = (T^\gamma{}_\gamma)^2 - T^\alpha{}_\beta(\eta^{\beta\delta})T^\beta{}_\alpha(\eta_{\beta\delta})\{(¼)\} = (T^\gamma{}_\gamma)^2 - T^{\alpha\delta}T_{\delta\alpha}\{(¼)\}$

and, since linear combinations of invariants are invariant:

Examine just the $(T^{\alpha\delta}T_{\delta\alpha})$ part, which for symm|asymm is $(\pm)(T^{\alpha\delta}T_{\alpha\delta})$ ie. the InnerProduct Invariant

a): Trace$[T^{\mu\nu}] = Tr[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T_\mu{}^\mu = T_\nu{}^\nu = (T_0{}^0 + T_1{}^1 + T_2{}^2 + T_3{}^3) = (T^{00} - T^{11} - T^{22} - T^{33}) = (T)$

for anti-symmetric: = 0

b): InnerProduct $T_{\mu\nu}T^{\mu\nu} = T_{00}T^{00} + T_{i0}T^{i0} + T_{0j}T^{0j} + T_{ij}T^{ij} = (T^{00})^2 - \Sigma_i[T^{i0}]^2 - \Sigma_j[T^{0j}]^2 + \Sigma_{i,j}[T^{ij}]^2$

for symmetric | anti-symmetric: $= (T^{00})^2 - 2\Sigma_i[T^{i0}]^2 + \Sigma_{i,j}[T^{ij}]^2 = \Sigma_{\mu=\nu}[T^{\mu\nu}]^2 - 2\Sigma_i[T^{i0}]^2 + 2\Sigma_{i>j}[T^{ij}]^2$

c): Antisymmetric Triple Product $T^\alpha{}_{[\alpha}T^\beta{}_\beta T^\gamma{}_{\gamma]} = Tr[T^{\mu\nu}]^3 - 3(Tr[T^{\mu\nu}])(T^\alpha{}_\beta T^\beta{}_\alpha) + T^\alpha{}_\beta T^\beta{}_\gamma T^\gamma{}_\alpha + T^\alpha{}_\gamma T^\beta{}_\alpha T^\gamma{}_\beta$

for anti-symmetric: = 0

d): Determinant $Det[T^{\mu\nu}] =?= -(1/2)\epsilon_{\alpha\beta\gamma\delta}T^{\alpha\beta}T^{\gamma\delta}$

If I got all the math right...

for anti-symmetric: $Det[T^{\mu\nu}] = Pfaffian[T^{\mu\nu}]^2$ (The Pfaffian is a special polynomial of the matrix entries)

Trace Tensor Invariant

$Tr[T^{\mu\nu}]=T_\nu{}^\nu=(T^{00}-T^{11}-T^{22}-T^{33})=T$

Set of 4 EigenValues$[T_\mu{}^\nu]$

Eigenvalues Tensor Invariants

4-Tensor

$$T^{\mu\nu} = \begin{bmatrix} T^{00}, T^{01}, T^{02}, T^{03} \\ T^{10}, T^{11}, T^{12}, T^{13} \\ T^{20}, T^{21}, T^{22}, T^{23} \\ T^{30}, T^{31}, T^{32}, T^{33} \end{bmatrix}$$

$T_\mu T^{\mu\nu}$

Inner Product Tensor Invariant

AsymmTri$[T^{\mu\nu}]$

Asymm Tri-Product Tensor Invariant

$Det[T^{\mu\nu}]$

Determinant Tensor Invariant

The lowered-indices form of a tensor just negativizes the (time-space) and (space-time) sections of the upper-indices tensor

Lowered 4-Tensor

$$T_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma}T^{\rho\sigma} = \begin{bmatrix} T_{00}, T_{01}, T_{02}, T_{03} \\ T_{10}, T_{11}, T_{12}, T_{13} \\ T_{20}, T_{21}, T_{22}, T_{23} \\ T_{30}, T_{31}, T_{32}, T_{33} \end{bmatrix} = \begin{bmatrix} +T^{00}, -T^{01}, -T^{02}, -T^{03} \\ -T^{10}, +T^{11}, +T^{12}, +T^{13} \\ -T^{20}, +T^{21}, +T^{22}, +T^{23} \\ -T^{30}, +T^{31}, +T^{32}, +T^{33} \end{bmatrix}$$

Invariants sometimes seen as

$I_1 = (1/1)Tr[(T^{\mu\nu})^1]$

$I_2 = (1/2)Tr[(T^{\mu\nu})^2]$

$I_3 = (1/3)Tr[(T^{\mu\nu})^3]$

$I_4 = (1/4)Tr[(T^{\mu\nu})^4]$

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$Det[T^\alpha{}_\alpha] = \Pi_k[\lambda_k]$; with $\{\lambda_k\}$ = Set of Eigenvalues
Characteristic Eqns: $Det[T^\alpha{}_\alpha - \lambda_k I_{(4)}]=0$

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$

$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0_o)^2$

= Lorentz Scalar Invariant

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Tensor Gymnastics

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Some Tensor Gymnastics:

Matrix **A** = Tensor $A^r{}_c$
with rows denoted by "r", columns by "c"

Example with dim=4: r,c={0..3}
Matrix **A** =
$[\ A^{r=0}{}_{c=0}\ \ A^{r=0}{}_{c=1}\ A^{r=0}{}_{c=2}\ A^{r=0}{}_{c=3}\ ]$
$[\ A^{r=1}{}_{c=0}\ \ A^{r=1}{}_{c=1}\ A^{r=1}{}_{c=2}\ A^{r=1}{}_{c=3}\ ]$
$[\ A^{r=2}{}_{c=0}\ \ A^{r=2}{}_{c=1}\ A^{r=2}{}_{c=2}\ A^{r=2}{}_{c=3}\ ]$
$[\ A^{r=3}{}_{c=0}\ \ A^{r=3}{}_{c=1}\ A^{r=3}{}_{c=2}\ A^{r=3}{}_{c=3}\ ]$

M = **A** × **B** = $A^d{}_e\, B^f{}_d = M^f{}_e$
,with the rows of **A** multiplied by the columns of **B**
due to the summation over index "d"

If we have sums over both indices:
$A^d{}_e\, B^e{}_d = M^e{}_e$ = Trace[**M**]
The sum over "d" gives the matrix multiplication and then the sum over "e" gives the Trace of the resulting matrix M

$A^d{}_e\, A^e{}_d = (\mathbf{A} \times \mathbf{A})^e{}_e = (N)^e{}_e$ = Trace[**N**] = Trace[$\mathbf{A}^2$] = Tr[$\mathbf{A}^2$]
$A^d{}_e\, A^e{}_d = (\eta_e{}^f A^d{}_f) A^e{}_d = \eta_e{}^f (A^d{}_f A^e{}_d) = \eta_e{}^f (N^e{}_f) = \delta_e{}^f (N^e{}_f)$ = Tr[**N**] = Tr[$\mathbf{A}^2$]

$A^d{}_{[d}\, A^e{}_{e]} = A^d{}_d\, A^e{}_e - A^d{}_e\, A^e{}_d$ = (Tr[**A**])^2 – Tr[$\mathbf{A}^2$]
,with brackets [..] around the indices indicating anti-symmetric product

The Trace formula's are independent of tensor dimension.

$A^a{}_a$ = Tr[**A**]

$A^a{}_{[a}\, A^b{}_{b]} = A^a{}_a\, A^b{}_b - A^a{}_b\, A^b{}_a$ = (Tr[**A**])^2 - Tr[$\mathbf{A}^2$]

$A^a{}_{[a}\, A^b{}_b\, A^c{}_{c]}$
$= +A^a{}_a A^b{}_b A^c{}_c - A^a{}_a A^b{}_c A^c{}_b + A^a{}_b A^b{}_c A^c{}_a - A^a{}_b A^b{}_a A^c{}_c + A^a{}_c A^b{}_a A^c{}_b - A^a{}_c A^b{}_b A^c{}_a$
$= +(A^a{}_a A^b{}_b A^c{}_c) - (A^a{}_a A^b{}_c A^c{}_b + A^a{}_b A^b{}_a A^c{}_c + A^a{}_c A^b{}_b A^c{}_a) + (A^a{}_b A^b{}_c A^c{}_a + A^a{}_c A^b{}_a A^c{}_b)$
$= +(A^a{}_a A^b{}_b A^c{}_c) - (A^a{}_a A^b{}_c A^c{}_b + A^c{}_c A^a{}_b A^b{}_a + A^b{}_b A^a{}_c A^c{}_a) + (A^a{}_b A^b{}_c A^c{}_a + A^a{}_c A^c{}_b A^b{}_a)$
= +(Tr[**A**])^3 - 3*(Tr[**A**])(Tr[$\mathbf{A}^2$]) + 2*(Tr[$\mathbf{A}^3$])

$A^a{}_{[a}\, A^b{}_b\, A^c{}_c\, A^d{}_{d]}$ =
$+A^a{}_a A^b{}_b A^c{}_c A^d{}_d - A^a{}_a A^b{}_b A^c{}_d A^d{}_c - A^a{}_a A^b{}_c A^c{}_b A^d{}_d + A^a{}_a A^b{}_c A^c{}_d A^d{}_b + A^a{}_a A^b{}_d A^c{}_b A^d{}_c - A^a{}_a A^b{}_d A^c{}_c A^d{}_b$
$- A^a{}_b A^b{}_a A^c{}_c A^d{}_d + A^a{}_b A^b{}_a A^c{}_d A^d{}_c + A^a{}_b A^b{}_c A^c{}_a A^d{}_d - A^a{}_b A^b{}_c A^c{}_d A^d{}_a - A^a{}_b A^b{}_d A^c{}_a A^d{}_c + A^a{}_b A^b{}_d A^c{}_c A^d{}_a$
$+ A^a{}_c A^b{}_a A^c{}_b A^d{}_d - A^a{}_c A^b{}_a A^c{}_d A^d{}_b - A^a{}_c A^b{}_b A^c{}_a A^d{}_d + A^a{}_c A^b{}_b A^c{}_d A^d{}_a + A^a{}_c A^b{}_d A^c{}_a A^d{}_b - A^a{}_c A^b{}_d A^c{}_b A^d{}_a$
$- A^a{}_d A^b{}_a A^c{}_b A^d{}_c + A^a{}_d A^b{}_a A^c{}_c A^d{}_b + A^a{}_d A^b{}_b A^c{}_a A^d{}_c - A^a{}_d A^b{}_b A^c{}_c A^d{}_a - A^a{}_d A^b{}_c A^c{}_a A^d{}_b + A^a{}_d A^b{}_c A^c{}_b A^d{}_a$
=
$+A^a{}_a A^b{}_b A^c{}_c A^d{}_d$
$- A^a{}_a A^b{}_b A^c{}_d A^d{}_c - A^a{}_a A^b{}_c A^c{}_b A^d{}_d - A^a{}_a A^b{}_d A^c{}_c A^d{}_b - A^a{}_b A^b{}_a A^c{}_c A^d{}_d - A^a{}_c A^b{}_b A^c{}_a A^d{}_d - A^a{}_d A^b{}_b A^c{}_c A^d{}_a$
$+ A^a{}_a A^b{}_c A^c{}_d A^d{}_b + A^a{}_a A^b{}_d A^c{}_b A^d{}_c + A^a{}_b A^b{}_c A^c{}_a A^d{}_d + A^a{}_b A^b{}_d A^c{}_c A^d{}_a + A^a{}_c A^b{}_a A^c{}_b A^d{}_d + A^a{}_c A^b{}_b A^c{}_d A^d{}_a + A^a{}_d A^b{}_a A^c{}_c A^d{}_b + A^a{}_d A^b{}_b A^c{}_a A^d{}_c$
$+ A^a{}_b A^b{}_a A^c{}_d A^d{}_c + A^a{}_c A^b{}_d A^c{}_a A^d{}_b + A^a{}_d A^b{}_c A^c{}_b A^d{}_a$
$- A^a{}_b A^b{}_c A^c{}_d A^d{}_a - A^a{}_b A^b{}_d A^c{}_a A^d{}_c - A^a{}_c A^b{}_a A^c{}_d A^d{}_b - A^a{}_c A^b{}_d A^c{}_b A^d{}_a - A^a{}_d A^b{}_a A^c{}_b A^d{}_c - A^a{}_d A^b{}_c A^c{}_a A^d{}_b$
=
+(Tr[**A**])^4
-6*(Tr[**A**])^2(Tr[$\mathbf{A}^2$])
+8*(Tr[**A**])(Tr[$\mathbf{A}^3$])
+3*(Tr[$\mathbf{A}^2$])^2
-6*(Tr[$\mathbf{A}^4$])
=
+(Tr[**A**])^4 -6*(Tr[**A**])^2(Tr[$\mathbf{A}^2$]) +8*(Tr[**A**])(Tr[$\mathbf{A}^3$]) +3*(Tr[$\mathbf{A}^2$])^2 -6*(Tr[$\mathbf{A}^4$])

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Characteristic Eqns: Det[$T^\alpha{}_\alpha - \lambda_k I_{(4)}$]=0

Trace[$T^{\mu\nu}$] = $\eta_{\mu\nu} T^{\mu\nu} = T^\mu{}_\mu$ = T
V·V = $V^\mu \eta_{\mu\nu} V^\nu$ = [$(v^0)^2$ - **v·v**] = $(v^0{}_o)^2$
= Lorentz Scalar Invariant

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Cayley-Hamilton Theorem

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General Cayley-Hamilton Theorem

$A^d + c_{d-1}A^{d-1} + \ldots + c_0 A^0 = 0_{(d)}$, with A = square matrix, d = dimension, A^0 = Identity(d) = $I_{(d)}$

Characteristic Polynomial: $p(\lambda) = \mathrm{Det}[A - \lambda I_{(d)}]$

The following are the Principle Tensor Invariants for dimensions 1..4

dim = 1: $A^1 + c_0 A^0 = 0$: $A - I_1 I_{(1)} = 0$

$I_1 = \mathrm{tr}[A] = \mathrm{Det}_{1D}[A] = \lambda_1$

dim = 2: $A^2 + c_1 A^1 + c_0 A^0 = 0$: $A^2 - I_1 A^1 + I_2 I_{(2)} = 0$

$I_1 = \mathrm{tr}[A] = \Sigma[\text{Eigenvalues}] = \lambda_1 + \lambda_2$

$I_2 = (\mathrm{tr}[A]^2 - \mathrm{tr}[A^2])/2 = \mathrm{Det}_{2D}[A] = \Pi[\text{Eigenvalues}] = \lambda_1\lambda_2$

dim = 3: $A^3 + c_2 A^2 + c_1 A^1 + c_0 A^0 = 0$: $A^3 - I_1 A^2 + I_2 A^1 - I_3 I_{(3)} = 0$

$I_1 = \mathrm{tr}[A] = \Sigma[\text{Eigenvalues}] = \lambda_1 + \lambda_2 + \lambda_3$

$I_2 = (\mathrm{tr}[A]^2 - \mathrm{tr}[A^2])/2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3$

$I_3 = [(\mathrm{tr}\,A)^3 - 3\,\mathrm{tr}(A^2)(\mathrm{tr}\,A) + 2\,\mathrm{tr}(A^3)]/6 = \mathrm{Det}_{3D}[A] = \Pi[\text{Eigenvalues}] = \lambda_1\lambda_2\lambda_3$

dim = 4: $A^4 + c_3 A^3 + c_2 A^2 + c_1 A^1 + c_0 A^0 = 0$: $A^4 - I_1 A^3 + I_2 A^2 - I_3 A^1 + I_4 I_{(4)} = 0$

$I_1 = \mathrm{tr}[A] = \Sigma[\text{Eigenvalues}] = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$

$I_2 = (\mathrm{tr}[A]^2 - \mathrm{tr}[A^2])/2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4$

$I_3 = [(\mathrm{tr}\,A)^3 - 3\,\mathrm{tr}(A^2)(\mathrm{tr}\,A) + 2\,\mathrm{tr}(A^3)]/6 = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4$

$I_4 = ((\mathrm{tr}\,A)^4 - 6\,\mathrm{tr}(A^2)(\mathrm{tr}\,A)^2 + 3(\mathrm{tr}(A^2))^2 + 8\,\mathrm{tr}(A^3)\,\mathrm{tr}\,A - 6\,\mathrm{tr}(A^4))/24 = \mathrm{Det}_{4D}[A] = \Pi[\text{Eigenvalues}] = \lambda_1\lambda_2\lambda_3\lambda_4$

4D Invariants

$I_0 = \Sigma[\text{Unique Eigenvalue Naughts}] = 1$ (1)

$I_1 = \Sigma[\text{Unique Eigenvalue Singles}] = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$ (4)

$I_2 = \Sigma[\text{Unique Eigenvalue Doubles}] = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4$ (6)

$I_3 = \Sigma[\text{Unique Eigenvalue Triples}] = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4$ (4)

$I_4 = \Sigma[\text{Unique Eigenvalue Quadruples}] = \lambda_1\lambda_2\lambda_3\lambda_4$ (1)

Each dimension gives the number of elements from it's row in Pascal's Triangle :)

SRQM Study: SR 4-Tensors

SR Tensor Invariants

Cayley-Hamilton Theorem

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

General Cayley-Hamilton Theorem $A^d + c_{d-1}A^{d-1} + \ldots + c_0 A^0 = 0_{(d)}$, with A = square matrix, d = dimension, A^0 = Identity(d) = $I_{(d)}$ $I_0 A^4 - I_1 A^3 + I_2 A^2 - I_3 A^1 + I_4 A^0 = 0$: for 4D Characteristic Polynomial: $p(\lambda) = \mathrm{Det}[A - \lambda I_{(d)}]$ **Tensor Invariants I_n**	**Dim = 1** A=[a] $= A^j{}_k$: j,k={1}	**Dim = 2** A=[a b] [c d] $= A^j{}_k$: j,k={1,2}	**Dim = 3** **Euclidean 3-Space** A=[a b c] [d e f] [g h i] $= A^j{}_k$: j,k={1,2,3}	**Dim = 4** **Minkowski SpaceTime** A=[a b c d] [e f g h] [i j k l] [m n o p] $= A^\mu{}_\nu$: μ,v={0,1,2,3}
$I_0 = 1/0! = 1$	(1) = 1	(1) = 1	(1) = 1	(1) = 1
$I_1 = \mathrm{tr}[A]/1!$ $= A^\alpha{}_\alpha$ = Σ[Unique Eigenvalue Singles]	(1) $= \lambda_1$ = (a) = Σ[Eigenvalues] $= \mathrm{Det}_{1D}[A]$ = Π[Eigenvalues]	(2) $= \lambda_1 + \lambda_2$ = (a + d) = Σ[Eigenvalues]	(3) $= \lambda_1 + \lambda_2 + \lambda_3$ = (a + e + i) = Σ[Eigenvalues]	(4) $= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$ = (a + f + k + p) = Σ[Eigenvalues]
$I_2 = (\mathrm{tr}[A]^2 - \mathrm{tr}[A^2])/2!$ $= A^\alpha{}_{[\alpha} A^\beta{}_{\beta]} / 2$ = Σ[Unique Eigenvalue Doubles]	=0	(1) $= \lambda_1\lambda_2$ = (ad - bc) $= \mathrm{Det}_{2D}[A]$ = Π[Eigenvalues]	(3) $= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3$ = (ae - bd)+(ai - cg)+(ei - fh)	(6) $= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4$ = (af - be) + (ak - ci) + (ap - dm) +(fk - gj) + (fp - hn) + (kp - lo)
$I_3 = [(\mathrm{tr}\,A)^3 - 3\,\mathrm{tr}(A^2)(\mathrm{tr}\,A) + 2\,\mathrm{tr}(A^3)]/3!$ $= A^\alpha{}_{[\alpha} A^\beta{}_\beta A^\gamma{}_{\gamma]} / 6$ = Σ[Unique Eigenvalue Triples]	=0	=0	(1) $= \lambda_1\lambda_2\lambda_3$ = a(ei-fh)-b(di-fg)+c(dh-eg) $= \mathrm{Det}_{3D}[A]$ = Π[Eigenvalues]	(4) $= \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4$ = ...
$I_4 = ((\mathrm{tr}\,A)^4 - 6\,\mathrm{tr}(A^2)(\mathrm{tr}\,A)^2 + 3(\mathrm{tr}(A^2))^2 + 8\,\mathrm{tr}(A^3)\,\mathrm{tr}\,A - 6\,\mathrm{tr}(A^4))/4!$ $= A^\alpha{}_{[\alpha} A^\beta{}_\beta A^\gamma{}_\gamma A^\delta{}_{\delta]} / 24$ = Σ[Unique Eigenvalue Quadruples]	=0	=0	=0	(1) $= \lambda_1\lambda_2\lambda_3\lambda_4$ =a(f(kp-lo)) + ... $= \mathrm{Det}_{4D}[A]$ = Π[Eigenvalues]

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: SR 4-Tensors
SR Tensor Invariants for Faraday EM Tensor

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

The Faraday EM Tensor $F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha = \partial \wedge \mathbf{A}$ is an anti-symmetric tensor that contains the Electric and Magnetic Fields, defined by the Exterior "Wedge" Product (^).
The 3-electric components ($\mathbf{e} = e^j = cF^{0j}$) are in the temporal-spatial sections.
The 3-magnetic components ($\mathbf{b} = b^k = -(1/2)\varepsilon_{ij}{}^{k} F^{ij}$) are in the only-spatial section.

(2,0)-Tensor = 4-Tensor $T^{\mu\nu}$: Has (4+) Tensor Invariants (though not all independent)
a) $T^\alpha{}_\alpha$ = Trace = Sum of EigenValues for (1,1)-Tensors (mixed)
b) $T^\alpha{}_{[\alpha}T^\beta{}_{\beta]}$ = Asymm Bi-Product → Inner Product
c) $T^\alpha{}_{[\alpha}T^\beta{}_\beta T^\gamma{}_{\gamma]}$ = Asymm Tri-Product → ?Name?
d) $T^\alpha{}_{[\alpha}T^\beta{}_\beta T^\gamma{}_\gamma T^\delta{}_{\delta]}$ = Asymm Quad-Product → 4D Determinant = Product of EigenValues for (1,1)-Tensors

a): Faraday Trace[$F^{\mu\nu}$] = $F_\nu{}^\nu = (F^{00}-F^{11}-F^{22}-F^{33}) = (0-0-0-0) = 0$
b): Faraday Inner Product $F_{\mu\nu}F^{\mu\nu} = \Sigma_{\mu=\nu}[F^{\mu\nu}]^2 - 2\Sigma_i[F^{i0}]^2 + 2\Sigma_{i>j}[F^{ij}]^2 = (0) - 2(\mathbf{e}\cdot\mathbf{e}/c^2) + 2(\mathbf{b}\cdot\mathbf{b}) = 2\{(\mathbf{b}\cdot\mathbf{b})-(\mathbf{e}\cdot\mathbf{e}/c^2)\}$
c): Faraday AsymmTri[$F^{\mu\nu}$] = $\mathrm{Tr}[F^{\mu\nu}]^3 - 3(\mathrm{Tr}[F^{\mu\nu}])(F^\alpha{}_\beta F^\beta{}_\alpha) + F^\alpha{}_\beta F^\beta{}_\gamma F^\gamma{}_\alpha + F^\alpha{}_\gamma F^\beta{}_\alpha F^\gamma{}_\beta = 0-3(0)+F^\alpha{}_\beta F^\beta{}_\gamma F^\gamma{}_\alpha + (-F^\alpha{}_\beta)(-F^\beta{}_\gamma)(-F^\gamma{}_\alpha) = 0$
d): Faraday Det[anti-symmetric $F^{\mu\nu}$] = $\mathrm{Pfaffian}[F^{\mu\nu}]^2 = [(-e^x/c)(-b^x) - (-e^y/c)(b^y) + (-e^z/c)(-b^z)]^2 = [(e^xb^x/c) + (e^yb^y/c) + (e^zb^z/c)]^2 = \{(\mathbf{e}\cdot\mathbf{b})/c\}^2$

Importantly, the Faraday EM Tensor has only (2) linearly-independent Lorentz invariants:
b) $2\{(\mathbf{b}\cdot\mathbf{b})-(\mathbf{e}\cdot\mathbf{e}/c^2)\}$
d) $\{(\mathbf{b}\cdot\mathbf{e})/c\}^2$
a) & c) give 0=0, and do not provide additional constraints

The 4-Gradient and 4-EMVectorPotential have (4) independent components each, for total of (8).
Subtract (2) due to the invariants which provide constraints to get a total of (6) independent components
= (6) independent components of a 4x4 anti-symmetric tensor
= (3) 3-electric **e** + (3) 3-magnetic **b** = (6) independent EM field components

Note: It is possible to have non-zero **e** and **b**, yet still have zeroes in the Tensor Invariants.
If **e** is orthogonal to **b**, then Det[$F^{\alpha\beta}$] = $\{(\mathbf{b}\cdot\mathbf{e})/c\}^2 = 0$.
If $(\mathbf{b}\cdot\mathbf{b})=(\mathbf{e}\cdot\mathbf{e}/c^2)$, then InnerProd[$F^{\alpha\beta}$] = $2\{(\mathbf{b}\cdot\mathbf{b})-(\mathbf{e}\cdot\mathbf{e}/c^2)\} = 0$.
These conditions lead to the properties of EM waves = photons = null 4-vectors, which have fields $|\mathbf{b}| = |\mathbf{e}|/c$ and **b** orthogonal to **e**, travelling at velocity c.

4-Gradient
$\partial = \partial^\mu = (\partial_t/c, -\nabla)$

$\mathrm{Tr}[F^{\mu\nu}] = F_\nu{}^\nu = 0$
Trace Tensor Invariant

$F_{\mu\nu}F^{\mu\nu} = 2\{(\mathbf{b}\cdot\mathbf{b})-(\mathbf{e}\cdot\mathbf{e}/c^2)\}$
Inner Product Tensor Invariant

$c^2 = 1/\varepsilon_o\mu_o$

AsymmTri[$F^{\mu\nu}$] = 0
Asymm Tri-Product Tensor Invariant

Det[$F^{\mu\nu}$] = $\{(\mathbf{e}\cdot\mathbf{b})/c\}^2$
Determinant Tensor Invariant

4-(EM)VectorPotential
$\mathbf{A} = A^\mu = (\varphi/c, \mathbf{a})$

Faraday EM Tensor
$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha = \partial \wedge \mathbf{A}$
$= -F^{\beta\alpha}$ (anti-symmetric)
→

$$\begin{bmatrix} F^{tt} & F^{tx} & F^{ty} & F^{tz} \\ F^{xt} & F^{xx} & F^{xy} & F^{xz} \\ F^{yt} & F^{yx} & F^{yy} & F^{yz} \\ F^{zt} & F^{zx} & F^{zy} & F^{zz} \end{bmatrix} =$$

$$\begin{bmatrix} 0 & \partial^0a^1-\partial^1a^0 & \partial^0a^2-\partial^2a^0 & \partial^0a^3-\partial^3a^0 \\ \partial^1a^0-\partial^0a^1 & 0 & \partial^1a^2-\partial^2a^1 & \partial^1a^3-\partial^3a^1 \\ \partial^2a^0-\partial^0a^2 & \partial^2a^1-\partial^1a^2 & 0 & \partial^2a^3-\partial^3a^2 \\ \partial^3a^0-\partial^0a^3 & \partial^3a^1-\partial^1a^3 & \partial^3a^2-\partial^2a^3 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & (\partial^ta^x+\nabla^x\varphi)/c & (\partial^ta^y+\nabla^y\varphi)/c & (\partial^ta^z+\nabla^z\varphi)/c \\ (-\nabla^x\varphi-\partial^ta^x/c) & 0 & -\nabla^xa^y+\nabla^ya^x & -\nabla^xa^z+\nabla^za^x \\ (-\nabla^y\varphi-\partial^ta^y/c) & -\nabla^ya^x+\nabla^xa^y & 0 & -\nabla^ya^z+\nabla^za^y \\ (-\nabla^z\varphi-\partial^ta^z/c) & -\nabla^za^x+\nabla^xa^z & -\nabla^za^y+\nabla^ya^z & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & -e^x/c & -e^y/c & -e^z/c \\ +e^x/c & 0 & -b^z & +b^y \\ +e^y/c & +b^z & 0 & -b^x \\ +e^z/c & -b^y & +b^x & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & , -e^j/c \\ +e^j/c, & -\varepsilon^{ij}{}_k b^k \end{bmatrix} =$$

$$\begin{bmatrix} 0 & , -\mathbf{e}/c \\ +\mathbf{e}^T/c, & -\nabla \wedge \mathbf{a} \end{bmatrix}$$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Fundamental EM Invariants:
$P = (1/2)F_{\mu\nu}F^{\mu\nu} = (-1/2)^*F_{\mu\nu}{}^*F^{\mu\nu} = \{(\mathbf{b}\cdot\mathbf{b})-(\mathbf{e}\cdot\mathbf{e}/c^2)\}$
$Q = (1/4)F_{\mu\nu}{}^*F^{\mu\nu} = (1/8)\varepsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta} = \{(\mathbf{e}\cdot\mathbf{b})/c\}$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: SR 4-Tensors
SR Tensor Invariants
for 4-AngularMomentum Tensor

4-Vector SRQM Interpretation of QM

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The 4-AngularMomentum Tensor $M^{\alpha\beta} = X^\alpha P^\beta - X^\beta P^\alpha = \mathbf{X \wedge P}$ is an anti-symmetric tensor
The 3-mass-moment components ($\mathbf{n} = n^j = M^{0j}/c$) are in the temporal-spatial sections.
The 3-angular-momentum components ($\mathbf{l} = l^k = +(1/2)\varepsilon_{ij}{}^k M^{ij}$) are in the only-spatial section.

(2,0)-Tensor = 4-Tensor $T^{\mu\nu}$: Has (4+) Tensor Invariants (though not all independent)
a) $T^\alpha{}_\alpha$ = Trace = Sum of EigenValues for (1,1)-Tensors (mixed)
b) $T^\alpha{}_{[\alpha}T^\beta{}_{\beta]}$ = Asymm Bi-Product → Inner Product
c) $T^\alpha{}_{[\alpha}T^\beta{}_\beta T^\gamma{}_{\gamma]}$ = Asymm Tri-Product → ?Name?
d) $T^\alpha{}_{[\alpha}T^\beta{}_\beta T^\gamma{}_\gamma T^\delta{}_{\delta]}$ = Asymm Quad-Product → 4D Determinant = Product of EigenValues for (1,1)-Tensors

a): 4-AngMom Trace$[M^{\mu\nu}] = M_\nu{}^\nu = (M^{00}-M^{11}-M^{22}-M^{33}) = (0\ -0\ -0\ -0) = 0$
b): 4-AngMom Inner Product $M_{\mu\nu}M^{\mu\nu} = \Sigma_{\mu=\nu}[M^{\mu\nu}]^2 - 2\Sigma_i[M^{i0}]^2 + 2\Sigma_{i>j}[M^{ij}]^2 = (0) - 2(c^2\mathbf{n}\cdot\mathbf{n}) + 2(\mathbf{l}\cdot\mathbf{l}) = 2\{(\mathbf{l}\cdot\mathbf{l})-(c^2\mathbf{n}\cdot\mathbf{n})\}$
c): 4-AngMom AsymmTri$[M^{\mu\nu}] = \mathrm{Tr}[M^{\mu\nu}]^3 - 3(\mathrm{Tr}[M^{\mu\nu}])(M^\alpha{}_\beta M^\beta{}_\alpha) + M^\alpha{}_\beta M^\beta{}_\gamma M^\gamma{}_\alpha + M^\alpha{}_\gamma M^\beta{}_\alpha M^\gamma{}_\beta = 0$
d): 4-AngMom Det[anti-symmetric $M^{\mu\nu}$] = Pfaffian$[M^{\mu\nu}]^2 = [(-cn^x)(+l^x) - (-cn^y)(-l^y) + (-cn^z)(+l^z)]^2 = [-(cn^xl^x) - (cn^yl^y) - (cn^zl^z)]^2 = \{c(\mathbf{n}\cdot\mathbf{l})\}^2$

Importantly, the 4-AngularMomentum Tensor has only (2) linearly-independent Lorentz Invariants:
b) $2\{(\mathbf{l}\cdot\mathbf{l})-(c^2\mathbf{n}\cdot\mathbf{n})\}$: see Wikipedia Laplace–Runge–Lenz_vector, sec. Casimir Invariants
d) $\{c(\mathbf{l}\cdot\mathbf{n})\}^2$
a) & c) give 0=0, and do not provide additional constraints

The 4-Position and 4-Momentum have (4) independent components each, for total of (8).
Subtract (2) due to the invariants which provide constraints to get a total of (6) independent components
= (6) independent components of a 4x4 anti-symmetric tensor
= (3) 3-mass-moment **n** + (3) 3-angular-momentum **l** = (6) independent 4-AngularMomentum components

3-massmoment $\mathbf{n} = \mathbf{x}m - t\mathbf{p} = m(\mathbf{x} - t\mathbf{u}) = m(\mathbf{r} - t\mathbf{u}) = m(\mathbf{r} - t(\boldsymbol{\omega} \times \mathbf{r}))$: Tangential velocity $\mathbf{u}_T = (\boldsymbol{\omega} \times \mathbf{r})$

$(-k/r)\mathbf{n} = -mk(\hat{\mathbf{r}} - t(\boldsymbol{\omega} \times \hat{\mathbf{r}})) = mkt(\boldsymbol{\omega} \times \hat{\mathbf{r}}) - mk\hat{\mathbf{r}} = t * d/dt(\mathbf{p}) \times \mathbf{L} - mk\hat{\mathbf{r}}$: $d/dt(\mathbf{p}) \times \mathbf{L} = mk(\boldsymbol{\omega} \times \hat{\mathbf{r}})$
n is related to the LRL = Laplace-Runge-Lenz 3-vector: $\mathbf{A} = \mathbf{p} \times \mathbf{L} - mk\hat{\mathbf{r}}$
which is another classical conserved vector. The invariance is shown here to be relativistic in origin.
Wikipedia article: Laplace-Runge-Lenz vector shows these as Casimir Invariants.
See Also: Relativistic Angular Momentum.

4-Position
$\mathbf{X} = X^\mu = (ct, \mathbf{x}) \in$ <Event>

$\mathrm{Tr}[M^{\mu\nu}] = M_\nu{}^\nu = 0$
Trace Tensor Invariant

$M_{\mu\nu}M^{\mu\nu} = 2\{(\mathbf{l}\cdot\mathbf{l})-(c^2\mathbf{n}\cdot\mathbf{n})\}$
Inner Product Tensor Invariant

AsymmTri$[M^{\mu\nu}] = 0$
Asymm Tri-Product Tensor Invariant

Det$[M^{\mu\nu}] = \{c(\mathbf{n}\cdot\mathbf{l})\}^2$
Determinant Tensor Invariant

4-Momentum
$\mathbf{P} = P^\mu = (mc, \mathbf{p}) = (E/c, \mathbf{p})$

4-AngularMomentum Tensor
$M^{\alpha\beta} = X^\alpha P^\beta - X^\beta P^\alpha = \mathbf{X \wedge P}$
→

$$\begin{bmatrix} M^{tt} & M^{tx} & M^{ty} & M^{tz} \\ M^{xt} & M^{xx} & M^{xy} & M^{xz} \\ M^{yt} & M^{yx} & M^{yy} & M^{yz} \\ M^{zt} & M^{zx} & M^{zy} & M^{zz} \end{bmatrix}$$
=
$$\begin{bmatrix} 0 & x^0p^1-x^1p^0 & x^0p^2-x^2p^0 & x^0p^3-x^3p^0 \\ x^1p^0-x^0p^1 & 0 & x^1p^2-x^2p^1 & x^1p^3-x^3p^1 \\ x^2p^0-x^0p^2 & x^2p^1-x^1p^2 & 0 & x^2p^3-x^3p^2 \\ x^3p^0-x^0p^3 & x^3p^1-x^1p^3 & x^3p^2-x^2p^3 & 0 \end{bmatrix}$$
=
$$\begin{bmatrix} 0 & ctp^x-xE/c & ctp^y-yE/c & ctp^z-zE/c \\ xE/c-ctp^x & 0 & xp^y-yp^x & xp^z-zp^x \\ yE/c-ctp^y & yp^x-xp^y & 0 & yp^z-zp^y \\ zE/c-ctp^z & zp^x-xp^z & zp^y-yp^z & 0 \end{bmatrix}$$
=
$$\begin{bmatrix} 0 & c(tp^x-xm) & c(tp^y-ym) & c(tp^z-zm) \\ c(xm-tp^x) & 0 & xp^y-yp^x & xp^z-zp^x \\ c(ym-tp^y) & yp^x-xp^y & 0 & yp^z-zp^y \\ c(zm-tp^z) & zp^x-xp^z & zp^y-yp^z & 0 \end{bmatrix}$$
=
$$\begin{bmatrix} 0 & -cn^x & -cn^y & -cn^z \\ +cn^x & 0 & +l^z & -l^y \\ +cn^y & -l^z & 0 & +l^x \\ +cn^z & +l^y & -l^x & 0 \end{bmatrix}$$
=
$$\begin{bmatrix} 0, & -cn^j \\ +cn^i, & \varepsilon^{ij}{}_k l^k \end{bmatrix}$$
=
$$\begin{bmatrix} 0, & -c\mathbf{n} \\ +c\mathbf{n}^T, & \mathbf{x} \wedge \mathbf{p} \end{bmatrix}$$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: SR 4-Tensors
SR Tensor Invariants for Minkowski Metric Tensor

4-Vector SRQM Interpretation of QM

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The Minkowksi Metric Tensor $\eta^{\mu\nu}$ is the tensor all SR 4-Vectors are measured by.

(2,0)-Tensor = 4-Tensor $T^{\mu\nu}$: Has (4+) Tensor Invariants (though not all independent)

a) $T^{\alpha}{}_{\alpha}$ = Trace = Sum of EigenValues for (1,1)-Tensors (mixed)
b) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta]}$ = Asymm Bi-Product → Inner Product
c) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma]}$ = Asymm Tri-Product → ?Name?
d) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma}T^{\delta}{}_{\delta]}$ = Asymm Quad-Product → 4D Determinant = Product of EigenValues for (1,1)-Tensors

a): Minkowksi Trace[$\eta^{\mu\nu}$] = 4
b): Minkowksi Inner Product $\eta_{\mu\nu}\eta^{\mu\nu}$ = 4
c): Minkowksi AsymmTri[$\eta^{\mu\nu}$] = 24 = 4!
d): Minkowksi Det[$\eta^{\mu\nu}$] = -1

a) $T^{\alpha}{}_{\alpha}$ = Tr[**A**] = 4
b) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta]} = (\mathrm{Tr}[\mathbf{A}])^2 - \mathrm{Tr}[\mathbf{A}^2] = 4^2 - 4 = 12$
c) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma]} = +(\mathrm{Tr}[\mathbf{A}])^3 - 3*(\mathrm{Tr}[\mathbf{A}])(\mathrm{Tr}[\mathbf{A}^2]) + 2*(\mathrm{Tr}[\mathbf{A}^3]) = 4^3 - 3*4*4 + 2*4 = 64 - 48 + 8 = 24$
d) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma}T^{\delta}{}_{\delta]} = +(\mathrm{Tr}[\mathbf{A}])^4 - 6*(\mathrm{Tr}[\mathbf{A}])^2(\mathrm{Tr}[\mathbf{A}^2]) + 8*(\mathrm{Tr}[\mathbf{A}])(\mathrm{Tr}[\mathbf{A}^3]) + 3*(\mathrm{Tr}[\mathbf{A}^2])^2 - 6*(\mathrm{Tr}[\mathbf{A}^4]) =$
$4^4 - 6*4^2*4 + 8*4*4 + 3*4^2 - 6*4 = 256 - 384 + 128 + 48 - 24 = 24$

$\Lambda^{\alpha}{}_{\mu}\Lambda^{\beta}{}_{\nu}\eta_{\alpha\beta} = \eta_{\mu\nu}$

Det(Exp[A])=Exp(Tr[A])

a) $T^{\alpha}{}_{\alpha}$ /1! = 4/1 = 4
b) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta]}$ /2! = 12/2 = 6
c) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma]}$ /3! = 24/6= 4
d) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma}T^{\delta}{}_{\delta]}$ /4! = 24/24 = 1

$\mathrm{Det}_{4D}(A) = ((\mathrm{tr}\,A)^4 - 6\,\mathrm{tr}(A^2)(\mathrm{tr}\,A)^2 + 3(\mathrm{tr}(A^2))^2 + 8\,\mathrm{tr}(A^3)\,\mathrm{tr}\,A - 6\,\mathrm{tr}(A^4))/24$

4

4-Gradient
$\partial = \partial^{\mu} = (\partial_t/c, -\nabla)$

Trace Tensor Invariant
$\mathrm{Tr}[\eta^{\mu\nu}] = (1) - (-1) - (-1) - (-1) = 4$
$\eta_{\mu\nu}\eta^{\mu\nu} = \eta^{\mu}{}_{\mu} = \delta^{\mu}{}_{\mu} = 1+1+1+1$

$\partial[\mathbf{R}] = \partial^{\mu}R^{\nu} = \eta^{\mu\nu}$
→
Diag[1,-1,-1,-1]
Diag[1,$-I_{(3)}$]
Diag[1,$-\delta^{jk}$]
=
[+1 0 0 0]
[0 -1 0 0]
[0 0 -1 0]
[0 0 0 -1]
{in Cartesian form}

$[\eta_{\mu\mu}] = 1/[\eta^{\mu\mu}] : \eta_{\mu}{}^{\nu} = \delta_{\mu}{}^{\nu}$
SR:Minkowski Metric
"Particle Physics" Convention

GR Trace Tensor Invariant
4D SpaceTime
In GR
$\mathrm{Tr}[g^{\mu\nu}] = g_{\mu\nu}g^{\mu\nu} = g^{\mu}{}_{\mu} = \delta^{\mu}{}_{\mu}$
$= 1+1+1+1 = 4$

EigenValues[$\eta^{\mu'}{}_{\nu}$]
=Set{1,1,1,1}
Eigenvalues Tensor Invariants

$\eta_{\mu\nu}\eta^{\mu\nu} = 4$
Inner Product Tensor Invariant

Signature[$\eta^{\mu\nu}$] = (+,-,-,-)
= {1,3,0} → (1-3)= -2
Signature Tensor Invariant

Det[$\eta^{\mu\nu}$] = -1
Det[$\eta^{\mu}{}_{\nu}$] = +1
Determinant Tensor Invariant

4-Position
$\mathbf{R} = R^{\mu} = (ct, \mathbf{r})$

AsymmTri[$\eta^{\mu\nu}$]=24
Asymm Tri-Product Tensor Invariant

EigenValues not defined for the standard Minkowski Metric Tensor since it is a type (2,0)-Tensor, all upper indices. However, they are defined for the mixed form (1,1)-Tensor
EigenValues are defined for the Lorentz Transforms since they are type (1,1)-Tensors, mixed indices

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Det}[T^{\alpha}{}_{\alpha}] = \Pi_k[\lambda_k]$; with $\{\lambda_k\}$ = EigenValues
Characteristic Eqns: $\mathrm{Det}[T^{\alpha}{}_{\alpha} - \lambda_k I_{(4)}]=0$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: SR 4-Tensors
SR Tensor Invariants
for Perfect Fluid Stress-Energy Tensor

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

The Perfect Fluid Stress-EnergyTensor $T^{\mu\nu}$ is the tensor of a non-torsional relativistic fluid.

(2,0)-Tensor = 4-Tensor $T^{\mu\nu}$: Has (4+) Tensor Invariants (though not all independent)

a) $T^{\alpha}{}_{\alpha}$ = Trace = Sum of EigenValues for (1,1)-Tensors (mixed)
b) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta]}$ = Asymm Bi-Product → Inner Product
c) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma]}$ = Asymm Tri-Product → ?Name?
d) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma}T^{\delta}{}_{\delta]}$ = Asymm Quad-Product → 4D Determinant = Product of EigenValues for (1,1)-Tensors

a): PerfectFluid Trace[$T^{\mu\nu}$] = $\rho_{eo}-3p_o$
b): PerfectFluid Inner Product $T_{\mu\nu}T^{\mu\nu} = (\rho_{eo})^2+3(p_o)^2$
c): PerfectFluid AsymmTri[$T^{\mu\nu}$] =
d): PerfectFluid Det[$T^{\mu\nu}$] = $\rho_{eo}(p_o)^3$

4-ForceDensity $\mathbf{F}_{density}$
$-\partial\cdot T^{\mu\nu} = F_{density}{}^{\mu}$

SR Conservation of StressEnergy $T^{\mu\nu}$ if $F_{density}{}^{\mu} = 0^{\mu}$

$\Lambda^{\alpha}{}_{\mu}\Lambda^{\beta}{}_{\nu}\eta_{\alpha\beta} = \eta_{\mu\nu}$

Det(Exp[A])=Exp(Tr[A])

$\mathrm{Det}_{4D}(A)=((\mathrm{tr}\,A)^4 - 6\,\mathrm{tr}(A^2)(\mathrm{tr}\,A)^2 + 3(\mathrm{tr}(A^2))^2 + 8\,\mathrm{tr}(A^3)\,\mathrm{tr}\,A - 6\,\mathrm{tr}(A^4))/24$

SR Perfect Fluid 4-Tensor

$T_{perfectfluid}{}^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} \rightarrow$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e=\rho_m c^2 & 0 & 0 & 0 \\ x & 0 & p & 0 & 0 \\ y & 0 & 0 & p & 0 \\ z & 0 & 0 & 0 & p \end{array}$$

$$\begin{matrix} \rho_e=\rho_m c^2 & 0^j \\ 0^i & p\delta^{ij} \end{matrix}$$

Units of [EnergyDensity=Pressure] **Symmetric**

EoS[$T^{\mu\nu}$]=w=p_o/ρ_{eo}

Tr[$T^{\mu\nu}$]=$\rho_{eo}-3p_o$

Trace Tensor Invariant

Tr[$T^{\mu\nu}$] = (ρ_{eo}) -(p_o) -(p_o) -(p_o) =
$\eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = \rho_{eo}-3p_o$

$T_{perfectfluid}{}^{\mu\nu}$
$\rightarrow_{\{MCRF\}}$
Diag[ρ_e,p,p,p]
Diag[ρ_e,p$I_{(3)}$]
Diag[ρ_e,pδ^{jk}]
=
$$\begin{bmatrix} \rho_e & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$
{in Cartesian form}

EigenValues[$T^{\mu}{}_{\nu}$]
=Set{ρ_{eo},$-p_o$,$-p_o$,$-p_o$}

Eigenvalues Tensor Invariants

$T_{\mu\nu}T^{\mu\nu} = (\rho_{eo})^2+3(p_o)^2$

Inner Product Tensor Invariant

Signature[$T^{\mu\nu}$] = (+,+,+,+)
= {4,0,0} = (4-0)= 4

Signature Tensor Invariant

Det[$T^{\mu\nu}$] = $\rho_{eo}(p_o)^3$
Det[$T^{\mu}{}_{\nu}$] = $-\rho_{eo}(p_o)^3$

Determinant Tensor Invariant

Equation of State
EoS[$T^{\mu\nu}$]=w=p_o/ρ_{eo}

Equation of State Tensor Invariant

AsymmTri[$T^{\mu\nu}$]=
not yet calc'd

Asymm Tri-Product Tensor Invariant

EigenValues not defined for the standard Perfect Fluid Tensor since it is a type (2,0)-Tensor, all upper indices. However, they are defined for the mixed form (1,1)-Tensor
EigenValues are defined for the Lorentz Transforms since they are type (1,1)-Tensors, mixed indices

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(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor V^{μ} = **V** = (v^0,**v**)
SR 4-CoVector:OneForm
(0,1)-Tensor V_{μ} = (v_0,-**v**)

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Det[$T^{\alpha}{}_{\alpha}$] = $\Pi_k[\lambda_k]$; with $\{\lambda_k\}$ = EigenValues
Characteristic Eqns: Det[$T^{\alpha}{}_{\alpha} - \lambda_k I_{(4)}$]=0

Trace[$T^{\mu\nu}$] = $\eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
V·V = $V^{\mu}\eta_{\mu\nu}V^{\nu}$ = [$(v^0)^2$ - **v·v**] = $(v^0{}_o)^2$
= Lorentz Scalar Invariant

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SR Perfect Fluid 4-Tensor

$T_{perfectfluid}^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} \rightarrow$

	t	x	y	z
t	$\rho_e=\rho_m c^2$	0	0	0
x	0	p	0	0
y	0	0	p	0
z	0	0	0	p

$\rho_e=\rho_m c^2$ 0^j; 0^i $p\delta^{ij}$

Units of [EnergyDensity=Pressure] **Symmetric**

EoS$[T^{\mu\nu}]=w=p_o/\rho_{eo}$

$Tr[T^{\mu\nu}]=\rho_{eo}-3p_o$

4-ForceDensity $\mathbf{F_{densityEM}}$
$-\partial\cdot T^{\mu\nu} = F_{densityEM}^{\mu}$
$= (\mathbf{j}\cdot\mathbf{e}/c, \rho\mathbf{e}+\mathbf{j}\times\mathbf{b})$
SR Conservation of StressEnergy $T^{\mu\nu}$
if $F_{density}^{\mu} = 0^{\mu}$

$T^{00} = ½(\varepsilon_o e^2+b^2/\mu_o) = ½(\varepsilon_o \mathbf{e}\cdot\mathbf{e}+\mathbf{b}\cdot\mathbf{b}/\mu_o)$
Note: this is positive-definite

Trace Tensor Invariant

$Tr[T^{\mu\nu}] = 0$
$= T^{00} - T^{11} - T^{22} - T^{33}$
$= T^{00} - 3T^{00}$(1 from δ^{ii}) + (2*T^{00} from $T^{xx}+T^{yy}+T^{zz}$)
$= T^{00} - 3T^{00} + 2*T^{00}$
$= 0$
=Sum of EigenValues

EigenValues$[T^{\mu\nu}]$ = Set of 4 values =
$\{ \pm½(\varepsilon_o e^2+b^2/\mu_o), \pm\sqrt{[¼(\varepsilon_o e^2-b^2/\mu_o)^2+(\varepsilon_o/\mu_o)(\mathbf{e}\cdot\mathbf{b})^2]} \}$

The Maxwell 4D EM Stress-EnergyTensor $T^{\mu\nu}$ is the tensor of an EM field.

Faraday EM Tensor

$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha = \partial \wedge \mathbf{A}$

	t	x	y	z
t	0	$-e^x/c$	$-e^y/c$	$-e^z/c$
x	$+e^x/c$	0	$-b^z$	$+b^y$
y	$+e^y/c$	$+b^z$	0	$-b^x$
z	$+e^z/c$	$-b^y$	$+b^x$	0

0 $-e^{0j}/c$; $+e^{i0}/c$ $-\varepsilon^{ij}{}_k b^k$

0 $-\mathbf{e}/c$; $+\mathbf{e}^T/c$ $-\nabla\wedge\mathbf{a}$

4-Tensor
Anti-symmetric

$Det[F^{\mu\nu}] = \{(\mathbf{e}\cdot\mathbf{b})/c\}^2$

$Tr[F^{\mu\nu}]=0$

$F_{\mu\nu}F^{\mu\nu}$
$=2\{(\mathbf{b}\cdot\mathbf{b})-(\mathbf{e}\cdot\mathbf{e}/c^2)\}$
$=2\{(\mathbf{b}\cdot\mathbf{b})-(\varepsilon_o\mu_o\mathbf{e}\cdot\mathbf{e})\}$

$Tr[T_{EM}^{\mu\nu}] = 0 = (\eta_{\mu\nu})[T_{EM}^{\mu\nu}]$
$=(\eta_{\mu\nu})(1/\mu_o)[F^{\mu\alpha}F^{\nu}{}_{\alpha}-(1/4)\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}]$
$=(1/\mu_o)[F^{\mu\alpha}(\eta_{\mu\nu})F^{\nu}{}_{\alpha}-(1/4)(\eta_{\mu\nu})\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}]$
$=(1/\mu_o)[F^{\mu\alpha}F_{\mu\alpha}-(1/4)(4)F_{\alpha\beta}F^{\alpha\beta}]$
$=(1/\mu_o)[F^{\mu\alpha}F_{\mu\alpha}- F_{\alpha\beta}F^{\alpha\beta}] = 0$

Eigenvalues Tensor Invariants

Maxwell 4D EM Stress-Energy Tensor

$T_{EM}^{\mu\nu} = -(1/\mu_o)[F^{\mu\alpha}F^{\nu}{}_{\alpha}-(1/4)\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}] \rightarrow_{\{No\ RestFrame,Light\text{-}Like,Null\}}$

	t	x	y	z
t	$½(\varepsilon_o e^2+b^2/\mu_o)$	s^x/c	s^y/c	s^z/c
x	s^x/c	$-\sigma^{xx}$	$-\sigma^{xy}$	$-\sigma^{xz}$
y	s^y/c	$-\sigma^{yx}$	$-\sigma^{yy}$	$-\sigma^{yz}$
z	s^z/c	$-\sigma^{zx}$	$-\sigma^{zy}$	$-\sigma^{zz}$

$½(\varepsilon_o e^2+b^2/\mu_o)$ s^{0j}/c; s^{i0}/c $-\sigma^{ij}$

4-Tensor in (+,-,-,-), neg of formula in (-,+,+,+)
Symmetric

$Tr[T^{\mu\nu}]=0$

$T_{MaxwellEM}^{\mu\nu} \rightarrow$

$½(\varepsilon_o e^2+b^2/\mu_o)$ s^{0j}/c; s^{i0}/c $-\sigma^{ij}$

w/ 3D Maxwell Stress Tensor

$\sigma^{ij} =$
$= \varepsilon_o e^i e^j+b^i b^j/\mu_o$
$-½(\varepsilon_o e^2+b^2/\mu_o)\delta^{ij}$
$= \varepsilon_o e^i e^j+b^i b^j/\mu_o$
$-T^{00}\delta^{ij}$

w/ 3D PoyntingVector

$T^{i0} = \mathbf{s} = (\mathbf{e}\times\mathbf{b}/\mu_o)$
Poynting Vector

$c^2=1/\varepsilon_o\mu_o$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

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(0,0)-Tensor S or S_o
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$Det[T^{\alpha}{}_{\alpha}] = \Pi_k[\lambda_k]$; with $\{\lambda_k\}$ = EigenValues
Characteristic Eqns: $Det[T^{\alpha}{}_{\alpha} - \lambda_k I_{(4)}]=0$

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

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Faraday EM Tensor

$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha = \partial \wedge \mathbf{A}$

	t	x	y	z
t	0	$-e^x/c$	$-e^y/c$	$-e^z/c$
x	$+e^x/c$	0	$-b^z$	$+b^y$
y	$+e^y/c$	$+b^z$	0	$-b^x$
z	$+e^z/c$	$-b^y$	$+b^x$	0

0	$-e^{0j}/c$
$+e^{i0}/c$	$-\varepsilon^{ij}{}_k b^k$

0	$-\mathbf{e}/c$
$+\mathbf{e}^T/c$	$-\nabla\wedge\mathbf{a}$

4-Tensor
Anti-symmetric

$\mathrm{Det}[F^{\mu\nu}] = \{(\mathbf{e}\cdot\mathbf{b})/c\}^2$

$\mathrm{Tr}[F^{\mu\nu}]=0$

$F_{\mu\nu}F^{\mu\nu}$
$=2\{(\mathbf{b}\cdot\mathbf{b})-(\mathbf{e}\cdot\mathbf{e}/c^2)\}$
$=2\{(\mathbf{b}\cdot\mathbf{b})-(\varepsilon_o\mu_o\mathbf{e}\cdot\mathbf{e})\}$

$T^{00} = \frac{1}{2}(\varepsilon_o e^2+b^2/\mu_o) = \frac{1}{2}(\varepsilon_o\mathbf{e}\cdot\mathbf{e}+\mathbf{b}\cdot\mathbf{b}/\mu_o)$
Note this is positive-definite

Maxwell 4D EM Stress-Energy Tensor

$T_{EM}{}^{\mu\nu} = -(1/\mu_o)[F^{\mu\alpha}F^\nu{}_\alpha-(1/4)\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}] \rightarrow_{\{\text{No RestFrame,Light-Like,Null}\}}$
$=(-1/\mu_o)F^{\mu\alpha}F^\nu{}_\alpha+\eta^{\mu\nu}(1/2\mu_o)\{(\mathbf{b}\cdot\mathbf{b})-(\varepsilon_o\mu_o\mathbf{e}\cdot\mathbf{e})\}$
$=(-1/\mu_o)F^{\mu\alpha}F^\nu{}_\alpha+\eta^{\mu\nu}(1/2)\{(\mathbf{b}\cdot\mathbf{b})/\mu_o-\varepsilon_o(\mathbf{e}\cdot\mathbf{e})\}$
$=(-1/\mu_o)\{F^{\mu0}F^\nu{}_0+F^{\mu1}F^\nu{}_1+F^{\mu2}F^\nu{}_2+F^{\mu3}F^\nu{}_3\}+\eta^{\mu\nu}(1/2)\{(\mathbf{b}\cdot\mathbf{b})/\mu_o-\varepsilon_o(\mathbf{e}\cdot\mathbf{e})\}$

00th comp$=(-1/\mu_o)\{F^{00}F^0{}_0+F^{01}F^0{}_1+F^{02}F^0{}_2+F^{03}F^0{}_3\}+\eta^{00}(1/2)\{(\mathbf{b}\cdot\mathbf{b})/\mu_o-\varepsilon_o(\mathbf{e}\cdot\mathbf{e})\}$
$=(-1/\mu_o)\{0+(-\mathbf{e}\cdot\mathbf{e}/c^2)\}+(1)(1/2)\{(\mathbf{b}\cdot\mathbf{b})/\mu_o-\varepsilon_o(\mathbf{e}\cdot\mathbf{e})\}$
$=\{(\varepsilon_o\mathbf{e}\cdot\mathbf{e})\}+(1/2)\{(\mathbf{b}\cdot\mathbf{b})/\mu_o-\varepsilon_o(\mathbf{e}\cdot\mathbf{e})\}$
$=+(1/2)\{\varepsilon_o(\mathbf{e}\cdot\mathbf{e})+(\mathbf{b}\cdot\mathbf{b})/\mu_o\}$

01th comp$=(-1/\mu_o)\{F^{00}F^1{}_0+F^{01}F^1{}_1+F^{02}F^1{}_2+F^{03}F^1{}_3\}+\eta^{01}(1/2)\{(\mathbf{b}\cdot\mathbf{b})/\mu_o-\varepsilon_o(\mathbf{e}\cdot\mathbf{e})\}$
$=(-1/c\mu_o)\{0+0-e^yb^z+e^zb^y\}+(0)$
$=(1/c\mu_o)\{+e^yb^z-e^zb^y\}$
$\rightarrow 0k^{th}$ comp $= (1/c)\{\varepsilon_{ij}{}^k e^i b^j/\mu_o\} \rightarrow (1/c)\{\mathbf{e}\times\mathbf{b}/\mu_o\} = \mathbf{s}/c$

	t	x	y	z
t	$\frac{1}{2}(\varepsilon_o e^2+b^2/\mu_o)$	s^x/c	s^y/c	s^z/c
x	s^x/c	$-\sigma^{xx}$	$-\sigma^{xy}$	$-\sigma^{xz}$
y	s^y/c	$-\sigma^{yx}$	$-\sigma^{yy}$	$-\sigma^{yz}$
z	s^z/c	$-\sigma^{zx}$	$-\sigma^{zy}$	$-\sigma^{zz}$

$\frac{1}{2}(\varepsilon_o e^2+b^2/\mu_o)$	s^{0j}/c
s^{i0}/c	$-\sigma^{ij}$

4-Tensor in (+,-,-,-), neg of formula in (-,+,+,+)
Symmetric

$\mathrm{Tr}[T^{\mu\nu}]=0$

$c^2=1/\varepsilon_o\mu_o$

Trace Tensor Invariant

$\mathrm{Tr}[T^{\mu\nu}] = 0$
$= T^{00} - T^{11} - T^{22} - T^{33}$
$= T^{00} - 3T^{00}$(1 from δ^{ii}) + (2*T^{00} from $T^{xx}+T^{yy}+T^{zz}$)
$= T^{00} - 3T^{00} + 2*T^{00}$
$= 0$
=Sum of EigenValues

$T_{MaxwellEM}{}^{\mu\nu}$
$\rightarrow$

$\frac{1}{2}(\varepsilon_o e^2+b^2/\mu_o)$	s^{0j}/c
s^{i0}/c	$-\sigma^{ij}$

w/ 3D Maxwell Stress Tensor

$\sigma^{ij} =$
$= \varepsilon_o e^ie^j+b^ib^j/\mu_o$
$-\frac{1}{2}(\varepsilon_o e^2+b^2/\mu_o)\delta^{ij}$
$= \varepsilon_o e^ie^j+b^ib^j/\mu_o$
$-T^{00}\delta^{ij}$

w/ 3D PoyntingVector

$T^{i0} = \mathbf{s} = (\mathbf{e}\times\mathbf{b}/\mu_o)$
Poynting Vector

$(|\mathbf{e}\times\mathbf{b}|)^2 + (\mathbf{e}\cdot\mathbf{b})^2 = e^2b^2$
$(|\mathbf{e}||\mathbf{b}|\sin)^2 + (|\mathbf{e}||\mathbf{b}|\cos)^2 = (|\mathbf{e}||\mathbf{b}|)^2$
$\sin^2 + \cos^2 = 1$

$\gamma = 1/\sqrt{[1-(\mathbf{u}\cdot\mathbf{u})/c^2]}$
$\gamma' = d\gamma/dt = \gamma^3(\mathbf{u}\cdot\mathbf{a})/c^2$
$(\gamma\beta)^2 = \gamma^2 - 1$
$\boldsymbol{\beta} = \mathbf{u}/c$

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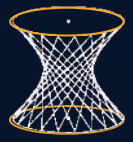
Characteristic Eqns: $\mathrm{Det}[T^\alpha{}_\alpha - \lambda_k I_{(4)}]=0$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics



A Tensor Study of Physical 4-Vectors

SRQM Study: SR 4-Tensors

Properties of Lorentz Transform Tensors Relation to 4D Kronecker Delta

4-Vector SRQM Interpretation of QM

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The Lorentz Transform Tensor { $\Lambda^{\mu'}{}_{\nu} = \partial x^{\mu'}/\partial x^{\nu} = \partial_{\nu}[X^{\mu'}]$ } is the tensor all SR 4-Vectors must transform by.

SR:Lorentz Transform
$\partial_{\nu}[R^{\mu'}] = \partial R^{\mu'}/\partial R^{\nu} = \Lambda^{\mu'}{}_{\nu}$

4D Kronecker Delta
$\Lambda^{\mu}{}_{\alpha}(\Lambda^{-1})^{\alpha}{}_{\nu} = \eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$

Inner Product Tensor Invariant
$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$

Rotation(0) = Identity = Boost(0)

Lorentz SR Rotation Tensor $\Lambda^{\mu}{}_{\nu}\rightarrow R^{\mu}{}_{\nu}$ =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[\theta] & -\sin[\theta] & 0 \\ 0 & \sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Lorentz SR Identity Tensor $\Lambda^{\mu}{}_{\nu}\rightarrow\eta^{\mu}{}_{\nu}$ $=R^{\mu}{}_{\nu}[0] = B^{\mu}{}_{\nu}[0] = \delta^{\mu}{}_{\nu} =$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

= Minkowski Delta

Lorentz SR Boost Tensor $\Lambda^{\mu}{}_{\nu}\rightarrow B^{\mu}{}_{\nu}$ =

$$\begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

↕ Double Minkowski-Metric Indexed-Altered Tensors (1 raised, 1 lowered) ↕

Lorentz SR Rotation Tensor $\Lambda_{\mu}{}^{\nu}\rightarrow R_{\mu}{}^{\nu}$ =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[\theta] & -\sin[\theta] & 0 \\ 0 & \sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Lorentz SR Identity Tensor $\Lambda_{\mu}{}^{\nu}\rightarrow\eta_{\mu}{}^{\nu}$ $=R_{\mu}{}^{\nu}[0] = B_{\mu}{}^{\nu}[0] = \delta_{\mu}{}^{\nu} =$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

= Minkowski Delta

Lorentz SR Boost Tensor $\Lambda_{\mu}{}^{\nu}\rightarrow B_{\mu}{}^{\nu}$ =

$$\begin{bmatrix} \gamma & +\beta\gamma & 0 & 0 \\ +\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The properties of the Lorentz Transforms give interesting relations to the 4D Kronecker Delta.
When taking an inner product of a Lorentz Transform, each index combo sums to either 0 or 1.
Different index combos give 0, same index combos give 1, the usual Delta rule {=1 for μ=ν, else =0}.

For Rotations:
0th with 0th : (1)(1)+(0)(0)+(0)(0)+(0)(0) = 1
0th with 1st : (1)(0)+(0)(cos[θ])+(0)(-sin[θ])+(1)(0) = 0
0th with 3rd : (1)(0)+(0)(0)+(0)(0)+(0)(1) = 0
...
1st with 1st : (0)(0)+(cos[θ])(cos[θ])+(-sin[θ])(-sin[θ])+(0)(0) = $\cos^2+\sin^2$ = 1
1st with 2nd : (0)(0)+(cos[θ])(sin[θ])+(-sin[θ])(cos[θ])+(0)(0) = cos*sin-cos*sin = 0
etc.

For Boosts:
0th with 0th : (γ)(γ)+(-γβ)(γβ)+(0)(0)+(0)(0) = $\gamma^2[1-\beta^2]$ = 1
0th with 1st : (γ)(+γβ)+(-γβ)(γ)+(0)(0)+(0)(0) = $\gamma^2[\beta-\beta]$ = 0
0th with 3rd : (γ)(0)+(-γβ)(0)+(0)(0)+(0)(1) = 0
...
2nd with 3rd : (0)(0)+(0)(0)+(1)(0)+(0)(1) = 0
etc.

The fact that each row gives a single (1) leads to the overall inner product $\Lambda_{\mu\nu}\Lambda^{\mu\nu} = 4 = \Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$ for 4D Lorentz Transforms (1+1+1+1 = 4).

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor V^{μ} = **V** = (v^0,**v**)
SR 4-CoVector:OneForm
(0,1)-Tensor V_{μ} = (v_0,-**v**)

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Det[$T^{\alpha}{}_{\alpha}$] = $\Pi_k[\lambda_k]$; with {λ_k} = EigenValues
Characteristic Eqns: Det[$T^{\alpha}{}_{\alpha} - \lambda_k I_{(4)}$]=0

Trace[$T^{\mu\nu}$] = $\eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu}$ = T
V·V = $V^{\mu}\eta_{\mu\nu}V^{\nu}$ = [$(v^0)^2$ - **v·v**] = $(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: SR 4-Tensors
SR EigenValues of Lorentz Transform Tensors

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

EigenValue Equation: $\det[\, R(\theta) - \lambda I_{(4)} \,] = 0$

$$\det \begin{bmatrix} 1-\lambda, & 0, & 0, & 0 \\ 0, & \cos-\lambda, & -\sin, & 0 \\ 0, & \sin, & \cos-\lambda, & 0 \\ 0, & 0, & 0, & 1-\lambda \end{bmatrix} = 0$$

$(1-\lambda)^2[(\cos\theta-\lambda)^2+\sin^2\theta] = 0$

$(1-\lambda)^2(\lambda^2-2\lambda\cos\theta+1) = 0$, from ($\sin^2+\cos^2=1$)

$(1-\lambda)^2(\lambda^2-\lambda[e^{+i\theta}+e^{-i\theta}]+1) = 0$, from ($\cos\theta = [e^{+i\theta}+e^{-i\theta}]/2$)

if $\lambda=1$ then 1st part $(1-\lambda)^2 = 0$, two 1's since $(1-\lambda)^2$

if $\lambda= e^{+i\theta}$ then 2nd part
$(\lambda^2-\lambda[e^{+i\theta}+e^{-i\theta}]+1)$
$=(e^{+i\theta 2}-e^{+i\theta}[e^{+i\theta}+e^{-i\theta}]+1)=(e^{+2i\theta}-e^{+2i\theta}-e^{+i\theta}e^{-i\theta}+1)=(0-1+1)$
$= 0$

if $\lambda= e^{-i\theta}$ then the 2nd part
$(\lambda^2-\lambda[e^{+i\theta}+e^{-i\theta}]+1)$
$=(e^{-i\theta 2}-e^{-i\theta}[e^{+i\theta}+e^{-i\theta}]+1)=(e^{-2i\theta}-e^{-2i\theta}-e^{-i\theta}e^{+i\theta}+1)=(0-1+1)$
$= 0$

SR Rotation EigenValues = { 1 , 1 , $e^{+i\theta}$, $e^{-i\theta}$ }

	Rotation(0)	= Identity	= Boost(0)
Inner Product Tensor Invariant: $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$	Lorentz SR Rotation Tensor $\Lambda^{\mu'}{}_{\nu}\to R^{\mu'}{}_{\nu}$ = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[\theta] & -\sin[\theta] & 0 \\ 0 & \sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	Lorentz SR Identity Tensor $\Lambda^{\mu'}{}_{\nu}\to\eta^{\mu'}{}_{\nu}$ $=R^{\mu'}{}_{\nu}[0] = B^{\mu'}{}_{\nu}[0]$ $= \delta^{\mu'}{}_{\nu}$ = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ = Minkowski Delta	Lorentz SR Boost Tensor $\Lambda^{\mu'}{}_{\nu}\to B^{\mu'}{}_{\nu}$ = $\begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Asymm Tri-Product Tensor Invariant: AsymmTri[$\Lambda^{\mu'}{}_{\nu}$]=? Not yet calc...			
	EigenValues[$R^{\mu'}{}_{\nu}$] =Set{1,$e^{i\theta}$,$e^{-i\theta}$,1}	EigenValues[$\eta^{\mu'}{}_{\nu}$] =Set{1,1,1,1}	EigenValues[$B^{\mu'}{}_{\nu}$] =Set{e^{θ},$e^{-\theta}$,1,1}
Trace Tensor Invariant: Tr[Cont. $\Lambda^{\mu'}{}_{\nu}$]={0..4..Infinity} Depends on "rotation" amount	Sum of EigenValues[$R^{\mu'}{}_{\nu}$] =Tr[$R^{\mu'}{}_{\nu}$]=$R^{\mu'}{}_{\mu}$ =$1+e^{i\theta}+e^{-i\theta}+1$ =$2+2\cos[\theta]$ ={0..4}	Sum of EigenValues[$\eta^{\mu'}{}_{\nu}$] =Tr[$\eta^{\mu'}{}_{\nu}$]=$\eta^{\mu'}{}_{\mu}$ =1+1+1+1 =4 ={4}	Sum of EigenValues[$B^{\mu'}{}_{\nu}$] =Tr[$B^{\mu'}{}_{\nu}$]=$B^{\mu'}{}_{\mu}$ =$e^{\theta}+e^{-\theta}+1+1$ =$2+2\cosh[\theta]=2+2\gamma$ ={4..Infinity}
Determinant Tensor Invariant: Det[Proper $\Lambda^{\mu'}{}_{\nu}$]=+1 Proper Transform always +1	Product of EigenValues[$R^{\mu'}{}_{\nu}$] =Det[$R^{\mu'}{}_{\nu}$] =$1\cdot e^{i\theta}\cdot e^{-i\theta}\cdot 1$ = +1	Product of EigenValues[$\eta^{\mu'}{}_{\nu}$] =Det[$\eta^{\mu'}{}_{\nu}$] =$1\cdot 1\cdot 1\cdot 1$ = +1	Product of EigenValues[$B^{\mu'}{}_{\nu}$] =Det[$B^{\mu'}{}_{\nu}$] =$e^{\theta}\cdot e^{-\theta}\cdot 1\cdot 1$ = +1
	Proper	Proper	Proper

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$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study: SR 4-Tensors
SR Tensor Invariants for Continuous Lorentz Transform Tensors

4-Vector SRQM Interpretation of QM

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The Lorentz Transform Tensor { $\Lambda^{\mu'}{}_{\nu} = \partial x^{\mu'}/\partial x^{\nu} = \partial_{\nu}[X^{\mu'}]$ } is the tensor all SR 4-Vectors must transform by.

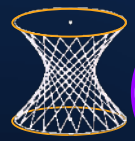
(2,0)-Tensor = 4-Tensor $T^{\mu\nu}$: Has (4+) Tensor Invariants (though not all independent)

a) $T^{\alpha}{}_{\alpha}$ = Trace = Sum of EigenValues for (1,1)-Tensors (mixed)
b) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta]}$ = Asymm Bi-Product → Inner Product
c) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma]}$ = Asymm Tri-Product → ?Name?
d) $T^{\alpha}{}_{[\alpha}T^{\beta}{}_{\beta}T^{\gamma}{}_{\gamma}T^{\delta}{}_{\delta]}$ = Asymm Quad-Product → 4D Determinant = Product of EigenValues for (1,1)-Tensors

a): Lorentz Trace[$\Lambda^{\mu\nu}$] = {0..4..Infinity} Lorentz Boost meets Rotation at Identity of 4
b): Lorentz Inner Product $\Lambda_{\mu\nu}\Lambda^{\mu\nu} = 4$ from $\{\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}\}$ and $\{\eta_{\mu\nu}\eta^{\mu\nu} = 4\}$
c): Lorentz AsymmTri[$\Lambda^{\mu\nu}$] =
d): Lorentz Det[$\Lambda^{\mu\nu}$] = +1 for Proper Transforms, Continuous Transforms Proper

An even more general version would be with a & b as arbitrary complex values:

could be 2 boosts, 2 rotations, or a boost:rotation combo



Tr[$\Lambda^{\mu}{}_{\nu}$]={-∞..+∞}
=Lorentz Transform Type

SR:Lorentz Transform
$\partial_{\nu}[R^{\mu'}] = \partial R^{\mu'}/\partial R^{\nu} = \Lambda^{\mu'}{}_{\nu}$
$\Lambda^{\mu}{}_{\nu} = (\Lambda^{-1})_{\nu}{}^{\mu} : \Lambda^{\mu}{}_{\alpha}(\Lambda^{-1})^{\alpha}{}_{\nu} = \eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$
$\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$
Det[$\Lambda^{\mu}{}_{\nu}$]=±1 $\Lambda_{\mu\nu}\Lambda^{\mu\nu}$=4

Inner Product Tensor Invariant

$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$

Asymm Tri-Product Tensor Invariant

AsymmTri[$\Lambda^{\mu'}{}_{\nu}$]=?
Not yet calc...

Trace Tensor Invariant

Tr[Cont. $\Lambda^{\mu'}{}_{\nu}$]={0..4..Infinity}
Depends on "rotation" amount

Determinant Tensor Invariant

Det[Proper $\Lambda^{\mu'}{}_{\nu}$]=+1
Proper Transform always +1

Rotation(0)	= Identity	= Boost(0)
Lorentz SR Rotation Tensor $\Lambda^{\mu'}{}_{\nu} \to R^{\mu'}{}_{\nu}$ = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[\theta] & -\sin[\theta] & 0 \\ 0 & \sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	Lorentz SR Identity Tensor $\Lambda^{\mu'}{}_{\nu} \to \eta^{\mu'}{}_{\nu}$ $= R^{\mu'}{}_{\nu}[0] = B^{\mu'}{}_{\nu}[0]$ $= \delta^{\mu'}{}_{\nu}$ = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ = Minkowski Delta	Lorentz SR Boost Tensor $\Lambda^{\mu'}{}_{\nu} \to B^{\mu'}{}_{\nu}$ = $\begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
EigenValues[$R^{\mu'}{}_{\nu}$] $=\text{Set}\{1,e^{i\theta},e^{-i\theta},1\}$	EigenValues[$\eta^{\mu'}{}_{\nu}$] $=\text{Set}\{1,1,1,1\}$	EigenValues[$B^{\mu'}{}_{\nu}$] $=\text{Set}\{e^{\theta},e^{-\theta},1,1\}$
Sum of EigenValues[$R^{\mu'}{}_{\nu}$] $=\text{Tr}[R^{\mu'}{}_{\nu}]=R^{\mu'}{}_{\mu}$ $=1+e^{i\theta}+e^{-i\theta}+1$ $=2+2\cos[\theta]$ $=\{0..4\}$	Sum of EigenValues[$\eta^{\mu'}{}_{\nu}$] $=\text{Tr}[\eta^{\mu'}{}_{\nu}]=\eta^{\mu'}{}_{\mu}$ $=1+1+1+1$ $=4$ $=\{4\}$	Sum of EigenValues[$B^{\mu'}{}_{\nu}$] $=\text{Tr}[B^{\mu'}{}_{\nu}]=B^{\mu'}{}_{\mu}$ $=e^{\theta}+e^{-\theta}+1+1$ $=2+2\cosh[\theta]=2+2\gamma$ $=\{4..\text{Infinity}\}$
Product of EigenValues[$R^{\mu'}{}_{\nu}$] $=\text{Det}[R^{\mu'}{}_{\nu}]$ $=1\cdot e^{i\theta}\cdot e^{-i\theta}\cdot 1$ $=+1$	Product of EigenValues[$\eta^{\mu'}{}_{\nu}$] $=\text{Det}[\eta^{\mu'}{}_{\nu}]$ $=1\cdot 1\cdot 1\cdot 1$ $=+1$	Product of EigenValues[$B^{\mu'}{}_{\nu}$] $=\text{Det}[B^{\mu'}{}_{\nu}]$ $=e^{\theta}\cdot e^{-\theta}\cdot 1\cdot 1$ $=+1$
Proper	Proper	Proper

EigenValues[$\Lambda^{\mu'}{}_{\nu}$] $=\text{Set}\{e^{a},e^{-a},e^{b},e^{-b}\}$

Sum of EigenValues[$\Lambda^{\mu'}{}_{\nu}$] $=\text{Tr}[\Lambda^{\mu'}{}_{\nu}]=\Lambda^{\mu'}{}_{\mu}$ $=\{e^{a}+e^{-a}+e^{b}+e^{-b}\}$ $=2(\cosh[a]+\cosh[b])$ $=\{-4..\text{Infinity}\}$

Product of EigenValues[$\Lambda^{\mu'}{}_{\nu}$] $=\text{Det}[\Lambda^{\mu'}{}_{\nu}]$ $=\{e^{a}\cdot e^{-a}\cdot e^{b}\cdot e^{-b}\}$ $=+1$

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(2,0)-Tensor $T^{\mu\nu}$
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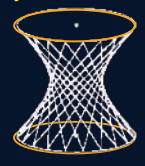
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics



A Tensor Study of Physical 4-Vectors

SRQM Diagram:
The Basis of Classical SR Physics
The Lorentz-Transform $\partial_\nu[R^{\mu'}]=\partial R^{\mu'}/\partial R^\nu=\Lambda^{\mu'}{}_\nu$

4-Vector SRQM Interpretation of QM

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$$\begin{matrix} \gamma & -\gamma\beta_j \\ -\gamma\beta^i & (\gamma-1)\beta^i\beta_j/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\delta^i_j \end{matrix}$$

$$\begin{matrix} 1 & 0_j \\ 0^i & (\delta^i_j-n^in_j)\cos(\theta)-(\varepsilon^i{}_{jk}n^k)\sin(\theta)+n^in_j \end{matrix}$$

$$\begin{matrix} 1 & 0_j \\ 0^i & \delta^i_j \end{matrix}$$

$Tr[B^{\mu'}{}_\nu]$
$= \gamma + (\gamma-1)[\beta^j\beta_j](\boldsymbol{\beta}\cdot\boldsymbol{\beta}) + \delta^j_j$
$= \gamma + (\gamma-1) + \delta^j_j$
$= \gamma + (\gamma-1) + 3$
$= 2(\gamma+1)$
$= \{4..\infty\}$

$Tr[R^{\mu'}{}_\nu]$
$= 1+(\delta^i_j - n^in_j)\cos(\theta)-(\varepsilon^i{}_{jk}n^k)\sin(\theta)+n^in_j$
$= 1+(3 - \mathbf{n}\cdot\mathbf{n})\cos(\theta)-0+\mathbf{n}\cdot\mathbf{n}$
$= 1+(3 - 1)\cos(\theta)+1$
$= 1+(2)\cos(\theta)+1$
$= 2(\cos(\theta)+1)$
$=\{0..4\}$

$Tr[I^{\mu'}{}_\nu]$
$= 1 + \delta^j_j$
$= 1 + 3$
$= 4$
$= \{4\}$

$[B^\mu{}_\alpha (B^{-1})^\alpha{}_\nu] = \delta^\mu{}_\nu$

for μ=ν=0

$[(B)^0{}_\alpha (B^{-1})^\alpha{}_0]$
$= \gamma^2 + (-\gamma)^2[-\beta^j\beta_j]$
$= \gamma^2(1-\boldsymbol{\beta}\cdot\boldsymbol{\beta})$
$= 1$

for μ=ν=1 : similar result for =2,=3

$=[(B)^1{}_0 (B^{-1})^0{}_1]+[(B)^1{}_1 (B^{-1})^1{}_1]+[(B)^1{}_2 (B^{-1})^2{}_1]+[(B)^1{}_3 (B^{-1})^3{}_1]$
$= \gamma^2[-\beta^1\beta_1] + \{(\gamma-1)[\beta^1\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\delta^1{}_1\}^2 + \{(\gamma-1)[\beta^1\beta_2]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\delta^1{}_2\}^2 + \{(\gamma-1)[\beta^1\beta_3]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\delta^1{}_3\}^2$
$= \gamma^2[-\beta^1\beta_1] + \{(\gamma-1)[\beta^1\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+1\}^2 + \{(\gamma-1)[\beta^1\beta_2]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}^2 + \{(\gamma-1)[\beta^1\beta_3]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}^2$
$= \gamma^2[-\beta^1\beta_1] + 2\{(\gamma-1)[\beta^1\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}+1+\{(\gamma-1)[\beta^1\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}^2 + \{(\gamma-1)[\beta^1\beta_2]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}^2 + \{(\gamma-1)[\beta^1\beta_3]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}^2$
$= \gamma^2[-\beta^1\beta_1] + 2\{(\gamma-1)[\beta^1\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}+1+(\gamma-1)^2(\beta^1)^2\{[\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}^2 + \{[\beta_2]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}^2 + \{[\beta_3]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}^2$
$= \gamma^2[-\beta^1\beta_1] + 2\{(\gamma-1)[\beta^1\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}+1+(\gamma-1)^2(\beta^1)^2/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})$
$= \gamma^2[-\beta^1\beta_1] + [2\{(\gamma-1)[\beta^1\beta_1]\}+(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+(\gamma-1)^2(\beta^1)^2]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})$
$= \gamma^2[-\beta^1\beta_1] + [-(\beta^1)^2+(\beta^1)^2+2\{(\gamma-1)[\beta^1\beta_1]\}+(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+(\gamma-1)^2(\beta^1)^2]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})$
$= \gamma^2[-\beta^1\beta_1] + (\beta^1)^2[-1+1+2\{(\gamma-1)\}+(\boldsymbol{\beta}\cdot\boldsymbol{\beta})/(\beta^1)^2+(\gamma-1)^2]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})$
$= \gamma^2[-\beta^1\beta_1] + (\beta^1)^2[\gamma^2-1+(\boldsymbol{\beta}\cdot\boldsymbol{\beta})/(\beta^1)^2]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})$
$= \gamma^2[-\beta^1\beta_1] + [(\beta^1)^2\gamma^2+1]$
$=1$

SR:Lorentz Transform

$\partial_\nu[R^{\mu'}] = \partial R^{\mu'}/\partial R^\nu = \Lambda^{\mu'}{}_\nu$
$\Lambda^\mu{}_\nu = (\Lambda^{-1})_\nu{}^\mu$
$\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta = \eta_{\alpha\beta}$
$Det[\Lambda^\mu{}_\nu]=\pm1$ $\quad \Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^\mu{}_\nu\Lambda_\mu{}^\nu$

$Tr[\Lambda^\mu{}_\nu]=\{-\infty..+\infty\}$
=Lorentz Transform Type

$\Lambda^\mu{}_\alpha(\Lambda^{-1})^\alpha{}_\nu = \Lambda^{\mu\alpha}(\Lambda^{-1})_{\alpha\nu} = \eta^\mu{}_\nu = \delta^\mu{}_\nu$

$Tr[R^{\mu'}{}_\nu]$
$2[1+\cos(\theta)]=\{0..4\}$
Lorentz Rotation

$Tr[\eta^{\mu'}{}_\nu]$
$=\{4\}$
Identity

$Tr[B^{\mu'}{}_\nu]$
$2[1+\gamma]=\{4..+\infty\}$
Lorentz Boost

$[B^\mu{}_\alpha (B^{-1})^\alpha{}_\nu] = \delta^\mu{}_\nu$

for μ=0, ν=1 : similar result for ν=2, ν=3

$=[(B)^0{}_\alpha (B^{-1})^\alpha{}_1]$
$=[(B)^0{}_0 (B^{-1})^0{}_1]+[(B)^0{}_1 (B^{-1})^1{}_1]+[(B)^0{}_2 (B^{-1})^2{}_1]+[(B)^0{}_3 (B^{-1})^3{}_1]$
$=[-\gamma\ \gamma\beta_1]+[\gamma\beta_1\ \{(\gamma-1)[\beta^1\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\delta^1{}_1\}]+[\gamma\beta_2\ (\gamma-1)[\beta^2\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})]+[\gamma\beta_3\ (\gamma-1)[\beta^3\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})]$
$=[-\gamma\ \gamma\beta_1]+[\gamma\beta_1\ \{(\gamma-1)[\beta^1\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+1\}]+[\gamma\beta_2\ (\gamma-1)[\beta^2\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})]+[\gamma\beta_3\ (\gamma-1)[\beta^3\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})]$
$=\gamma\beta_1\{[-\gamma\ ]+[\ \{(\gamma-1)[\beta^1\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+1\}]+[\beta_2\ (\gamma-1)[\beta^2]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})]+[\beta_3\ (\gamma-1)[\beta^3]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})]\}$
$=\gamma\beta_1\{[-\gamma\ ]+1+\ \{(\gamma-1)[\beta^1\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}+\beta_2\ (\gamma-1)[\beta^2]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\beta_3\ (\gamma-1)[\beta^3]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}$
$=\gamma\beta_1\{[-\gamma\ ]+1+\ (\gamma-1)\{[\beta^1\beta_1]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}+\beta_2\ [\beta^2]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})+\beta_3\ [\beta^3]/(\boldsymbol{\beta}\cdot\boldsymbol{\beta})\}$
$=\gamma\beta_1\{[-\gamma\ ]+1+\ (\gamma-1)\}$
$=0$

for μ=2, ν=3 : similar result for other μ<>ν let a=(γ-1)/(**β·β**)

$=[(B)^2{}_\alpha (B^{-1})^\alpha{}_3]$
$=[(B)^2{}_0 (B^{-1})^0{}_3]+[(B)^2{}_1 (B^{-1})^1{}_3]+[(B)^2{}_2 (B^{-1})^2{}_3]+[(B)^2{}_3 (B^{-1})^3{}_3]$
$=[(-\gamma\beta^2)(\gamma\beta_3)]+[(a[\beta^2\beta_1])(a[\beta^1\beta_3])]+[\{a[\beta^2\beta_2]+\delta^2{}_2\}(a[\beta^2\beta_3])]+[(a[\beta^2\beta_3])\{a[\beta^3\beta_3]+\delta^3{}_3\}]$
$=[(-\gamma^2\beta^2\beta_3)]+[(a^2[\beta^2\beta_1][\beta^1\beta_3])]+[\{a[\beta^2\beta_3]+(a^2[\beta^2\beta_3][\beta^2\beta_3])\}]+[(a^2[\beta^2\beta_3][\beta^3\beta_3]+(a[\beta^2\beta_3])\}]$
$=(-\gamma^2\beta^2\beta_3)+(a^2[\beta^2\beta_1][\beta^1\beta_3])+(a^2[\beta^2\beta_3][\beta^2\beta_3])+(a^2[\beta^2\beta_3][\beta^3\beta_3]+(2a[\beta^2\beta_3])$
$=(-\beta^2\beta_3)\{(-\gamma^2)+(a^2[\beta_1][\beta^1])+(a^2[\beta^2\beta_3])+(a^2[\beta^3\beta_3]+(2a)\}$
$=(-\beta^2\beta_3)\{(-\gamma^2)+(a^2(\boldsymbol{\beta}\cdot\boldsymbol{\beta}))+(2a)\}$

$=...$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

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(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
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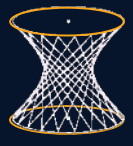
SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Invariant $Tr[\Lambda^{\mu'}{}_\nu] \rightarrow$
$-\infty,..,(-4),..,-2,..,(0),..,+2,..,(+4),\ldots,+\infty$
Trace identifies CPT Symmetry
in the Lorentz Transform

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics



A Tensor Study of Physical 4-Vectors

SRQM Study: SR 4-Tensors

SR Tensor Invariants for Discrete Lorentz Transform Tensors

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

SR:Lorentz Transform

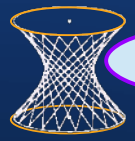
$\partial_\nu[R^\mu] = \partial R^{\mu'}/\partial R^\nu = \Lambda^{\mu'}{}_\nu$

$\Lambda^\mu{}_\nu = (\Lambda^{-1})_\nu{}^\mu : \Lambda^\mu{}_\alpha(\Lambda^{-1})^\alpha{}_\nu = \eta^\mu{}_\nu = \delta^\mu{}_\nu$

$\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta = \eta_{\alpha\beta}$

$\mathrm{Det}[\Lambda^\mu{}_\nu]=\pm1$ $\quad \Lambda_{\mu\nu}\Lambda^{\mu\nu}=4$

$\mathrm{Tr}[\Lambda^\mu{}_\nu]=\{-\infty..+\infty\}$
=Lorentz Transform Type



Inner Product Tensor Invariant

$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^\mu{}_\nu\Lambda_\mu{}^\nu$

Asymm Tri-Product Tensor Invariant

AsymmTri[$\Lambda^{\mu'}{}_\nu$]=?
Not yet calc...

Lorentz SR TPcombo Tensor $\Lambda^{\mu'}{}_\nu \to TP^{\mu'}{}_\nu$ = $-\eta^{\mu'}{}_\nu = -\delta^{\mu'}{}_\nu$ =	Lorentz SR Parity-Inversion Tensor $\Lambda^{\mu'}{}_\nu \to P^{\mu'}{}_\nu$ =	Lorentz SR Flip-xy-Combo Tensor $\Lambda^{\mu'}{}_\nu \to Fxy^{\mu'}{}_\nu$ = $-\eta^{\mu'}{}_\nu = -\delta^{\mu'}{}_\nu$ =	Lorentz SR Time-Reversal Tensor $\Lambda^{\mu'}{}_\nu \to T^{\mu'}{}_\nu$ =	Lorentz SR Identity Tensor $\Lambda^{\mu'}{}_\nu \to \eta^{\mu'}{}_\nu$ = $\delta^{\mu'}{}_\nu$ =
$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
= Negative Identity	= Flip-xyz	= Rotation-z (π)	= Flip-t	= Minkowski Delta
EigenValues[$TP^{\mu'}{}_\nu$] =Set{-1,-1,-1,-1}	EigenValues[$P^{\mu'}{}_\nu$] =Set{1,-1,-1,-1}	EigenValues[$Fxy^{\mu'}{}_\nu$] =Set{1,-1,-1,1}	EigenValues[$T^{\mu'}{}_\nu$] =Set{-1,1,1,1}	EigenValues[$\eta^{\mu'}{}_\nu$] =Set{1,1,1,1}
Sum of EigenValues[$TP^{\mu'}{}_\nu$] =Tr[$TP^{\mu'}{}_\nu$]=$TP^{\mu'}{}_\mu$ = -1-1-1-1 = -4	Sum of EigenValues[$P^{\mu'}{}_\nu$] =Tr[$P^{\mu'}{}_\nu$]=$P^{\mu'}{}_\mu$ = 1-1-1-1 = -2	Sum of EigenValues[$Fxy^{\mu'}{}_\nu$] =Tr[$Fxy^{\mu'}{}_\nu$]=$Fxy^{\mu'}{}_\mu$ = 1-1-1+1 = 0	Sum of EigenValues[$T^{\mu'}{}_\nu$] =Tr[$T^{\mu'}{}_\nu$]=$T^{\mu'}{}_\mu$ = -1+1+1+1 = 2	Sum of EigenValues[$\eta^{\mu'}{}_\nu$] =Tr[$\eta^{\mu'}{}_\nu$]=$\eta^{\mu'}{}_\mu$ = 1+1+1+1 = 4
Product of EigenValues[$TP^{\mu'}{}_\nu$] =Det[$TP^{\mu'}{}_\nu$] = -1·-1·-1·-1 = +1	Product of EigenValues[$P^{\mu'}{}_\nu$] =Det[$P^{\mu'}{}_\nu$] = 1·-1·-1·-1 = -1	Product of EigenValues[$Fxy^{\mu'}{}_\nu$] =Det[$Fxy^{\mu'}{}_\nu$] = -1·-1·-1·1 = +1	Product of EigenValues[$T^{\mu'}{}_\nu$] =Det[$T^{\mu'}{}_\nu$] = -1·1·1·1 = -1	Product of EigenValues[$\eta^{\mu'}{}_\nu$] =Det[$\eta^{\mu'}{}_\nu$] = 1·1·1·1 = +1
Proper	Improper	Proper	Improper	Proper

The Trace of various discrete Lorentz transforms varies in steps from {-4,-2,0,2,4}

This includes Mirror Flips, Time Reversal, and Parity Inverse – essentially taking all combinations of ±1 on the diagonal of the transform.

Trace Tensor Invariant

Tr[Discrete $\Lambda^{\mu'}{}_\nu$]={-4,-2,0,2,4}
Depends on transform

Determinant Tensor Invariant

Det[$\Lambda^{\mu'}{}_\nu$]=±1
Proper Transform = +1
Improper Transform = -1

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

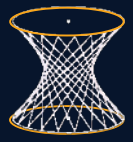
SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Det[$T^\alpha{}_\alpha$] = $\Pi_k[\lambda_k]$; with $\{\lambda_k\}$ = EigenValues
Characteristic Eqns: Det[$T^\alpha{}_\alpha - \lambda_k I_{(4)}$]=0

Trace[$T^{\mu\nu}$] = $\eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics



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More SR Tensor Invariants for Discrete Lorentz Transform Tensors

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$\mathrm{Det}[\Lambda^\mu{}_\nu]=\pm1$ $\quad \Lambda_{\mu\nu}\Lambda^{\mu\nu}=4$

$\mathrm{Tr}[\Lambda^\mu{}_\nu]=\{-\infty..+\infty\}$
=Lorentz Transform Type

The Flip-xy-Combo is the equivalent of a π-Rotation-z.

I suspect that this may be related to exchange symmetry and the Spin-Statistics idea that a particle-exchange is the equivalent of a spin-rotation.

A single Flip would not be an exchange because it leaves a mirror-inversion of <right-|-left>.

But the extra Flip along an orthogonal axis corrects the mirror-inversion, and would be an overall exchange because the particle is in a different location.

Lorentz SR 0-Rotation-z

Tensor $\Lambda^{\mu'}{}_\nu \to R^{\mu'}{}_\nu$ =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[0] & -\sin[0] & 0 \\ 0 & \sin[0] & \cos[0] & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

EigenValues$[R^{\mu'}{}_\nu]$ =Set$\{1,e^{i0},e^{-i0},1\}$

Sum of EigenValues$[R^{\mu'}{}_\nu]$ =Tr$[R^{\mu'}{}_\nu]$=$R^{\mu'}{}_\mu$ =$1+e^{i0}+e^{-i0}+1$ =$2+2\cos[0]$ =4

Product of EigenValues$[R^{\mu'}{}_\nu]$ =Det$[R^{\mu'}{}_\nu]$ =$1\cdot e^{i0}\cdot e^{-i0}\cdot 1$ = +1

Proper

Lorentz SR Identity

Tensor $\Lambda^{\mu'}{}_\nu \to \eta^{\mu'}{}_\nu$ = $\delta^{\mu'}{}_\nu$ =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

= Minkowski Delta

EigenValues$[\eta^{\mu'}{}_\nu]$ =Set$\{1,1,1,1\}$

Sum of EigenValues$[\eta^{\mu'}{}_\nu]$ =Tr$[\eta^{\mu'}{}_\nu]$=$\eta^{\mu'}{}_\mu$ = 1+1+1+1 =$2+2\cos[0]$ = 4

Product of EigenValues$[\eta^{\mu'}{}_\nu]$ =Det$[\eta^{\mu'}{}_\nu]$ = 1·1·1·1 = +1

Proper

Lorentz SR Flip-x

Tensor $\Lambda^{\mu'}{}_\nu \to Fx^{\mu'}{}_\nu$ =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

EigenValues$[Fx^{\mu'}{}_\nu]$ =Set$\{1,-1,1,1\}$

Sum of EigenValues$[Fx^{\mu'}{}_\nu]$ =Tr$[Fx^{\mu'}{}_\nu]$=$Fx^{\mu'}{}_\mu$ = 1-1+1+1 = 2

Product of EigenValues$[Fx^{\mu'}{}_\nu]$ =Det$[Fx^{\mu'}{}_\nu]$ = 1·-1·1·1 = -1

Improper

Lorentz SR Flip-y

Tensor $\Lambda^{\mu'}{}_\nu \to Fy^{\mu'}{}_\nu$ =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

EigenValues$[Fy^{\mu'}{}_\nu]$ =Set$\{1,1,-1,1\}$

Sum of EigenValues$[Fy^{\mu'}{}_\nu]$ =Tr$[Fy^{\mu'}{}_\nu]$=$Fy^{\mu'}{}_\mu$ = 1+1-1+1 = 2

Product of EigenValues$[Fy^{\mu'}{}_\nu]$ =Det$[Fy^{\mu'}{}_\nu]$ = 1·1·-1·1 = -1

Improper

Lorentz SR Flip-xy-Combo

Tensor $\Lambda^{\mu'}{}_\nu \to Fxy^{\mu'}{}_\nu$ = $-\eta^{\mu'}{}_\nu$ = $-\delta^{\mu'}{}_\nu$ =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

= Rotation-z (π)

EigenValues$[Fxy^{\mu'}{}_\nu]$ =Set$\{1,-1,-1,1\}$

Sum of EigenValues$[Fxy^{\mu'}{}_\nu]$ =Tr$[Fxy^{\mu'}{}_\nu]$=$Fxy^{\mu'}{}_\mu$ = 1-1-1+1 =$2+2\cos[\pi]$ = 0

Product of EigenValues$[Fxy^{\mu'}{}_\nu]$ =Det$[Fxy^{\mu'}{}_\nu]$ = -1·-1·-1·1 = +1

Proper

Lorentz SR π-Rotation-z

Tensor $\Lambda^{\mu'}{}_\nu \to R^{\mu'}{}_\nu$ =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[\pi] & -\sin[\pi] & 0 \\ 0 & \sin[\pi] & \cos[\pi] & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

EigenValues$[R^{\mu'}{}_\nu]$ =Set$\{1,e^{i\pi},e^{-i\pi},1\}$

Sum of EigenValues$[R^{\mu'}{}_\nu]$ =Tr$[R^{\mu'}{}_\nu]$=$R^{\mu'}{}_\mu$ =$1+e^{i\pi}+e^{-i\pi}+1$ =$2+2\cos[\pi]$ =0

Product of EigenValues$[R^{\mu'}{}_\nu]$ =Det$[R^{\mu'}{}_\nu]$ =$1\cdot e^{i\pi}\cdot e^{-i\pi}\cdot 1$ = +1

Proper

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor V^μ = **V** = $(v^0,\mathbf{v})$
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SRQM Study:
More Tensor Invariants

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Riemann Curvature Tensor = Riem
$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}$
→ $0^{\rho}{}_{\sigma\mu\nu}$ for SR "Flat" Minkowski SpaceTime

Contraction 1 pair of indices

Ricci Tensor = Ric
$R_{\mu\nu}$

Metric
$g^{\mu\nu}$

Ricci Scalar
Scalar Curvature
R or S
$= g^{\mu\nu}R_{\mu\nu} = R_\mu{}^\mu$
$= Tr_g[Ric]$

Trace-Free Ricci Tensor
$S_{\mu\nu}$
$= R_{\mu\nu} - (1/4)Rg_{\mu\nu}$

Contraction 2 pair of indices

Kretschmann scalar
K
$= R^{\rho\sigma\mu\nu} R_{\rho\sigma\mu\nu}$

$\rightarrow 48G^2M^2/(c^4r^6)$
for Schwarzschild BH

K
$=8/(r^2+a^2\cos^2\theta)^6$
$*[$
$6m^2(r^6-15a^2r^4\cos^2\theta+ 15a^4r^2\cos^4\theta-a^6\cos^6\theta)$
$-12mQ^2r(r^4-10a^2r^2\cos^2\theta+ 5a^4\cos^4\theta)$
$+Q^4(7r^4-34a^2r^2\cos^2\theta+ 7a^4\cos^4\theta)$
$]$
for Kerr-Newman BH
with $G=c=4\pi\varepsilon_o=1$

Carminati–McLenaghan invariants or CM scalars
Below are just the 5 real ones, there are some complex ones too

R
$= g^{\mu\nu}R_{\mu\nu} = R^\mu{}_\mu$

R_1
$= (1/4)S^a{}_bS^b{}_a$

R_2
$= (-1/8)S^a{}_bS^b{}_cS^c{}_a$

R_3
$= (1/16)S^a{}_bS^b{}_cS^c{}_dS^d{}_a$

M_3
$= (1/16)S^{bc}S_{ef}(C_{abcd}C^{aefd} + {}^*C_{abcd}{}^*C^{aefd})$

M_4
$= (-1/32)S^{ag}S^{ef}S^c{}_d(C_{ac}{}^{db}C_{befg} + {}^*C_{ac}{}^{db}{}^*C_{befg})$

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(0,0)-Tensor S or S_o
Lorentz Scalar

see also: Zakhary–McIntosh curvature invariants

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= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

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SR 4-Scalars, 4-Vectors, 4-Tensors

Elegantly join many dual physical properties and relations

4-Vector SRQM Interpretation of QM

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SR 4-Scalars, 4-Vectors, and 4-Tensors beautifully and elegantly display the relations between lots of different physical properties and relations. Their notation makes navigation through the physics very simple.

They also devolve very nicely into the limiting/approximate Newtonian cases of { $|\mathbf{v}| << c$ } by letting { $\gamma \to 1$ and $\gamma' = d\gamma/dt \to 0$ }.

SR tells us that several different physical properties are actually dual aspects of the same thing, with the only real difference being one's point of view, or reference frame.

Examples of 4-Vectors = (1,0)-Tensors include:
(Time , Space), (Energy , Momentum), (Power , Force), (Frequency , WaveNumber), (Time Differential , Spatial Gradient),(NumberDensity , NumberFlux), (ChargeDensity , CurrentDensity), (EM-ScalarPotential , EM-VectorPotential), etc.

4-Scalar
S

SR 4-Vector $\mathbf{V} = V^\alpha$
$=(v^t,\mathbf{v}) \to (v^t,v^x,v^y,v^z)$
$=(\text{temporal} * c^{\pm 1},\text{spatial})$

4-Tensor $T^{\alpha\beta}$
=
$[T^{tt}\ T^{tx}\ T^{ty}\ T^{tz}]$
$[T^{xt}\ T^{xx}\ T^{xy}\ T^{xz}]$
$[T^{yt}\ T^{yx}\ T^{yy}\ T^{yz}]$
$[T^{zt}\ T^{zx}\ T^{zy}\ T^{zz}]$
=
[temporal,mixed]
[mixed ,spatial]

One can also examine 4-Tensors, which are type (2,0)-Tensors.
The Faraday EM Tensor similarly combines EM fields:
Electric { $\mathbf{e} = e^{i0} = e^{0j} = (e^x,e^y,e^z)$ } and Magnetic { $\mathbf{b} = b^k = (b^x,b^y,b^z)$ }

$$F^{\alpha\beta} = \begin{bmatrix} 0 & -e^{0j}/c \\ +e^{i0}/c & -(\varepsilon^{ij}{}_k\, b^k) \end{bmatrix}$$

Also, things are even more related than that.
The 4-Momentum is just a constant times 4-Velocity.
The 4-WaveVector is just a constant times 4-Velocity.

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

m_o
E_o/c^2

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

ω_o/c^2

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$

Faraday EM Tensor $F^{\alpha\beta}$
=
$[0\ \ -e^x/c\ \ -e^y/c\ \ -e^z/c]$
$[+e^x/c\ \ 0\ \ -b^z\ \ +b^y]$
$[+e^y/c\ \ +b^z\ \ 0\ \ -b^x]$
$[+e^z/c\ \ -b^y\ \ +b^x\ \ 0]$
=
$[\ 0\ ,\ -e^j/c\ ]$
$[+e^i/c,\ -\varepsilon^{ij}{}_k b^k]$

In addition, the very important conservation/continuity equations seem to just fall out of the notation.
The universe apparently has some simple laws which can be easy to write down by using a little math and a super notation.

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= Lorentz Scalar Invariant

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SRQM Diagram: SR 4-Vectors and Lorentz Scalars / Physical Constants

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Heart of SR

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})\in$<Event>

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

ProperTime Derivative

4-Acceleration
$\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$
$=d\mathbf{U}/d\tau$

$\mathbf{U}\cdot\mathbf{A}=\mathbf{U}\cdot\mathbf{U}'=0$

4-UnitTemporal
$\bar{\mathbf{T}}=\gamma(1,\boldsymbol{\beta})$

$\bar{\mathbf{T}}\cdot\bar{\mathbf{T}}=+1$

Time:Space Orthogonal

$\bar{\mathbf{T}}\cdot\bar{\mathbf{S}}=0$

$\bar{\mathbf{S}}\cdot\bar{\mathbf{S}}=-1$

4-UnitSpatial
$\bar{\mathbf{S}}=\gamma_{\beta n}(\hat{\mathbf{n}}\cdot\boldsymbol{\beta},\hat{\mathbf{n}})_\perp$

Speed of Light c

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

$\mathbf{U}\cdot\mathbf{U}=c^2$

Rest Number Density n_o

Rest Charge Density ρ_o

EM φ_o/c^2

Rest Scalar Potential

4-NumberFlux
$\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$

EM Charge q

4-ChargeFlux
4-CurrentDensity
$\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

4-Polarization
$\mathbf{E}=(\varepsilon^0,\boldsymbol{\varepsilon})=(\boldsymbol{\varepsilon}\cdot\boldsymbol{\beta},\boldsymbol{\varepsilon})$

$\mathbf{U}\cdot\mathbf{E}=0$ Polarization is Rest Spatial

ω_o/c^2

Wave Velocity
$v_{group}*v_{phase}=c^2$
Rest AngFrequency

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$

$\{\omega_o=0\}\leftrightarrow\{\mathbf{K}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{K}$ is null$\}$

E_o/ω_o

m_o E_o/c^2

$E=mc^2$ Rest Energy:Mass

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

$\{m_o=0\}\leftrightarrow\{\mathbf{P}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{P}$ is null$\}$

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

ProperTime Derivative

4-Force
$\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$
$=d\mathbf{P}/d\tau$

4-EMVectorPotential
$\mathbf{A}=(\varphi/c,\mathbf{a})$

$\{\varphi_o=0\}\leftrightarrow\{\mathbf{A}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{A}$ is null$\}$

EM Charge q

4-EMPotentialMomentum
$\mathbf{Q}=(U/c,\mathbf{q})=q\mathbf{A}$

Minimal Coupling $\mathbf{P}+\mathbf{Q}$

Minkowski
$\partial[\mathbf{R}]=\partial^\mu[R^\nu]=\eta^{\mu\nu}$
Metric

Lorentz
$\partial_\nu[R^{\mu'}]=\Lambda^{\mu'}{}_\nu$
Transform

SpaceTime Dim
$Tr[\eta^{\mu\nu}]=4=\Lambda_{\mu\nu}\Lambda^{\mu\nu}$

Soul of SR

$..[\mathbf{K}_T\cdot\mathbf{R}]$ $..\int[\mathbf{K}_T\cdot d\mathbf{R}]$ $..[-\Phi_{phase}]$

Conservation of 4-TotalWaveVector Sum of Plane-Waves

$\sum_n[..]$

4-TotalWaveVector
$\mathbf{K}_T=(\omega_T/c,\mathbf{k}_T)$
$=-\partial[\Phi_{phase}]$

E_{To}/ω_{To}

4-Gradient
$\partial=(\partial_t/c,-\nabla)$
$=(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

Complex Plane-Waves
$\mathbf{K}_T=-\partial[\Phi]$

Hamilton-Jacobi
$\mathbf{P}_T=-\partial[S]$

$..[\mathbf{P}_T\cdot\mathbf{R}]$ $..\int[\mathbf{P}_T\cdot d\mathbf{R}]$ $..[-S_{action}]$

T

4-TotalMomentum
$\mathbf{P}_T=(E_T/c,\mathbf{p}_T)=(H/c,\mathbf{p}_T)$
$=-\partial[S_{action}]$

Conservation of 4-TotalMomentum Sum of Momenta

$\sum_n[..]$

+

4-MomentumIncField
$\mathbf{P}_f=(E_f/c,\mathbf{p}_f)=\mathbf{P}+\mathbf{Q}=\mathbf{P}+q\mathbf{A}$

SRQM Diagram

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$Trace[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
SRQM 4-Vectors and Lorentz Scalars / Physical Constants

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Heart of SR

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})\in$<Event>

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$

4-Acceleration
$\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$
$=d\mathbf{U}/d\tau$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

ProperTime Derivative

$\mathbf{U}\cdot\mathbf{A}=\mathbf{U}\cdot\mathbf{U}'=0$

4-UnitTemporal
$\bar{\mathbf{T}}=\gamma(1,\boldsymbol{\beta})$

$\bar{\mathbf{T}}\cdot\bar{\mathbf{T}}=+1$

Time:Space Orthogonal

$\bar{\mathbf{T}}\cdot\bar{\mathbf{S}}=0$

$\bar{\mathbf{S}}\cdot\bar{\mathbf{S}}=-1$

4-UnitSpatial
$\bar{\mathbf{S}}=\gamma_{\beta n}(\hat{\mathbf{n}}\cdot\boldsymbol{\beta},\hat{\mathbf{n}})_{\perp}$

Speed of Light c

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

$\mathbf{U}\cdot\mathbf{U}=c^2$

Rest Number Density n_o

$\rho_{prob^o}=\chi^*\psi$
$=|\langle\chi|\psi\rangle|^2$

Born Probability Rule
Rest Prob Density

Rest Charge Density ρ_o

EM φ_o/c^2

Rest Scalar Potential

4-NumberFlux
$\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$

4-ProbCurrDensity
4-ProbabilityFlux
$\mathbf{J}_{prob}=(\rho_{prob}c,\mathbf{j}_{prob})$

EM Charge q

4-ChargeFlux
4-CurrentDensity
$\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

4-Polarization
$\mathbf{E}=(\varepsilon^0,\boldsymbol{\varepsilon})=(\boldsymbol{\varepsilon}\cdot\boldsymbol{\beta},\boldsymbol{\varepsilon})$

4-Spin
$\mathbf{S}=(s^0,\mathbf{s})=(\mathbf{s}\cdot\boldsymbol{\beta},\mathbf{s})$

Spin is actually an outcome of Poincaré Invariance, not QM

$\mathbf{U}\cdot\mathbf{E}=0$
$\mathbf{U}\cdot\mathbf{S}=0$

Polarization:Spin is Rest Spatial

ω_o/c^2

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$

Wave Velocity
$v_{group}*v_{phase}=c^2$
Rest AngFrequency

$\{\omega_o=0\}\leftrightarrow\{\mathbf{K}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{K}$ is null$\}$

E_o/ω_o ($\hbar$)

Einstein de Broglie
$\mathbf{P}=\hbar\mathbf{K}$

m_o
E_o/c^2

$E=mc^2$
Rest Energy:Mass

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

$\{m_o=0\}\leftrightarrow\{\mathbf{P}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{P}$ is null$\}$

4-EMVectorPotential
$\mathbf{A}=(\varphi/c,\mathbf{a})$

$\{\varphi_o=0\}\leftrightarrow\{\mathbf{A}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{A}$ is null$\}$

EM Charge q

4-EMPotentialMomentum
$\mathbf{Q}=(U/c,\mathbf{q})=q\mathbf{A}$

Minimal Coupling
$\mathbf{P}+\mathbf{Q}$

$+$

4-MomentumIncField
$\mathbf{P}_f=(E_f/c,\mathbf{p}_f)=\mathbf{P}+\mathbf{Q}=\mathbf{P}+q\mathbf{A}$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

ProperTime Derivative

4-Force
$\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$
$=d\mathbf{P}/d\tau$

Minkowski
$\partial[\mathbf{R}]=\partial^\mu[R^\nu]=\eta^{\mu\nu}$
Metric

Lorentz
$\partial_\nu[R^{\mu'}]=\Lambda^{\mu'}{}_\nu$
Transform

SpaceTime Dim
$Tr[\eta^{\mu\nu}]=4=\Lambda_{\mu\nu}\Lambda^{\mu\nu}$

Soul of SR

$..[\mathbf{K}_T\cdot\mathbf{R}]$
$..\int[\mathbf{K}_T\cdot d\mathbf{R}]$
$..[-\Phi_{phase}]$
(i)

Complex Plane-Waves
$\mathbf{K}_T=-\partial[\Phi],\mathbf{K}=i\partial$

Conservation of 4-TotalWaveVector Sum of Plane-Waves

$\sum_n[..]$

4-TotalWaveVector
$\mathbf{K}_T=(\omega_T/c,\mathbf{k}_T)$
$=-\partial[\Phi_{phase}]$

E_{To}/ω_{To} ($\hbar$)

Einstein de Broglie
$\mathbf{P}_T=\hbar\mathbf{K}_T$

4-Gradient
$\partial=(\partial_t/c,-\nabla)$
$=(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

Hamilton-Jacobi
$\mathbf{P}_T=-\partial[S]$

$..[\mathbf{P}_T\cdot\mathbf{R}]$
$..\int[\mathbf{P}_T\cdot d\mathbf{R}]$
$..[-S_{action}]$

T

4-TotalMomentum
$\mathbf{P}_T=(E_T/c,\mathbf{p}_T)=(H/c,\mathbf{p}_T)$
$=-\partial[S_{action}]$

Conservation of 4-TotalMomentum Sum of Momenta

$\sum_n[..]$

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
Quantum Principles

$Trace[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
$=$ Lorentz Scalar Invariant

SRQM Study:

SR Gradient 4-Vectors = 4D (1,0)-Tensors
SR Gradient One-Forms = 4D (0,1)-Tensors

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4-Vector = Type 4D (1,0)-Tensor	**[Temporal : Spatial] components**
4-Position $\mathbf{R} = R^\mu = (ct,\mathbf{r})$	[Time (t) : Space (**r**)]
4-Gradient $\boldsymbol{\partial}_R = \boldsymbol{\partial} = \partial^\mu = \partial/\partial R_\mu = (\partial_t/c,-\nabla)$	[Time Differential (∂_t) : Spatial Gradient(∇)]

Standard 4-Vector	**Related Gradient 4-Vector (from index-raised Gradient One-Form)**
4-Position $\mathbf{R} = R^\mu = (ct,\mathbf{r})$	4-PositionGradient $\boldsymbol{\partial}_R = \partial_R{}^\mu = \partial/\partial R_\mu = (\partial_{R^t}/c,-\nabla_R) = \boldsymbol{\partial} = \partial^\mu$ = 4-Gradient
4-Velocity $\mathbf{U} = U^\mu = \gamma(c,\mathbf{u})$	4-VelocityGradient $\boldsymbol{\partial}_U = \partial_U{}^\mu = \partial/\partial U_\mu = (\partial_{U^t}/c,-\nabla_U)$
4-Momentum $\mathbf{P} = P^\mu = (E/c,\mathbf{p})$	4-MomentumGradient $\boldsymbol{\partial}_P = \partial_P{}^\mu = \partial/\partial P_\mu = (\partial_{P^t}/c,-\nabla_P)$
4-WaveVector $\mathbf{K} = K^\mu = (\omega/c,\mathbf{k})$	4-WaveGradient $\boldsymbol{\partial}_K = \partial_K{}^\mu = \partial/\partial K_\mu = (\partial_{K^t}/c,-\nabla_K)$

In each case, the (Whichever)Gradient 4-Vector is derived from a 4D SR One-Form or 4-CoVector, which is a type (0,1)-Tensor
ex. One-Form PositionGradient $\partial_{R^\nu} = \partial/\partial R^\nu = (\partial_{R^t}/c,\nabla_R)$

The (Whichever)Gradient 4-Vector is the index-raised version of the SR One-Form (Whichever)Gradient
ex. 4-PositionGradient $\partial_R{}^\mu = \partial/\partial R_\mu = (\partial_{R^t}/c,-\nabla_R) = \eta^{\mu\nu}\partial_{R^\nu} = \eta^{\mu\nu}\partial/\partial R^\nu = \eta^{\mu\nu}(\partial_{R^t}/c,\nabla_R)_\nu = \eta^{\mu\nu}$(One-Form PositionGradient)$_\nu$

This is why the 4-Gradient is commonly seen with a minus sign in the spatial component, unlike the other regular 4-Vectors, which have all positive components.

4-Tensors can be constructed from the Tensor Outer Product of 4-Vectors

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

Some Basic 4-Vectors
Minkowski SpaceTime Diagram
Events & Dimensions

4-Vector SRQM Interpretation of QM

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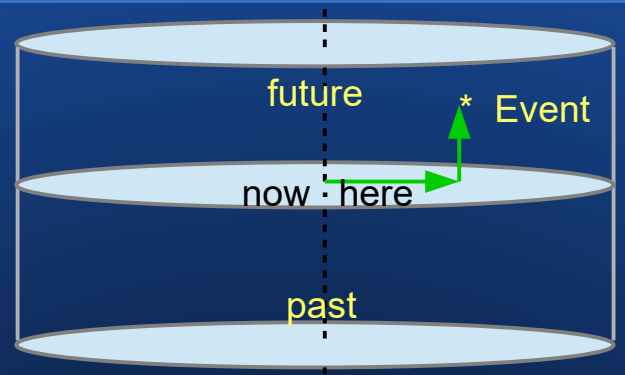
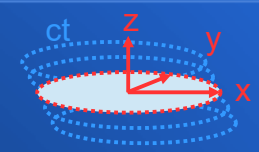


"Stack of Motion Picture Photos"

Δt time-like interval

Δr space-like interval



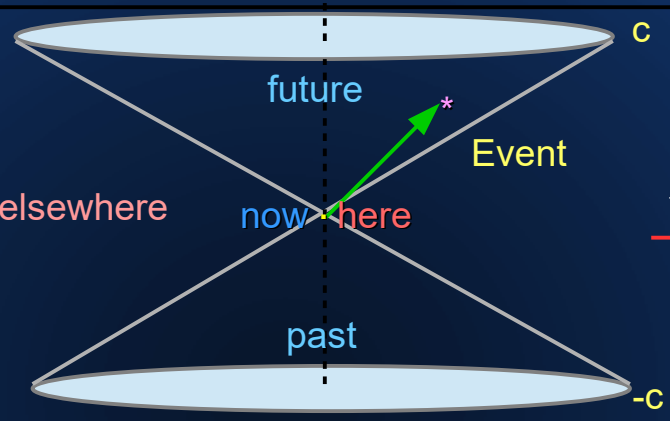
Classical Mechanics

4-Displacement$_{CM}$
$\mathbf{\Delta R}_{CM}=(c\Delta t, \mathbf{\Delta r})$

1/c → time displacement Δt

→ 3-displacement $\mathbf{\Delta r} = \Delta r^i \rightarrow (\Delta x, \Delta y, \Delta z)$

Note the separate dimensional units: (time + 3D space)
Δt is [time, SI→s], |**Δr**| is [length, SI→m]



LightCone

Δt time-like interval (+)

c light-like interval (0) = null

Δr space-like interval (-)

Special Relativity

4-Displacement
$\mathbf{\Delta R}=(c\Delta t, \mathbf{\Delta r})$

4-Position
$\mathbf{R}=(ct, \mathbf{r}) \in$ <Event>

$\mathbf{\Delta R}\cdot\mathbf{\Delta R} = [(c\Delta t)^2 - \mathbf{\Delta r}\cdot\mathbf{\Delta r}] = 0$

$(c\Delta\tau)^2$ Time-Like (+)
Light-like:Null (0)
$-(\Delta r_o)^2$ Space-like (-)

Note the matching dimensional units: (4D Time·Space)
(cΔt) is [length/time]*[time] = [length], |**Δr**| is [length], |**ΔR**| is [length]
τ is the Proper Time = "rest-time", time as measured by something not moving spatially
The Minkowski Diagram provides a great visual representation of SpaceTime

SR 4-Tensor
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(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Classical (scalar | 3-vector)
Galilean Invariant | Not Lorentz Invariant

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

Some Basic 4-Vectors

Minkowski SpaceTime Diagram, WorldLines, LightSpeed to the Future!

4-Vector SRQM Interpretation of QM

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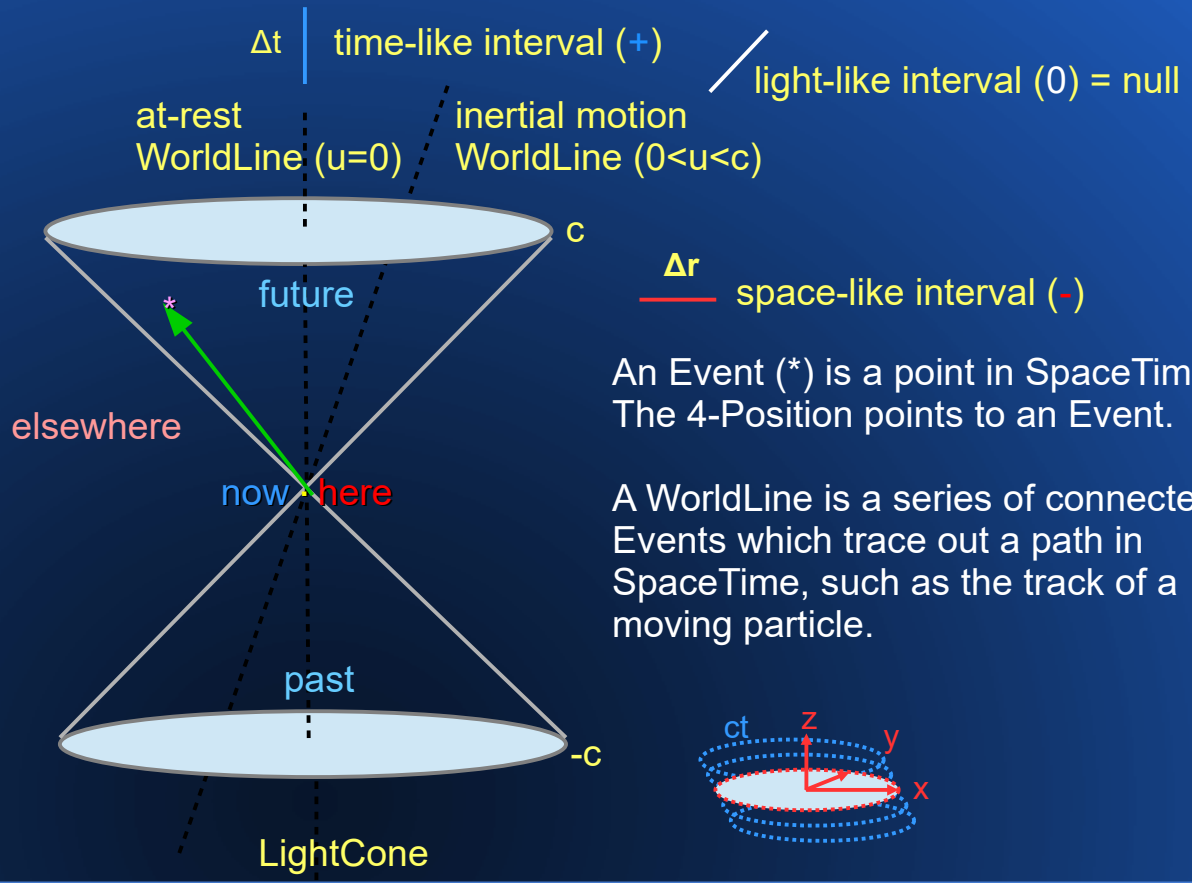

An Event (*) is a point in SpaceTime
The 4-Position points to an Event.

A WorldLine is a series of connected Events which trace out a path in SpaceTime, such as the track of a moving particle.

4-Displacement
ΔR=(cΔt,**Δr**)

4-Position
R=(ct,**r**)∈<Event>

The 4-Position is a particular type of 4-Displacement, for which the vector base is at the <Origin> = (0,0,0,0) = 4-Zero.

4-Position is Lorentz Invariant, but not Poincaré Invariant.
A standard 4-Displacement is both.

$$\mathbf{\Delta R}\cdot\mathbf{\Delta R} = [(c\Delta t)^2 - \mathbf{\Delta r}\cdot\mathbf{\Delta r}] = \begin{cases} (c\Delta\tau)^2 & \text{for time-like } (+) \\ 0 & \text{for light-like } (0) \\ -(\Delta r_o)^2 & \text{for space-like } (-) \end{cases}$$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})=d\mathbf{R}/d\tau$
$\mathbf{U}\cdot\mathbf{U}=c^2$

4-Velocity(rest-frame)
$\mathbf{U}_o=(c,\mathbf{0})$
$\mathbf{U}_o\cdot\mathbf{U}_o=c^2$

4-Velocity(photonic)
$\mathbf{U}_c=\gamma_c(c,c\hat{\mathbf{n}})$
$\mathbf{U}_c\cdot\mathbf{U}_c=c^2$

$$\mathbf{U}\cdot\mathbf{U} = \gamma(c,\mathbf{u})\cdot\gamma(c,\mathbf{u}) = \gamma^2(c^2-\mathbf{u}\cdot\mathbf{u}) = (c^2)$$
$$\gamma = 1/\sqrt{[1-(u/c)^2]} = 1/\sqrt{[1-(\beta)^2]}$$

Massive particles move temporally into the future at the speed-of-light (c) in their own rest-frame.

Massless particles (photonic) move nully into the future at the speed-of-light (c), and have no rest-frame.

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$$\text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$$
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= Lorentz Scalar Invariant

A Tensor Study of Physical 4-Vectors

SR Invariant Intervals

Minkowski Diagram:Lorentz Transform

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SciRealm@aol.com
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SR:Lorentz Transform

$\partial_\nu[R^{\mu'}] = \partial R^{\mu'}/\partial R^\nu = \Lambda^{\mu'}{}_\nu$

$\Lambda^\mu{}_\nu = (\Lambda^{-1})_\nu{}^\mu : \Lambda^\mu{}_\alpha(\Lambda^{-1})^\alpha{}_\nu = \eta^\mu{}_\nu = \delta^\mu{}_\nu$

$\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta = \eta_{\alpha\beta}$

$\mathrm{Det}[\Lambda^\mu{}_\nu]=\pm1$ $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4$

SR:Minkowski Metric

$\partial[\mathbf{R}] = \partial^\mu R^\nu = \eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu} \rightarrow$

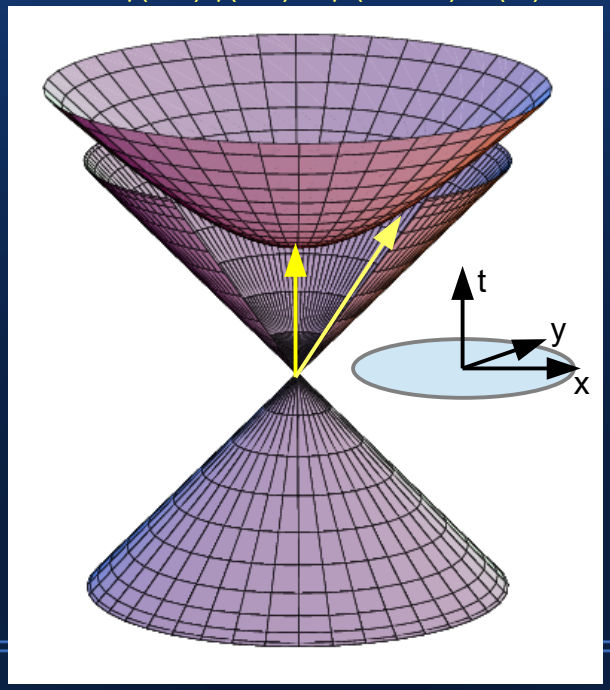
$\mathrm{Diag}[1,-1,-1,-1] = \mathrm{Diag}[1,-I_{(3)}] = \mathrm{Diag}[1,-\delta^{jk}]$

{in Cartesian form} "Particle Physics" Convention

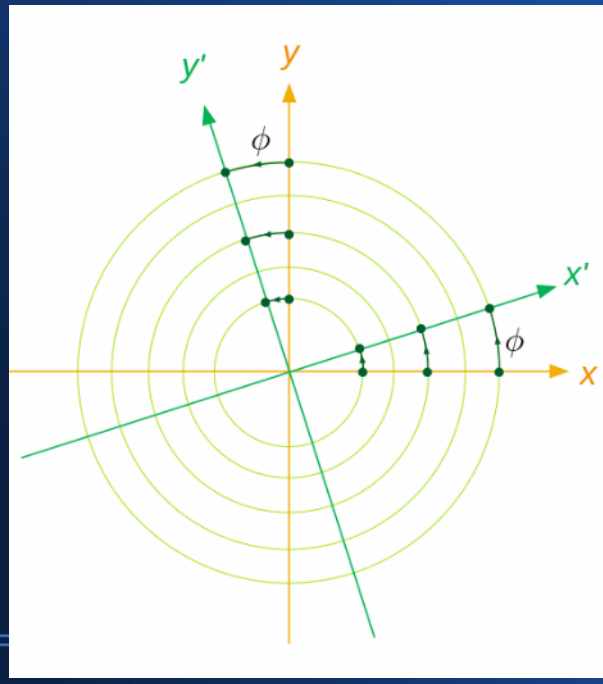
$\{\eta_{\mu\mu}\} = 1/\{\eta^{\mu\mu}\} : \eta_\mu{}^\nu = \delta_\mu{}^\nu$ $\mathrm{Tr}[\eta^{\mu\nu}]=4$

Since the SpaceTime magnitude of **U** is a constant (c), changes in the components of **U** are like rotating the 4-Vector without changing its length. It keeps the same magnitude.
Rotations, purely spatial changes, {eg. along x,y} result in circular displacements.
Boosts, or temporal-spatial changes, {eg. along x,t} result in hyperbolic displacements.
The interval between the origin and a given topograph-line is a Lorentz Invariant Constant.

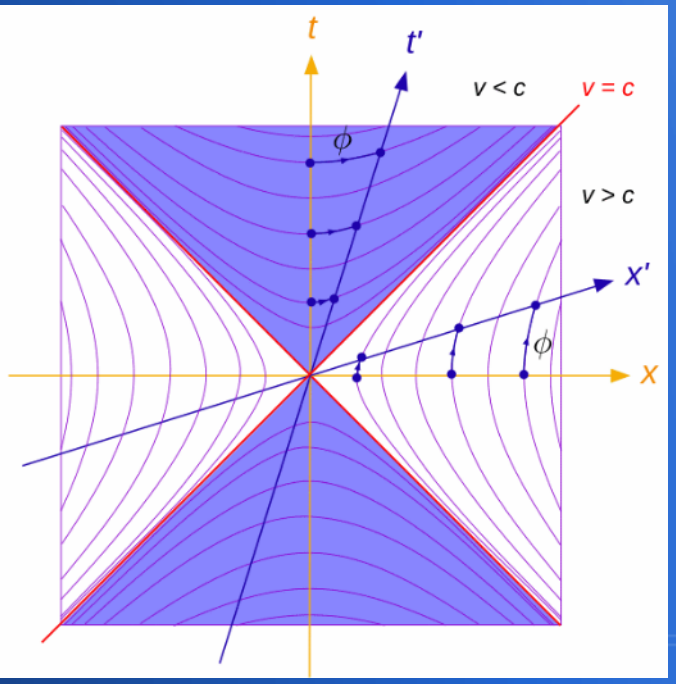
$\mathbf{U}\cdot\mathbf{U} = \gamma(c,\mathbf{u})\cdot\gamma(c,\mathbf{u}) = \gamma^2(c^2-\mathbf{u}\cdot\mathbf{u}) = (c^2)$



Rotation (x,y): Purely Spatial



Boost (x,t): Spatial-Temporal



The Light Cone / Minkowski Diagram provides a great visual representation of SpaceTime

A Tensor Study of Physical 4-Vectors

SR Invariant Intervals
Minkowski Diagram

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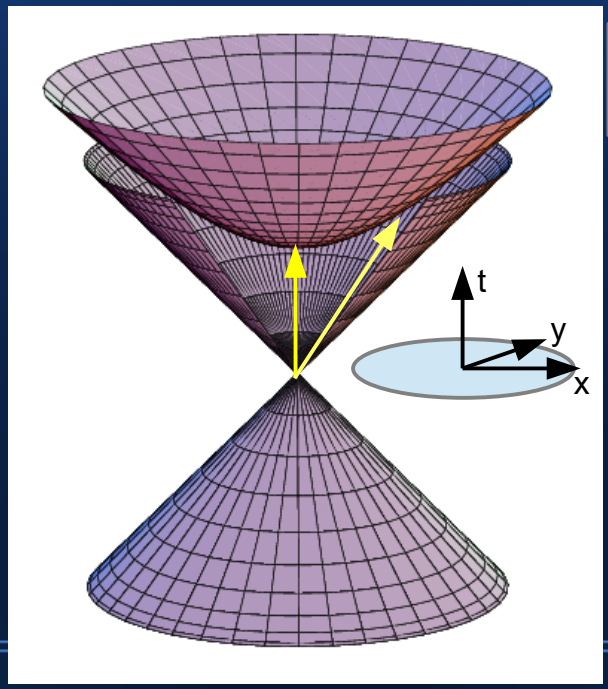
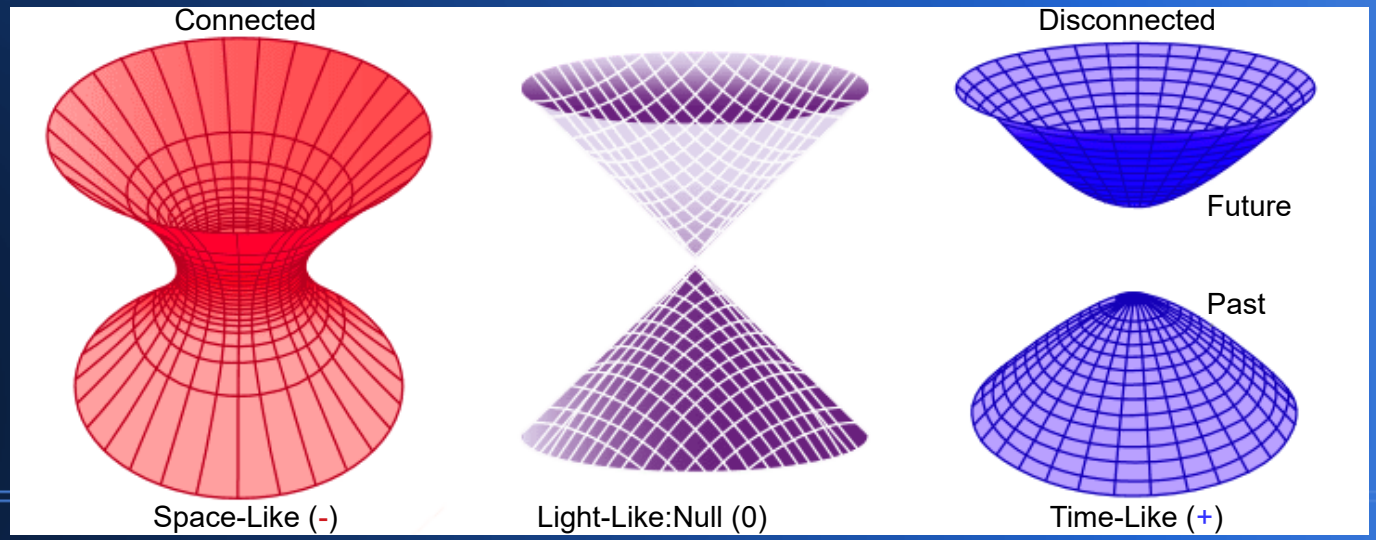


$\mathbf{\Delta R}\cdot\mathbf{\Delta R} = [(c\Delta t)^2 - \mathbf{\Delta r}\cdot\mathbf{\Delta r}] = (0)$

$(c\Delta\tau)^2$	Time-like:Temporal	(+) {causal = 1D temporally-ordered, *spatially relative*}
(0)	Light-like:Null:Photonic	(0) {causal & topological, maximum signal speed ($\lvert\mathbf{\Delta r}/\Delta t\rvert$=c)}
$-(\Delta r_o)^2$	Space-like:Spatial	(–) {*temporally relative*, topological = 3D spatially-ordered}



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(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM: Some Basic 4-Vectors

4-Position, 4-Velocity, 4-Acceleration SpaceTime Kinematics

4-Vector SRQM Interpretation of QM

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ProperTime
$\mathbf{R}\cdot\mathbf{U}/\mathbf{U}\cdot\mathbf{U}=(ct,\mathbf{r})\cdot\gamma(c,\mathbf{u})/c^2=\gamma(c^2t - \mathbf{r}\cdot\mathbf{u})/c^2=(c^2t_o)/c^2$
$= t_o = \tau$

4-Gradient
$\partial=(\partial_t/c,-\nabla)\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)=\gamma d/dt$
$= d/d\tau$

Special Relativity

$|\mathbf{v}| = |\mathbf{u}| = \{0 \leftrightarrow c\}$
$\gamma = 1/\sqrt{[1-(v/c)^2]}$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

4-Acceleration
$\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$
$=\gamma^2(\gamma^2(\mathbf{u}\cdot\mathbf{a})/c,\gamma^2(\mathbf{u}\cdot\mathbf{a})\mathbf{u}/c^2+\mathbf{a})$

4-Vectors:
$\mathbf{R} \in$ <Event>
$\mathbf{U} = d\mathbf{R}/d\tau$
$\mathbf{A} = d\mathbf{U}/d\tau$

↓ Newtonian/Classical Limit ↓

Classical Mechanics

$|\mathbf{v}| = |\mathbf{u}| << c$
$\gamma\rightarrow 1+O[(v/c)^2]$
$\gamma'\rightarrow 0$

4-Position$_{CM}$
$\mathbf{R}_{CM}=(ct\ \ \mathbf{r})$

$d/dt\,[..]$

4-Velocity$_{CM}$
$\mathbf{U}_{CM}=(c\ \ \mathbf{u})$

$d/dt\,[..]$

4-Acceleration$_{CM}$
$\mathbf{A}_{CM}=(0\ \ \mathbf{a})$

Since time:space don't mix in CM, Typically use time t & 3-position **r** separately

Since temporal velocity (c) always constant in CM Typically use just 3-velocity **u**

Since temporal acceleration (0) always constant in CM, Typically use just 3-acceleration **a**

time
$t=(t)$

3-position
$\mathbf{r} = (r^i)\rightarrow(x,y,z)$

$d/dt\,[..]$

3-velocity
$\mathbf{u} = (u^i)\rightarrow(u^x,u^y,u^z)$

$d/dt\,[..]$

3-acceleration
$\mathbf{a} = (a^i)\rightarrow(a^x,a^y,a^z)$

scalar:
$t =$ <time>
3-vectors:
$\mathbf{r} =$ <location>
$\mathbf{u} = d\mathbf{r}/dt$
$\mathbf{a} = d\mathbf{u}/dt$

The relativistic Gamma factor $\gamma = 1/\sqrt{[1-(v/c)^2]}$
The 1st order Newtonian Limit gives $\gamma \sim 1 + O[(v/c)^2]$
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For historical reasons, velocity can be represented by either (v) or (u)

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Classical (scalar / 3-vector)
Galilean Invariant / Not Lorentz Invariant

$\gamma' = d\gamma/dt = \gamma^3(\mathbf{u}\cdot\mathbf{a})/c^2$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
$=$ Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM: Some Basic 4-Vectors

4-Position, 4-Velocity, 4-Acceleration, (RestMass), 4-Momentum, 4-Force SpaceTime Dynamics

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Special Relativity

$|\mathbf{v}| = |\mathbf{u}| = \{0 \leftrightarrow c\}$

$\gamma = 1/\sqrt{[1-(v/c)^2]}$

ProperTime
$\mathbf{R}\cdot\mathbf{U}/\mathbf{U}\cdot\mathbf{U}=(ct,\mathbf{r})\cdot\gamma(c,\mathbf{u})/c^2$
$=\gamma(c^2t - \mathbf{r}\cdot\mathbf{u})/c^2=(c^2t_o)/c^2$
$= t_o = \tau$

ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)$
$=\gamma(\partial_t + \mathbf{u}\cdot\nabla)=\gamma d/dt$
$= d/d\tau$

4-Gradient
$\partial=(\partial_t c,-\nabla)=\partial/\partial R_\mu$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})=d\mathbf{R}/d\tau$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

4-Acceleration
$\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})=d\mathbf{U}/d\tau$

4-Vectors:
$\mathbf{R} \in$ <Event>
$\mathbf{U} = d\mathbf{R}/d\tau$
$\mathbf{A} = d\mathbf{U}/d\tau$

$E_o/c^2 = m_o$

4-Momentum
$\mathbf{P}=(E/c,\mathbf{p})=(mc,\mathbf{p})=m_o\mathbf{U}$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

4-Force
$\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})=d\mathbf{P}/d\tau$

$\mathbf{P} = m_o\mathbf{U}$
$\mathbf{F} = d\mathbf{P}/d\tau$

This group of 4-Vectors are the main ones that are connected by the ProperTime Derivative.
$\mathbf{U}\cdot\partial = d/d\tau = \gamma d/dt = \gamma(c\partial_t/c+\mathbf{u}\cdot\nabla) = \gamma(\partial_t + \mathbf{u}\cdot\nabla)$

The classical part of it, the convective derivative, $(\partial_t + \mathbf{u}\cdot\nabla)$, is known by many different names: The convective derivative is a derivative taken with respect to a moving coordinate system. It is also called the advective derivative, derivative following the motion, hydrodynamic derivative, Lagrangian derivative, material derivative, particle derivative, substantial derivative, substantive derivative, Stokes derivative, or total derivative

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4-Velocity, 4-Momentum, $E=mc^2$

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$\gamma = 1/\sqrt{[1-(v/c)^2]}$

$\mathbf{P}\cdot\mathbf{U} = \gamma(E - \mathbf{p}\cdot\mathbf{u}) = E_o = m_oc^2$

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$ — $\mathbf{U}\cdot\mathbf{U} = (c)^2$

$E_o/c^2 = m_o$ — RestEnergy, RestMass

4-Momentum $\mathbf{P}=(E/c,\mathbf{p})=(mc,\mathbf{p})$ — $\mathbf{P}\cdot\mathbf{P} = (E_o/c)^2 = (m_oc)^2$

$\mathbf{U} = \gamma(c,\mathbf{u})$

$\mathbf{P} = (E/c,\mathbf{p}) = m_o\mathbf{U} = \gamma m_o(c,\mathbf{u}) = m(c,\mathbf{u})$

Temporal part: {energy} $E = mc^2 = \gamma m_oc^2 = \gamma E_o$

$E = m_oc^2 + (\gamma-1)m_oc^2$

$E = E_o + (\gamma-1)E_o$

(rest) + (kinetic)

Spatial part: **{momentum}** $\mathbf{p} = \gamma m_o\mathbf{u} = m\mathbf{u}$

↓ Newtonian/Classical Limit ↓

Classical Mechanics

$|\mathbf{v}| = |\mathbf{u}| << c$

4-Velocity$_{CM}$ $\mathbf{U}_{CM}=(1+(v/c)^2)(c\ \mathbf{u})$ — m_o Classical Mass — 4-Momentum$_{CM}$ $\mathbf{P}_{CM}=(E/c\ \mathbf{p})=(mc\ \mathbf{p})$

c

energy $E=(E)$

3-momentum $\mathbf{p}\rightarrow(p^x,p^y,p^z)$

$\mathbf{u} \rightarrow (u^x,u^y,u^z)$

$\mathbf{P} = (E/c,\mathbf{p}) \sim (1+(v/c)^2/2)m_o(c,\mathbf{u})$

Temporal part: {energy} $E \sim (1+(v/c)^2/2)m_oc^2 = m_oc^2 + m_ov^2/2$

$E_o + |\mathbf{p}|^2/2m_o$

(rest) + (kinetic)

Spatial part: **{momentum}** $\mathbf{p} \sim (1)m_o\mathbf{u} = m_o\mathbf{u} \rightarrow m\mathbf{u}$

Since time:space don't mix in CM,
Typically use energy E & 3-momentum **p** separately

The relativistic Gamma factor $\gamma = 1/\sqrt{[1-(v/c)^2]}$

The 1st order Newtonian Limit gives $\gamma \sim 1 + O[(v/c)^2]$

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4-Gradient
$\partial=(\partial_t/c,-\nabla)\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)=\gamma d/dt$
$= d/d\tau$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

ProperTime Derivative
$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{U}\cdot\mathbf{U}=c^2$

ProperTime Derivative
$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

4-Acceleration
$\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

$\mathbf{U}\cdot\mathbf{A}=\mathbf{U}\cdot\mathbf{U}'=0$
$\leftrightarrow \mathbf{U}\perp\mathbf{A}$

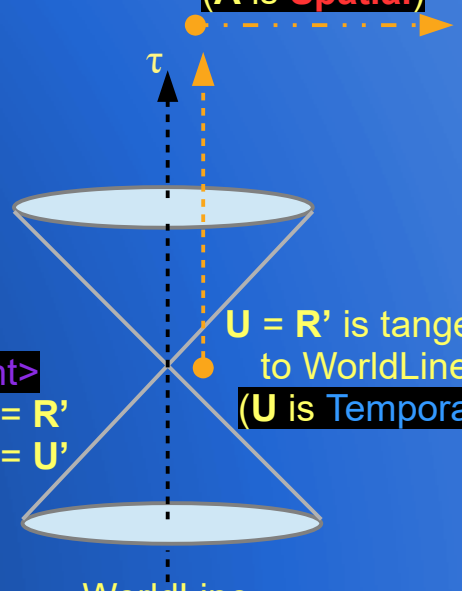
SpaceTime Orthogonality

4-Velocity **U** (a Temporal 4-Vector) is orthogonal to its own 4-Acceleration **A** = **U'** (a Spatial 4-Vector)

4-Vectors
$\mathbf{R}\in$ <Event>
$\mathbf{U}=d\mathbf{R}/d\tau=\mathbf{R}'$
$\mathbf{A}=d\mathbf{U}/d\tau=\mathbf{U}'$

A = U' = R" is normal to WorldLine (**A** is Spatial)

U = R' is tangent to WorldLine (**U** is Temporal)



R moves along Worldline

The Lorentz Scalar Product can be used to show SpaceTime orthogonality when the result is zero.

$\mathbf{U}\cdot\mathbf{U}=c^2$

$d/d\tau[\mathbf{U}\cdot\mathbf{U}]=d/d\tau[c^2]=0$

$d/d\tau[\mathbf{U}\cdot\mathbf{U}]=d/d\tau[\mathbf{U}]\cdot\mathbf{U}+\mathbf{U}\cdot d/d\tau[\mathbf{U}]=\mathbf{A}\cdot\mathbf{U}+\mathbf{U}\cdot\mathbf{A}=2(\mathbf{U}\cdot\mathbf{A})=0$

$\mathbf{U}\cdot\mathbf{A}=\mathbf{U}\cdot\mathbf{U}'=0$: The 4-Velocity is SpaceTime orthogonal to it's 4-Acceleration.

4-Velocity is the direction along a WorldLine.
4-Acceleration is the thing which causes a WorldLine to bend/curve.

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(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM: Some Basic 4-Vectors

4-Displacement, 4-Velocity, Relativity of Simultaneity

4-Vector SRQM Interpretation of QM

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$\partial\cdot\mathbf{X}=4$ SpaceTime Dimension

$\partial_\nu[X^{\mu'}]=\partial X^{\mu'}/\partial X^\nu=\Lambda^{\mu'}{}_\nu$ Lorentz Transform

$\partial[\mathbf{X}]=\eta^{\mu\nu}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$ Minkowski Metric

4-Gradient $\partial=(\partial_t/c,-\nabla)$

4-Displacement
$\mathbf{\Delta X}=(c\Delta t,\mathbf{\Delta x})$
$\mathbf{dX}=(cdt,\mathbf{dx})$

4-Position
$\mathbf{X}=(ct,\mathbf{x})$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

ProperTime Derivative

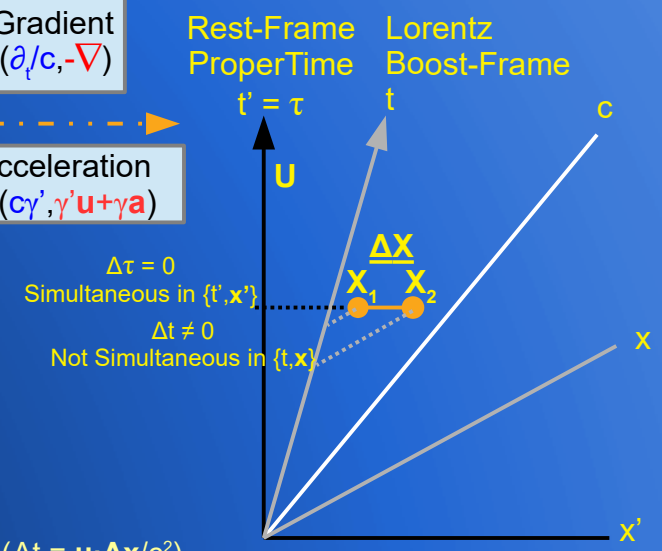
4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

ProperTime Derivative

4-Acceleration
$\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

$\mathbf{U}\cdot\mathbf{\Delta X}=\gamma(c,\mathbf{u})\cdot(c\Delta t,\mathbf{\Delta x})=\gamma(c^2\Delta t-\mathbf{u}\cdot\mathbf{\Delta x})$
$=c^2\Delta t_o=c^2\Delta\tau$



If Lorentz Scalar ($\mathbf{U}\cdot\mathbf{\Delta X}=0=c^2\Delta\tau$), then the ProperTime displacement ($\Delta\tau$) is zero, and the event separation ($\mathbf{\Delta X}=\mathbf{X}_2-\mathbf{X}_1$) is orthogonal to the worldline $\mathbf{U}$.
$\mathbf{X}_1$ and $\mathbf{X}_2$ are therefore simultaneous for the observer on this worldline $\mathbf{U}$.

Examining the equation we get $\gamma(c^2\Delta t-\mathbf{u}\cdot\mathbf{\Delta x})=0$. The coordinate time difference between the events is ($\Delta t=\mathbf{u}\cdot\mathbf{\Delta x}/c^2$)
The condition for simultaneity in an alternate frame (moving at 3-velocity $\mathbf{u}$ wrt. the worldline $\mathbf{U}$) is $\Delta t=0$, which implies ($\mathbf{u}\cdot\mathbf{\Delta x}$) = 0.

This can be met by:
($|\mathbf{u}|=0$), the alternate observer is not moving wrt. the events, i.e. is on worldline $\mathbf{U}$ or on a worldline parallel to $\mathbf{U}$.
($|\mathbf{\Delta x}|=0$), the events are at the same spatial location (co-local).
($\mathbf{u}\cdot\mathbf{\Delta x}=0$), the alternate observer's motion is perpendicular (orthogonal) to the spatial separation $\mathbf{\Delta x}$ of the events in that frame.

If none of these conditions is met, then the events will not be simultaneous in the alternate reference frame.
This is the mathematics behind the concept of Relativity of Simultaneity.

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A Tensor Study of Physical 4-Vectors

SR Diagram:
SR Motion * Lorentz Scalar = Interesting Physical 4-Vector

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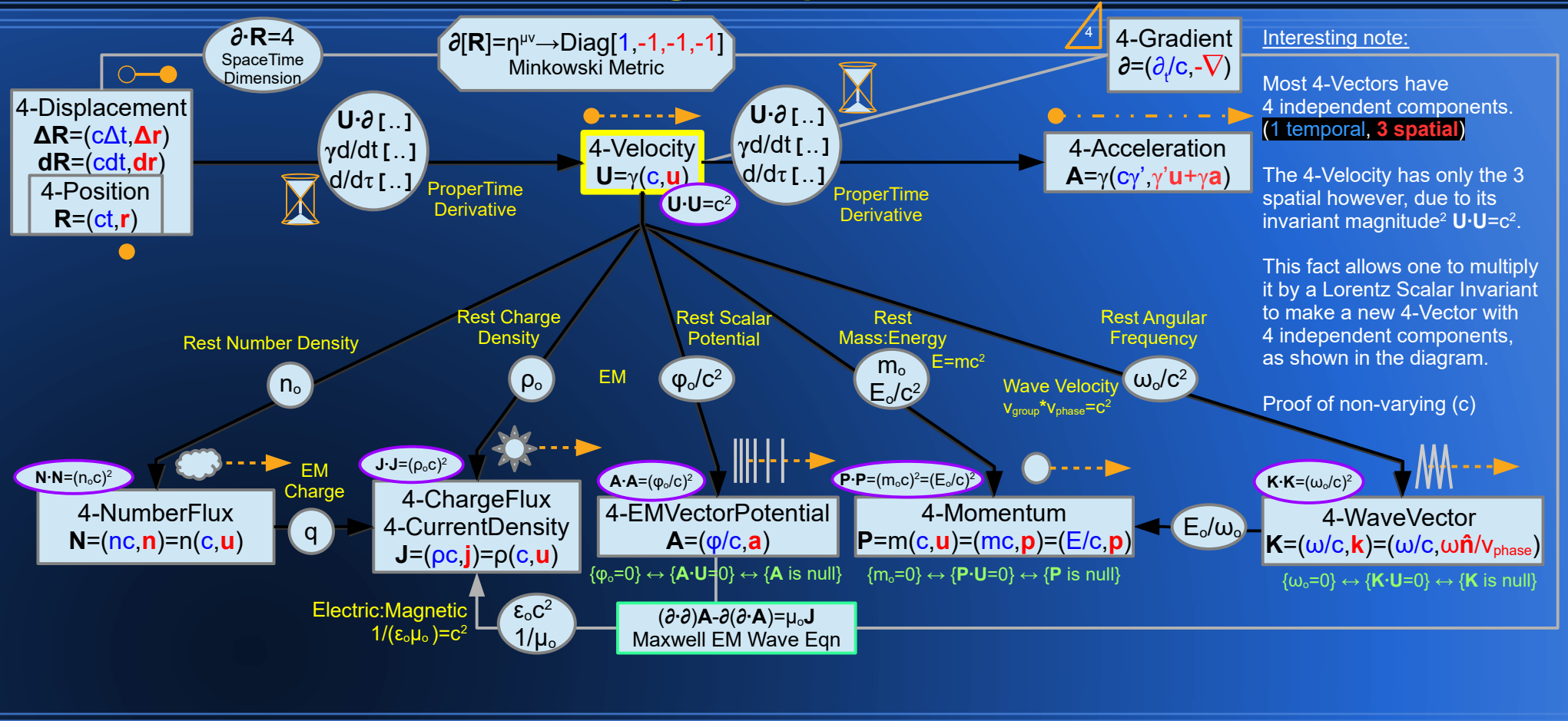

Interesting note:

Most 4-Vectors have 4 independent components. (1 temporal, **3 spatial**)

The 4-Velocity has only the 3 spatial however, due to its invariant magnitude² $\mathbf{U}\cdot\mathbf{U}=c^2$.

This fact allows one to multiply it by a Lorentz Scalar Invariant to make a new 4-Vector with 4 independent components, as shown in the diagram.

Proof of non-varying (c)

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4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

ProperTime Derivative

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{U}\cdot\mathbf{U}=c^2$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

ProperTime Derivative

4-Gradient
$\partial=(\partial_t/c,-\nabla)$

4-Acceleration
$\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

Rest Number Density
Rest Probabilty Density

n_o

Born Rule

$\rho_{prob^o}=\chi^*\psi$
$=|\langle\chi|\psi\rangle|^2$

Rest Charge Density

ρ_o

EM

Rest Scalar Potential

φ_o/c^2

Rest Mass:Energy

m_o
E_o/c^2

$E=mc^2$

Wave Velocity
$v_{group}*v_{phase}=c^2$

Rest Angular Frequency

ω_o/c^2

$\mathbf{N}\cdot\mathbf{N}=(n_oc)^2$
$\mathbf{J}_{prob}\cdot\mathbf{J}_{prob}=(\rho_{prob^o}c)^2$

4-NumberFlux
$\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$

4-ProbCurrDensity
4-ProbabilityFlux
$\mathbf{J}_{prob}=(\rho_{prob}c,\mathbf{j}_{prob})$

EM Charge

q

$\mathbf{J}\cdot\mathbf{J}=(\rho_oc)^2$

4-ChargeFlux
4-CurrentDensity
$\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

$\mathbf{A}\cdot\mathbf{A}=(\varphi_o/c)^2$

4-EMVectorPotential
$\mathbf{A}=(\varphi/c,\mathbf{a})$

$\{\varphi_o=0\}\leftrightarrow\{\mathbf{A}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{A}\text{ is null}\}$

$\mathbf{P}\cdot\mathbf{P}=(m_oc)^2=(E_o/c)^2$

4-Momentum
$\mathbf{P}=m(c,\mathbf{u})=(mc,\mathbf{p})=(E/c,\mathbf{p})$

$\{m_o=0\}\leftrightarrow\{\mathbf{P}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{P}\text{ is null}\}$

Einstein
de Broglie
$\mathbf{P}=\hbar\mathbf{K}$

E_o/ω_o
$(\hbar)$

$\mathbf{K}\cdot\mathbf{K}=(\omega_o/c)^2$
$=(m_oc/\hbar)^2$

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$

$\{\omega_o=0\}\leftrightarrow\{\mathbf{K}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{K}\text{ is null}\}$

Electric:Magnetic
$1/(\varepsilon_o\mu_o)=c^2$

ε_oc^2
$1/\mu_o$

$(\partial\cdot\partial)\mathbf{A}-\partial(\partial\cdot\mathbf{A})=\mu_o\mathbf{J}$
Maxwell EM Wave Eqn

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Existing SR Rules
Quantum Principles

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SRQM Diagram: ProperTime Derivative Very Fundamental Results

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4-Displacement $\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$ $d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position $\mathbf{R}=(ct,\mathbf{r})$

ProperTime Derivative $\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

Continuity of 4-Velocity Flow: $\partial\cdot\mathbf{U}=0$

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{U}\cdot\mathbf{U}=c^2$

ProperTime Derivative $\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

Acceleration of Event is perpendicular to Event WorldLine $\mathbf{U}\cdot\mathbf{A}=\mathbf{U}\cdot\mathbf{U}'=0$

4-Acceleration $\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

E_o/c^2 m_o

4-Momentum $\mathbf{P}=(E/c,\mathbf{p})=(mc,\mathbf{p})$

ProperTime Derivative $\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

4-Force $\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$

$\partial\cdot\mathbf{R}$ = 4: SpaceTime Dimension is 4

$d/d\tau(\partial\cdot\mathbf{R}) = d/d\tau(4) = 0$
$(\mathbf{U}\cdot\partial)(\partial\cdot\mathbf{R}) = (\mathbf{U}\cdot\partial)(4) = 0$

$d/d\tau(\partial\cdot\mathbf{R}) = d/d\tau(\partial)\cdot\mathbf{R} + \partial\cdot d/d\tau(\mathbf{R}) = 0$
$d/d\tau(\partial\cdot\mathbf{R}) = d/d\tau[\partial]\cdot\mathbf{R} + \partial\cdot\mathbf{U} = 0$
$\partial\cdot\mathbf{U} = -d/d\tau[\partial]\cdot\mathbf{R}$
$\partial\cdot\mathbf{U} = -(\mathbf{U}\cdot\partial)[\partial]\cdot\mathbf{R}$
$\partial\cdot\mathbf{U} = -(U_\nu\partial^\nu)[\partial_\mu]R^\mu$
$\partial\cdot\mathbf{U} = -U_\nu\partial^\nu\partial_\mu R^\mu$
$\partial\cdot\mathbf{U} = -U_\nu\partial_\mu\partial^\nu R^\mu$
$\partial\cdot\mathbf{U} = -U_\nu\partial_\mu\eta^{\nu\mu}$
$\partial\cdot\mathbf{U} = -U_\nu(0^\nu)$
$\partial\cdot\mathbf{U}$ = 0: Conservation of the 4-Velocity Flow (4-Velocity Flow-Field)

$\mathbf{U}\cdot\mathbf{U} = c^2$: Tensor Invariant of 4-Velocity

$d/d\tau[\mathbf{U}\cdot\mathbf{U}] = d/d\tau[c^2] = 0$
$(\mathbf{U}\cdot\partial)[\mathbf{U}\cdot\mathbf{U}] = (\mathbf{U}\cdot\partial)[c^2] = 0$

$d/d\tau[\mathbf{U}\cdot\mathbf{U}] = d/d\tau[\mathbf{U}]\cdot\mathbf{U} + \mathbf{U}\cdot d/d\tau[\mathbf{U}] = \mathbf{A}\cdot\mathbf{U} + \mathbf{U}\cdot\mathbf{A} = 2(\mathbf{U}\cdot\mathbf{A}) = 0$
$\mathbf{U}\cdot\mathbf{A} = \mathbf{U}\cdot\mathbf{U}' = 0$: The 4-Velocity is SpaceTime orthogonal to it's 4-Acceleration.

4-Velocity is the direction of an Event along a WorldLine.
4-Acceleration of an Event is the thing which causes a WorldLine to bend.

4-Vectors:
$\mathbf{R} \in$ <Event>
$\mathbf{U} = d\mathbf{R}/d\tau$
$\mathbf{A} = d\mathbf{U}/d\tau$

$\mathbf{P} = m_o\mathbf{U}$
$\mathbf{F} = d\mathbf{P}/d\tau$

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(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Local Continuity of 4-Velocity leads to all of the Conservation Laws

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\eta^{\mu\nu}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$ Minkowski Metric

4 4-Gradient $\partial=(\partial_t/c,-\nabla)$

ProperTime Derivative $\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$ $=\gamma d/dt=d/d\tau$

4-Displacement $\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$ $d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position $\mathbf{R}=(ct,\mathbf{r})$

ProperTime Derivative $\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

Continuity of 4-Velocity Flow: $\partial\cdot\mathbf{U}=0$

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$ ProperTime Derivative

4-Acceleration $\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

$\partial\cdot\mathbf{R} = 4$
$d/d\tau(\partial\cdot\mathbf{R}) = d/d\tau(4) = 0$

$d/d\tau(\partial\cdot\mathbf{R}) = d/d\tau(\partial)\cdot\mathbf{R} + \partial\cdot d/d\tau(\mathbf{R}) = 0$
$d/d\tau(\partial\cdot\mathbf{R}) = d/d\tau[\partial]\cdot\mathbf{R} + \partial\cdot\mathbf{U} = 0$
$\partial\cdot\mathbf{U} = -d/d\tau[\partial]\cdot\mathbf{R}$
$\partial\cdot\mathbf{U} = -(\mathbf{U}\cdot\partial)[\partial]\cdot\mathbf{R}$
$\partial\cdot\mathbf{U} = -(U_\nu\partial^\nu)[\partial_\mu]R^\mu$
$\partial\cdot\mathbf{U} = -U_\nu\partial^\nu\partial_\mu R^\mu$
$\partial\cdot\mathbf{U} = -U_\nu\partial_\mu\partial^\nu R^\mu$: I believe this is legit, partials commute
$\partial\cdot\mathbf{U} = -U_\nu\partial_\mu\eta^{\nu\mu}$
$\partial\cdot\mathbf{U} = -U_\nu(0^\nu)$
$\partial\cdot\mathbf{U} = 0$
Conservation of the 4-Velocity Flow
(4-Velocity Flow-Field)

$\partial\cdot\mathbf{U} = 0$
$\partial\cdot(\text{Lorentz Scalar})\mathbf{U} = 0(\text{Lorentz Scalar})$
$\partial\cdot(\text{Lorentz Scalar})\mathbf{U} = 0$
$\partial\cdot(\textbf{Interesting 4-Vector}) = 0$

Example:
$\partial\cdot(\rho_o)\mathbf{U} = 0$
$\partial\cdot\mathbf{J} = 0$
$(\partial_t/c\ \rho c + \nabla\cdot\mathbf{j}) = 0$
$(\partial_t\rho + \nabla\cdot\mathbf{j}) = 0$
= Conservation of Charge
= A Continuity Equation

Conservation Laws:

All of the Physical Conservation Laws are in the form of a 4-Divergence, which is a Lorentz Invariant Scalar equation.

These are local continuity equations which basically say that the temporal change in a quantity is balanced by the flow of that quantity into or out of a local spatial region.

Conservation of Charge:
$\partial\cdot\mathbf{J} = (\partial_t\rho + \nabla\cdot\mathbf{j}) = 0$

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A Tensor Study of Physical 4-Vectors

SRQM Diagram:
SRQM Motion * Lorentz Scalar Conservation Laws, Continuity Eqns

4-Vector SRQM Interpretation of QM

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4-Displacement $\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$ $d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position $\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$ ProperTime Derivative

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$ $\mathbf{U}\cdot\mathbf{U}=c^2$

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$ ProperTime Derivative

Continuity of 4-Velocity Flow: $\partial\cdot\mathbf{U}=0$

4-Acceleration $\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

Rest Number Density
Rest Probabilty Density

n_o $\rho_{prob^o}=\chi^*\psi=|\langle\chi|\psi\rangle|^2$

Born Rule

$\mathbf{N}\cdot\mathbf{N}=(n_oc)^2$ $\mathbf{J}_{prob}\cdot\mathbf{J}_{prob}=(\rho_{prob^o}c)^2$

Rest Charge Density ρ_o

EM

Rest Scalar Potential φ_o/c^2

Rest Mass:Energy m_o E_o/c^2 $E=mc^2$

Wave Velocity $v_{group}*v_{phase}=c^2$

Rest Angular Frequency ω_o/c^2

EM Charge q

$\mathbf{J}\cdot\mathbf{J}=(\rho_oc)^2$

4-ChargeFlux
4-CurrentDensity
$\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

4-NumberFlux
$\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$
4-ProbCurrDensity
4-ProbabilityFlux
$\mathbf{J}_{prob}=(\rho_{prob}c,\mathbf{j}_{prob})$

m_o Rest Mass:Energy

$\mathbf{G}\cdot\mathbf{G}=(\rho_mc)^2$

4-MassFlux
4-MomentumDensity
$\mathbf{G}=(\rho_mc,\mathbf{g})=\rho_m(c,\mathbf{u})$

$\mathbf{A}\cdot\mathbf{A}=(\varphi_o/c)^2$

4-EMVectorPotential
$\mathbf{A}=(\varphi/c,\mathbf{a})$

$\{\varphi_o=0\}\leftrightarrow\{\mathbf{A}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{A}$ is null$\}$

$\mathbf{P}\cdot\mathbf{P}=(m_oc)^2=(E_o/c)^2$

4-Momentum
$\mathbf{P}=m(c,\mathbf{u})=(mc,\mathbf{p})=(E/c,\mathbf{p})$

$\{m_o=0\}\leftrightarrow\{\mathbf{P}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{P}$ is null$\}$

Einstein de Broglie $\mathbf{P}=\hbar\mathbf{K}$

E_o/ω_o $(\hbar)$

$\mathbf{K}\cdot\mathbf{K}=(\omega_o/c)^2=(m_oc/\hbar)^2$

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$

$\{\omega_o=0\}\leftrightarrow\{\mathbf{K}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{K}$ is null$\}$

Conservation of Particle #: $\partial\cdot\mathbf{N}=0$
Probability: $\partial\cdot\mathbf{J}_{prob}=0$

Conservation of Mass: $\partial\cdot\mathbf{G}=0$

Conservation of Charge: $\partial\cdot\mathbf{J}=0$

Lorenz Gauge Conservation of EM Potential: $\partial\cdot\mathbf{A}=0$

Conservation of 4-Momentum: $\partial\cdot\mathbf{P}=0$

Conservation of 4-WaveVector: $\partial\cdot\mathbf{K}=0$

These are Fluid or Density -type Conservation/Continuity Laws

These are Individual Particle/Wave/Delta-function Conservation/Continuity Laws

Conservation Laws:

All of the Physical Conservation Laws are in the form of a 4-Divergence, which is a Lorentz Invariant Scalar equation.

These are local continuity equations which basically say that the temporal change in a quantity is balanced by the flow of that quantity into or out of a local spatial region.

Conservation of Charge:
$\partial\cdot\mathbf{J}=(\partial_t\rho+\nabla\cdot\mathbf{j})=0$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
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SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu}=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
Quantum Principles

$\mathrm{Trace}[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^{\mu}{}_{\mu}=T$
$\mathbf{V}\cdot\mathbf{V}=V^{\mu}\eta_{\mu\nu}V^{\nu}=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM: Some Basic 4-Vectors
4-Velocity, 4-Gradient, Time Dilation

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John B. Wilson
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at-rest
worldline $\mathbf{U}_o$
(u=0)
fully temporal

const inertial motion
worldline **U**
(0<u<c)
trades some time for space

4-Gradient
$\partial=(\partial_t/c,-\nabla)$

ProperTime
$\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$
Derivative

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

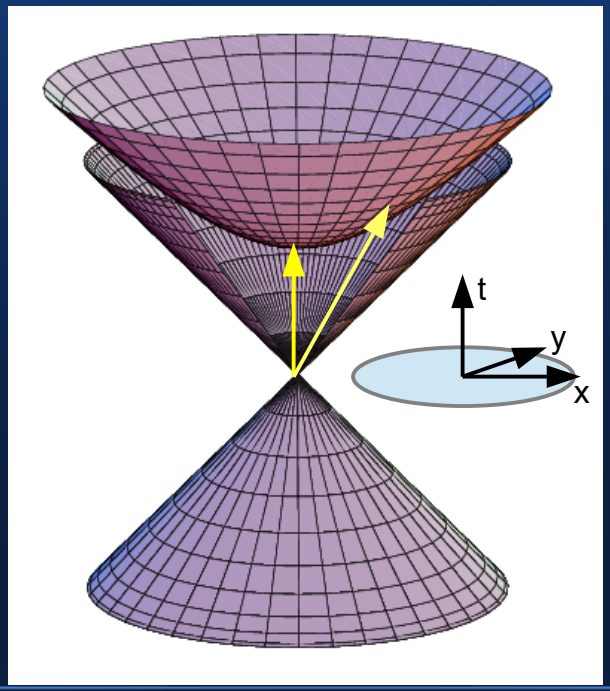
$\mathbf{U}\cdot\mathbf{U} = \gamma(c,\mathbf{u})\cdot\gamma(c,\mathbf{u}) = \gamma^2(c^2-\mathbf{u}\cdot\mathbf{u}) = (c^2)$

$\gamma = 1/\sqrt{[1-(u/c)^2]} = 1/\sqrt{[1-\beta^2]}$

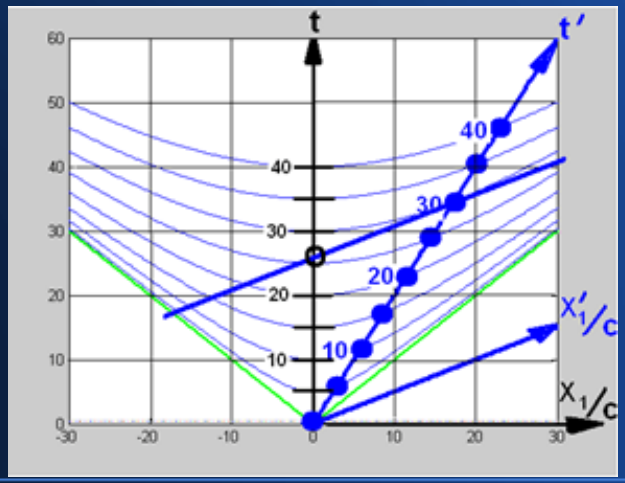
ProperTime
$d\tau=(1/\gamma)dt$
Differential

4-Velocity$_{(at\text{-}rest)}$
$\mathbf{U}_o=(c,\mathbf{0})$

Everything moves into future (+t)
at the speed-of-light (c)
in its own spatial rest-frame



The Minkowski Diagram provides
a great visual representation
of SpaceTime



Since the SpaceTime magnitude of **U** is a constant, changes in the components of **U** are like "rotating" the 4-Vector without changing its length. However, as **U** gains some spatial velocity, it loses some "relative" temporal velocity. Objects that move in some reference frame "age" more slowly relative to those at rest in the same reference frame.

Time Dilation!

$$\Delta t = \gamma\Delta\tau = \gamma\Delta t_o$$
$$dt = \gamma d\tau$$
$$d/d\tau = \gamma d/dt$$

Each observer will see the other as aging more slowly; similarly to two people moving oppositely along a train track, seeing the other as appearing smaller in the distance.

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$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM: Some Basic 4-Vectors

Lorentz Invariant d'Alembertian (∂·∂)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

The d'Alembertian { $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla$ } is a 4D Lorentz Scalar Invariant

It is used as the basis of many "wave-type" equations.

$(\partial\cdot\partial)\varphi[\mathbf{X}] = (\partial\cdot\partial)\varphi[(t,\mathbf{x})] = 0$ is the standard relativistic wave equation
$(\partial\cdot\partial)\mathbf{A}[\mathbf{X}] = (\partial\cdot\partial)\mathbf{A}[(t,\mathbf{x})] = \mathbf{0}$ is the Maxwell EM Wave equation In Lorenz Gauge $(\partial\cdot\mathbf{A})=0$
$(\partial\cdot\partial)\mathbf{A}[\mathbf{X}] = (\partial\cdot\partial)\mathbf{A}[(t,\mathbf{x})] = \mu_o\mathbf{J}$ is the Maxwell EM Wave equation In Lorenz Gauge $(\partial\cdot\mathbf{A})=0$ with a Source term $\mathbf{J}$

$(\partial\cdot\partial)\varphi[\mathbf{X}] = -(m_o c/\hbar)^2\varphi[\mathbf{X}]$ is the standard relativistic quantum Klein-Gordon equation

$(\partial\cdot\partial)G[\mathbf{X}-\mathbf{X'}] = \delta^{(4)}[\mathbf{X}-\mathbf{X'}]$: $G[\mathbf{X}-\mathbf{X'}]$ is a 4D Green's Function, $\delta^{(4)}[\mathbf{X}-\mathbf{X'}]$ is a 4D Dirac Delta function

$\delta^{(4)}[\mathbf{X}-\mathbf{X'}] = 1/(2\pi)^4 \int d^4\mathbf{K}\, e^{-i(\mathbf{K}\cdot(\mathbf{X}-\mathbf{X'}))} = \delta[ct-ct']\,\delta^{(3)}[x-x'] = \delta[ct-ct']\,\delta[x-x']\,\delta[y-y']\,\delta[z-z']$
$\int_{[\text{Some 4D Volume}]} \delta^{(4)}[X - X']\, d^4\mathbf{X} = \{1 \text{ if } \mathbf{X'} \text{ in the 4D Volume, 0 otherwise}\}$

4-Gradient
$\partial = (\partial_t/c, -\nabla)$

d'Alembertian
$\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla$
$= (\partial/c\partial t_o)^2 = (\partial/c\partial\tau)^2$
$= (\partial_{to}/c)^2 = (\partial_\tau/c)^2$

The Covariant 4D versions of the Green's Function and the Dirac Delta Function.
Given a linear ordinary differential equation (ODE), L(solution) = source, one can first solve L(green) = δ[s], for each s, and realizing that, since the source is a sum of delta functions, the solution is a sum of Green's functions as well, by linearity of L.

4D Green's Function
$G[\mathbf{X}-\mathbf{X'}]$

4D Dirac Delta
$\delta^{(4)}[\mathbf{X}-\mathbf{X'}]$

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= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM: Some Basic 4-Vectors

SR 4-WaveVector K

Solution to d'Alembertian (∂·∂)

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
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4-WaveVector, aka. Wave 4-Vector: {solution of d'Alembertian Wave Eqn. ∂·∂}

$\mathbf{K} = (\omega_o/c^2)\mathbf{U} = (\omega/c,\mathbf{k}) = (\omega/c,\omega\hat{\mathbf{n}}/v_{phase}) = (\omega/c,\omega\mathbf{u}/c^2) = (\omega/c^2)(c,\mathbf{u}) = (\omega/c)(1,\boldsymbol{\beta}) = (1/c\mathrm{T},\hat{\mathbf{n}}/\lambda\!\!\!^{-}) = -\partial[\Phi_{phase,plane}]$

There are multiple ways of writing out the components of the 4-WaveVector, with each one giving an interesting take on what the 4-WaveVector means.

An SR wave Ψ is actually composed of two tensors:
(1) 4-Vector propagation part = K^α (the engine), in $e^{\wedge}(-iK^\alpha X_\alpha)$
(2) Variable amplitude part = A (the load), depends on what is waving...

4-Scalar A: $\Psi = A\, e^{\wedge}(-iK^\alpha X_\alpha)$
ex. KG Quantum Wave

4-Vector A^μ: $\Psi^\mu = A^\mu\, e^{\wedge}(-iK^\alpha X_\alpha)$
ex. Maxwell Photon Wave

4-Tensor $A^{\mu\nu}$: $\Psi^{\mu\nu} = A^{\mu\nu}\, e^{\wedge}(-iK^\alpha X_\alpha)$
ex. Gravitational Wave Approx.

The Ψ tensor-type will match the A tensor-type, as the propagation part $e^{\wedge}(-iK^\alpha X_\alpha)$ is overall dimensionless.

One comparison I find very interesting is:
$\mathbf{R}\cdot\mathbf{R} = (ct_o)^2 = (c\tau)^2$
$\mathbf{K}\cdot\mathbf{K} = (1/c\mathrm{T}_o)^2$
$\partial\cdot\partial = (\partial/c\partial t_o)^2 = (\partial/c\partial\tau)^2$

$\psi_n(\mathbf{X}) = A_n\, e^{\wedge}-i(\mathbf{K}_n\cdot\mathbf{X})$: Explicit form of an SR plane wave
$\psi(\mathbf{X}) = \Sigma_n[\ \psi_n(\mathbf{X})\ ]$: Complete wave is a superposition of multiple plane waves.
$\partial[\ \psi(\mathbf{X})\ ] = \partial[\ Ae^{\wedge}-i(\mathbf{K}\cdot\mathbf{X})\ ] = -i\mathbf{K}\ [\ Ae^{\wedge}-i(\mathbf{K}\cdot\mathbf{X})\ ] = -i\mathbf{K}[\ \psi(\mathbf{X})\ ]$
$\partial = -i\mathbf{K}$ as the condition for a complex-valued plane wave.
$\partial\cdot\partial[\ \psi(\mathbf{X})\ ] = (-i)(-i)(\mathbf{K}\cdot\mathbf{K})[\ \psi(\mathbf{X})\ ] = -(\mathbf{K}\cdot\mathbf{K})[\ \psi(\mathbf{X})\ ]$

Invariant Phase
$\mathbf{K}\cdot\mathbf{R}$
$= (\omega/c,\mathbf{k})\cdot(ct,\mathbf{r})$
$=(\omega t - \mathbf{k}\cdot\mathbf{r})$
$=(t/\mathrm{T} - \hat{\mathbf{n}}\cdot\mathbf{r}/\lambda\!\!\!^{-})$
$= -\Phi_{phase,plane}$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}$
$=(c\tau)^2$
$=(ct_o)^2$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

4-Velocity $\mathbf{U}$
$=\gamma(c,\mathbf{u})$
$=\gamma c(1,\boldsymbol{\beta})$

$\mathbf{U}\cdot\mathbf{U}$
$=c^2$
$=\lambda\!\!\!^{-2}(\omega^2-\omega_o^2)$
$= \lambda\!\!\!^{-2}\omega^2_{(for\ photon)}$
$= \lambda^2\nu^2_{(for\ photon)}$
$=|v_{phase}{}^*v_{group}|$
$= \lambda\!\!\!^{-}_C{}^2\omega_o^2$

ω_o/c^2

$\mathbf{K}\cdot\mathbf{U} = \gamma(\omega - \mathbf{k}\cdot\mathbf{u}) = \omega_o$
RestAngularFrequency

4-WaveVector $\mathbf{K}$
$=(\omega/c,\mathbf{k})$
$=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$
$=(\omega/c,\omega\mathbf{u}/c^2)$
$=(\omega/c^2)(c,\mathbf{u})$
$=(\omega/c)(1,\boldsymbol{\beta})$
$=(1/c\mathrm{T},\hat{\mathbf{n}}/\lambda\!\!\!^{-})$
$\mathbf{K}=-\partial[\Phi_{phase,plane}]$

$\mathbf{K}\cdot\mathbf{K}$
$=(\omega/c)^2-\mathbf{k}\cdot\mathbf{k}$
$=(\omega_o/c)^2$
$=(1/c\mathrm{T}_o)^2$
$=(1/\lambda\!\!\!^{-}_C)^2$

$..[\mathbf{K}_T\cdot\mathbf{R}]$
$..\int[\mathbf{K}_T\cdot d\mathbf{R}]$
$..[-\Phi_{phase}]$

4-Gradient
$\partial=(\partial_t/c,-\nabla)$

d'Alembertian
$\partial\cdot\partial = (\partial_t/c)^2-\nabla\cdot\nabla$
$= (\partial/c\partial t_o)^2 = (\partial/c\partial\tau)^2$
$= (\partial_{to}/c)^2 = (\partial_\tau/c)^2$

I believe the last one is correct: $(\partial\cdot\partial)[\mathbf{R}] = \mathbf{0} = (\partial/c\partial\tau)^2[\mathbf{R}] = \mathbf{A}_o/c^2 = \mathbf{0}$: The 4-Acceleration seen in the ProperTime Frame = RestFrame = **0**
Normally $(d/d\tau)^2[\mathbf{R}] = \mathbf{A}$, which could be non-zero. But that is for the total derivative, not the partial derivative.

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4-Velocity, 4-WaveVector
Wave Properties, Relativistic Doppler Effect

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4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$\mathbf{U}\cdot\mathbf{U} = (c)^2$

ω_o/c^2
RestAngularFrequency

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$
$\mathbf{K}\cdot\mathbf{K} = (\omega_o/c)^2$

$\mathbf{K} = (\omega/c,\mathbf{k}) = (\omega/c,\omega\hat{\mathbf{n}}/v_{phase}) = (\omega_o/c^2)\mathbf{U}$
$= (\omega_o/c^2)\gamma(c,\mathbf{u}) = (\omega/c^2)(c,\mathbf{u}) = (\omega/c,(\omega/c^2)\mathbf{u})$

$(\omega/c,\omega\hat{\mathbf{n}}/v_{phase}) = (\omega/c,(\omega/c^2)\mathbf{u})$
Taking just the spatial components of the 4-WaveVector:
$\omega\mathbf{n}/v_{phase} = (\omega/c^2)\mathbf{u}$
$\hat{\mathbf{n}}/v_{phase} = (\mathbf{u}/c^2)$

$u * v_{phase} = c^2$
$v_{group} * v_{phase} = c^2$, with $u = v_{group}$

Wave Group velocity (v_{group}) is mathematically the same as Particle velocity (u).
Wave Phase velocity (v_{phase}) is the speed of an individual plane-wave, also the speed of signal synchronicity, the speed of the wave of coordinated flashes.

Relativistic SR Doppler Effect

($\hat{\mathbf{n}}$) here is the unit-directional 3-vector of the photon

Choose an observer frame for which:
$\mathbf{K} = (\omega/c,\mathbf{k})$, with $\mathbf{k},\hat{\mathbf{n}}$ pointing toward observer
$\mathbf{U}_{obs} = (c,\mathbf{0})$ $\quad \mathbf{K}\cdot\mathbf{U}_{obs} = (\omega/c,\mathbf{k})\cdot(c,\mathbf{0}) = \omega = \omega_{obs^o}$
$\mathbf{U}_{emit} = \gamma(c,\mathbf{u})$ $\quad \mathbf{K}\cdot\mathbf{U}_{emit} = (\omega/c,\mathbf{k})\cdot\gamma(c,\mathbf{u}) = \gamma(\omega - \mathbf{k}\cdot\mathbf{u}) = \omega_{emit^o}$

$\mathbf{K}\cdot\mathbf{U}_{obs} / \mathbf{K}\cdot\mathbf{U}_{emit} = \omega_{obs^o}/\omega_{emit^o} = \omega/[\gamma(\omega - \mathbf{k}\cdot\mathbf{u})]$
For photons, $\mathbf{K}$ is null → $\mathbf{K}\cdot\mathbf{K} = 0$ → $\mathbf{k} = (\omega/c)\hat{\mathbf{n}}$
$\omega_{obs^o}/\omega_{emit^o} = \omega/[\gamma(\omega - (\omega/c)\hat{\mathbf{n}}\cdot\mathbf{u})] = 1/[\gamma(1 - \hat{\mathbf{n}}\cdot\boldsymbol{\beta})] = 1/[\gamma(1 - |\boldsymbol{\beta}|\cos[\theta_{obs}])]$
$\omega_{obs}/\omega_{emit} = \gamma\omega_{obs^o}/(\gamma\omega_{emit^o}) = \omega_{obs^o}/\omega_{emit^o}$

$\omega_{obs} = \omega_{emit}/[\gamma(1 - \hat{\mathbf{n}}\cdot\boldsymbol{\beta})] = \omega_{emit}*\sqrt{[1+|\beta|]}*\sqrt{[1-|\beta|]}/(1 - \hat{\mathbf{n}}\cdot\boldsymbol{\beta})$
with $\gamma = 1/\sqrt{[1-\beta^2]} = 1/(\sqrt{[1+|\beta|]}*\sqrt{[1-|\beta|]})$

For motion of emitter $\boldsymbol{\beta}$: (in observer frame of reference)
Away from obs, $(\hat{\mathbf{n}}\cdot\boldsymbol{\beta}) = -\beta$, $\omega_{obs} = \omega_{emit}*\sqrt{[1-|\beta|]}/\sqrt{(1 + |\beta|)}$ = Red Shift
Toward obs, $(\hat{\mathbf{n}}\cdot\boldsymbol{\beta}) = +\beta$, $\omega_{obs} = \omega_{emit}*\sqrt{[1+|\beta|]}/\sqrt{(1 - |\beta|)}$ = Blue Shift
Transverse, $(\hat{\mathbf{n}}\cdot\boldsymbol{\beta}) = 0$, $\omega_{obs} = \omega_{emit}/\gamma$ = Transverse Doppler Shift

The Phase Velocity of a Photon {$v_{phase} = c$} equals the Particle Velocity of a Photon {$u = c$}
The Phase Velocity of a Massive Particle {$v_{phase} > c$} is greater than the Velocity of a Massive Particle {$u < c$}

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SR 4-CoVector:OneForm
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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM: Some Basic 4-Vectors
4-Velocity, 4-WaveVector
Wave Properties, Relativistic Aberration

4-Vector SRQM Interpretation of QM

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$\mathbf{K}\cdot\mathbf{U} = \gamma(\omega - \mathbf{k}\cdot\mathbf{u}) = \omega_o$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$\mathbf{U}\cdot\mathbf{U} = (c)^2$

ω_o/c^2
RestAngularFrequency

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$
$\mathbf{K}\cdot\mathbf{K} = (\omega_o/c)^2$

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$\omega_{obs} = \omega_{emit}/[\gamma(1 - \hat{\mathbf{n}}\cdot\boldsymbol{\beta})] = \omega_{emit}/[\gamma(1 - |\boldsymbol{\beta}|\cos[\theta_{obs}])]$

Change reference frames with {obs→emit} &{ $\boldsymbol{\beta} \rightarrow -\boldsymbol{\beta}$ }

$\omega_{emit} = \omega_{obs}/[\gamma(1 + \hat{\mathbf{n}}\cdot\boldsymbol{\beta})] = \omega_{obs}/[\gamma(1 + |\boldsymbol{\beta}|\cos[\theta_{emit}])]$

$(\omega_{obs})*(\omega_{emit}) =(\omega_{emit}/[\gamma(1 - |\boldsymbol{\beta}|\cos[\theta_{obs}])])*(\omega_{obs}/[\gamma(1 + |\boldsymbol{\beta}|\cos[\theta_{emit}])])$

$1 = (1/[\gamma(1 - |\boldsymbol{\beta}|\cos[\theta_{obs}])])*(1/[\gamma(1 + |\boldsymbol{\beta}|\cos[\theta_{emit}])])$
$1 = (\gamma(1 - |\boldsymbol{\beta}|\cos[\theta_{obs}]))*(\gamma(1 + |\boldsymbol{\beta}|\cos[\theta_{emit}]))$
$1 = \gamma^2(1 - |\boldsymbol{\beta}|\cos[\theta_{obs}])*(1 + |\boldsymbol{\beta}|\cos[\theta_{emit}])$

Solve for $|\boldsymbol{\beta}|\cos[\theta_{obs}]$ and use $\{(\gamma^2-1) = \beta^{2\gamma2}\}$

$\cos[\theta_{obs}] = (\cos[\theta_{emit}] + |\boldsymbol{\beta}|) / (1 + |\boldsymbol{\beta}|\cos[\theta_{emit}])$

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$\mathbf{P}\cdot\mathbf{P} = (m_o c)^2 = (E_o/c)^2$

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$
$\mathbf{P}=-\partial[S_{action,free}]$

$\int \mathbf{P}\cdot d\mathbf{R} = -S_{action,free}$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

Rest Mass:Energy Einstein
$E=\gamma E_o=\gamma m_o c^2=mc^2$

m_o
E_o/c^2

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{U}\cdot\mathbf{U} = (c)^2$

$\mathbf{P}\cdot\mathbf{U} = E_o$

Einstein
de Broglie
$\mathbf{P} = \hbar\mathbf{K}$

$\omega_o/E_o = (1/\hbar)$
or
$E_o/\omega_o = (\hbar)$

$\mathbf{K}\cdot\mathbf{U} = \omega_o$

ω_o/c^2

Wave Velocity
$v_{group} * v_{phase} = c^2$
RestAngFrequency

$\int \mathbf{K}\cdot d\mathbf{R} = -\Phi_{phase,plane}$

$\mathbf{P}\cdot\mathbf{K} = m_o\omega_o$

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$
$\mathbf{K}=-\partial[\Phi_{phase,plane}]$

$\mathbf{K}\cdot\mathbf{K} = (\omega_o/c)^2$

$..[-S_{action,free}]$

Hamilton-Jacobi
$\mathbf{P} = -\partial[S_{action,free}]$

$..[-\Phi_{phase,plane}]$

$\mathbf{K} = -\partial[\Phi_{phase,plane}]$
WaveVector Gradient

4-Gradient
$\partial=(\partial_t/c,-\nabla)\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

d'Alembertian
$\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = (\partial_\tau/c)^2$

SpaceTime
$\partial\cdot\mathbf{R}=4$
Dimension

$\partial[\mathbf{R}]=\eta^{\mu\nu}\rightarrow \mathrm{Diag}[1,-1,-1,-1]$
Minkowski Metric

ProperTime
$\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$
Derivative

Treating motion like a particle
Moving particles have a 4-Velocity
4-Momentum is the negative 4-Gradient of the SR Action (S)

Treating motion like a wave
Moving waves have a 4-Velocity
4-WaveVector is the negative 4-Gradient of the SR Phase (Φ)

See Hamilton-Jacobi Formulation of Mechanics
for info on the Lorentz Scalar Invariant SR Action.
{ $\mathbf{P} = (E/c,\mathbf{p}) = -\partial[S] = (-\partial/c\partial t[S],\nabla[S])$ }
{temporal component} $E = -\partial/\partial t[S] = -\partial_t[S]$
{spatial component} $\mathbf{p} = \nabla[S]$
Note This is the Action (S_{action}) for a free particle.
Generally Action is for the 4-TotalMomentum $\mathbf{P}_T$ of a system.

See SR Wave Definition
for info on the Lorentz Scalar Invariant SR WavePhase.
{ $\mathbf{K} = (\omega/c,\mathbf{k}) = -\partial[\Phi] = (-\partial/c\partial t[\Phi],\nabla[\Phi])$ }
{temporal component} $\omega = -\partial/\partial t[\Phi] = -\partial_t[\Phi]$
{spatial component} $\mathbf{k} = \nabla[\Phi]$
Note This is the Phase (Φ) for a single free plane-wave.
Generally WavePhase is for the 4-TotalWaveVector $\mathbf{K}_T$ of a system.

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A Tensor Study of Physical 4-Vectors

Some Cool Minkowski Metric Tensor Tricks

4-Gradient, 4-Position, 4-Velocity SpaceTime is 4D

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ProperTime Derivative
$\mathbf{U}\cdot\partial=\gamma(c,\mathbf{u})\cdot(\partial_t/c,-\nabla)=\gamma(\partial_t+\mathbf{u}\cdot\nabla)$
$= d/d\tau = \gamma d/dt$

4-Gradient
$\partial=(\partial_t/c,-\nabla)\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

$\partial[\mathbf{R}] = \eta^{\mu\nu}$
$\rightarrow$Diag[1,-1,-1,-1]
Minkowski Metric

$\partial\cdot\mathbf{R}=4$
SpaceTime Dimension

$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^{\mu}{}_{\nu}\Lambda_{\mu}{}^{\nu}$

$Tr[\eta^{\mu\nu}]=\eta_{\mu}{}^{\mu}=4$
SpaceTime Dimension

=

$\partial[\mathbf{R}] = \eta^{\mu\nu}$
$\rightarrow$Diag[1,-1,-1,-1]
Index-Raised
Minkowski Metric

Index Raise

$\mathbf{U}\cdot\partial$ [..]
$\gamma d/dt$ [..]
$d/d\tau$ [..]

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\eta_{\mu}{}^{\nu}$
$\rightarrow$Diag[1,1,1,1]
Index-Mixed
Minkowski Metric

=

$\delta_{\mu}{}^{\nu}$
=Diag[1,1,1,1]
Kronecker Delta

$\{\eta^{\mu\mu}\}$
=
$1/\{\eta_{\mu\mu}\}$

Index Lower

$\eta_{\mu\nu}$
$\rightarrow$Diag[1,-1,-1,-1]
Index-Lowered
Minkowski Metric

$\eta_{\alpha\beta}(\eta^{\gamma\beta}) = \eta_{\alpha}{}^{\gamma}$ = Diag[1,-1,-1,-1]*Diag[1,-1,-1,-1] = Diag[1,1,1,1]
thus
Single Index-Lowering the Minkowski Metric ($\eta^{\gamma\beta}$) gives the Kronecker Delta

$(\partial\cdot\mathbf{R}) = (\partial^{\alpha}\cdot R^{\beta}) = (\partial^{\alpha}\eta_{\alpha\beta}R^{\beta}) = \eta_{\alpha\beta}(\partial^{\alpha}R^{\beta}) = \eta_{\alpha\beta}(\eta^{\alpha\beta}) = \eta_{\alpha}{}^{\alpha} = \delta_{\alpha}{}^{\alpha} = 4$
Trace[Minkowski Metric] = $Tr[\eta^{\alpha\beta}] = \eta_{\alpha\beta}[\eta^{\alpha\beta}] = \eta_{\alpha}{}^{\alpha} = \delta_{\alpha}{}^{\alpha} = 4$
thus
The Divergence of 4-Position ($\partial\cdot\mathbf{R}$) = "Magnitude" of the Minkowski Metric $Tr[\eta^{\alpha\beta}]$ = the Dimension of SpaceTime (4)

$(\mathbf{U}\cdot\partial)[\mathbf{R}] = (U^{\alpha}\cdot\partial^{\beta})[R^{\gamma}] = (U^{\alpha}\eta_{\alpha\beta}\partial^{\beta})[R^{\gamma}] = (U_{\beta}\partial^{\beta})[R^{\gamma}] = (U_{\beta})\partial^{\beta}[R^{\gamma}] = (U_{\beta})\eta^{\beta\gamma} = U^{\gamma} = \mathbf{U} = (d/d\tau)[\mathbf{R}]$
thus
Lorentz Scalar Product ($\mathbf{U}\cdot\partial$) = Derivative wrt. ProperTime ($d/d\tau$) = Relativistic Factor * Derivative wrt. CoordinateTime $\gamma(d/dt)$:

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SRQM+EM Diagram: 4-Vectors

4-Vector SRQM Interpretation of QM

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4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

4-Acceleration
$\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

4-Polarization:Spin
$\mathbf{E}=(\varepsilon^0,\boldsymbol{\varepsilon})=(\boldsymbol{\varepsilon}\cdot\boldsymbol{\beta},\boldsymbol{\varepsilon})$
$\mathbf{S}=(s^0,\mathbf{s})=(\mathbf{s}\cdot\boldsymbol{\beta},\mathbf{s})$
Complex

4-Gradient
$\partial=(\partial_t/c,-\nabla)$

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})$

4-TotalWaveVector
$\mathbf{K}_T=(\omega_T/c,\mathbf{k}_T)$

4-UnitTemporal
$\bar{\mathbf{T}}=\gamma(1,\boldsymbol{\beta})$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

4-Force
$\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$

4-TotalMomentum
$\mathbf{P}_T=(E_T/c,\mathbf{p}_T)=(H/c,\mathbf{p}_T)$

4-UnitSpatial
$\bar{\mathbf{S}}=\gamma_{\beta n}(\hat{\mathbf{n}}\cdot\boldsymbol{\beta},\hat{\mathbf{n}})_\perp$

4-MassFlux
4-MomentumDensity
$\mathbf{G}=(\rho_m c,\mathbf{g})=(\rho_e/c,\mathbf{g})$

4-ForceDensity
$\mathbf{F}_{den}=\gamma(\dot{E}_{den}/c,\mathbf{f}_{den})$

4-MomentumField
$\mathbf{P}_f=(E_f/c,\mathbf{p}_f)$
$=\mathbf{P}+\mathbf{Q}=\mathbf{P}+q\mathbf{A}$

4-NumberFlux
$\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$

4-ProbCurrDensity
4-ProbabilityFlux
$\mathbf{J}_{prob}=(\rho_{prob}c,\mathbf{j}_{prob})$
$=(i\hbar/2m_o)(\psi^*\partial[\psi]-\partial[\psi^*]\psi)+(q/m_o)(\psi^*\psi)\mathbf{A}$
Complex

4-ChargeFlux
4-CurrentDensity
$\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

4-EMVectorPotential
$\mathbf{A}=(\varphi/c,\mathbf{a})$

4-EMPotentialMomentum
$\mathbf{Q}=(U/c,\mathbf{q})=q\mathbf{A}$

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Complex

SR Perfect Fluid
$T^{\mu\nu}=((\rho_{eo}+p_o)/c^2)U^\mu U^\nu-(p_o)\eta^{\mu\nu}$
$T^{\mu\nu}=(\rho_{eo})V^{\mu\nu}+(-p_o)H^{\mu\nu}$
StressEnergy 4-Tensor

Einstein GR
$G^{\mu\nu}=R^{\mu\nu}-g^{\mu\nu}R/2$
4-Tensor

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$\partial=(\partial_t/c,-\nabla)$

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$\mathbf{K}=(\omega/c,\mathbf{k})$

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$\eta_{\mu\nu}$

EM Faraday
$F^{\alpha\beta}=\partial^\alpha A^\beta-\partial^\beta A^\alpha$
$=[\ 0\ ,\ -e^j/c]$
$[+e^i/c,-\varepsilon^{ij}{}_k b^k]$
4-Tensor

$\eta_{\mu\nu}$

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$\bar{\mathbf{T}}=\gamma(1,\boldsymbol{\beta})$

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$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\eta^{\mu\nu}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$ Minkowski Metric

$-p_o$

$-\partial\cdot T^{\mu\nu}=\mathbf{F}_{den}$ SR Conservation of StressEnergy if $\mathbf{F}_{den}=0^\mu$

Einstein GR $G^{\mu\nu}=R^{\mu\nu}-g^{\mu\nu}R/2$ 4-Tensor

$\partial\cdot G^{\mu\nu}=0^\mu$ SR Conservation of Einstein Tensor

4-Acceleration $\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

4-Polarization:Spin $\mathbf{E}=(\varepsilon^0,\boldsymbol{\varepsilon})=(\boldsymbol{\varepsilon}\cdot\boldsymbol{\beta},\boldsymbol{\varepsilon})$ $\mathbf{S}=(s^0,\mathbf{s})=(\mathbf{s}\cdot\boldsymbol{\beta},\mathbf{s})$ Complex

$(\rho_{eo}+p_o)/c^2$

SR Perfect Fluid $T^{\mu\nu}=((\rho_{eo}+p_o)/c^2)U^\mu U^\nu-(p_o)\eta^{\mu\nu}$ $T^{\mu\nu}=(\rho_{eo})V^{\mu\nu}+(-p_o)H^{\mu\nu}$ StressEnergy 4-Tensor

$8\pi G/c^4$

4-Gradient $\partial=(\partial_t/c,-\nabla)$

4-Displacement $\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$ $d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position $\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]=\gamma d/dt[..]$

$\mathbf{K}\cdot\mathbf{R}=-\Phi_{phase}$

4-WaveVector $\mathbf{K}=(\omega/c,\mathbf{k})$

4-TotalWaveVector $\mathbf{K}_T=(\omega_T/c,\mathbf{k}_T)$

$..[\mathbf{K}_T\cdot\mathbf{R}]$ $..[-\Phi_{T\,phase}]$ (i)

$\mathbf{T}\cdot\Delta\mathbf{R}/c=\Delta\tau$ ProperTime

$\mathbf{U}\cdot\mathbf{A}=\mathbf{U}\cdot\mathbf{U}'=0$

$\mathbf{U}\cdot\mathbf{E}=0$ $\mathbf{U}\cdot\mathbf{S}=0$

ProperTime $\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$ Derivative

E_{To}/ω_{To} (ħ)

$..[\mathbf{P}_T\cdot\mathbf{R}]$ $..[-S_{action}]$

EM Faraday $F^{\alpha\beta}=\partial^\alpha A^\beta-\partial^\beta A^\alpha$ $=[\ 0\ ,\ -e^j/c]$ $[+e^j/c,-\varepsilon^{ij}{}_k b^k]$ 4-Tensor

$\eta_{\mu\nu}$ ·

$\varepsilon_o c^2$ $1/\mu_o$

4-UnitTemporal $\bar{\mathbf{T}}=\gamma(1,\boldsymbol{\beta})$

$\mathbf{T}\cdot\mathbf{S}=0$

c

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$

ω_o/c^2

m_o E_o/c^2

E_o/ω_o (ħ)

$\eta_{\mu\nu}$ ·

q

$\mathbf{P}_T\cdot\mathbf{R}$ $=\int\mathbf{P}_T\cdot d\mathbf{R}$ $=-S_{action}$

4-TotalMomentum $\mathbf{P}_T=(E_T/c,\mathbf{p}_T)=(H/c,\mathbf{p}_T)$

n_o $\rho_{prob^o}=\chi^*\psi$ $=|\langle\chi|\psi\rangle|^2$

4-UnitSpatial $\bar{\mathbf{S}}=\gamma_{\beta n}(\hat{\mathbf{n}}\cdot\boldsymbol{\beta},\hat{\mathbf{n}})_\perp$

ρ_o

φ_o/c^2

4-Momentum $\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

$\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]$

4-Force $\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$

n_o

n_o

m_o E_o/c^2

4-MassFlux 4-MomentumDensity $\mathbf{G}=(\rho_m c,\mathbf{g})=(\rho_e/c,\mathbf{g})$

$\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]$

4-ForceDensity $\mathbf{F}_{den}=\gamma(\dot{E}_{den}/c,\mathbf{f}_{den})$

4-MomentumField $\mathbf{P}_f=(E_f/c,\mathbf{p}_f)$ $=\mathbf{P}+\mathbf{Q}=\mathbf{P}+q\mathbf{A}$

4-NumberFlux $\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$

4-ProbCurrDensity 4-ProbabilityFlux $\mathbf{J}_{prob}=(\rho_{prob}c,\mathbf{j}_{prob})$ $=(i\hbar/2m_o)(\psi^*\partial[\psi]-\partial[\psi^*]\psi)+(q/m_o)(\psi^*\psi)\mathbf{A}$ Complex

q

4-ChargeFlux 4-CurrentDensity $\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

4-EMVectorPotential $\mathbf{A}=(\varphi/c,\mathbf{a})$

q

$\varepsilon_o c^2$ $1/\mu_o$

$(\partial\cdot\partial)\mathbf{A}-\partial(\partial\cdot\mathbf{A})=\mu_o\mathbf{J}$ Maxwell EM Wave Eqn

$\partial\cdot\mathbf{A}=0$ Conservation of EM Field = Lorenz Gauge

4-EMPotentialMomentum $\mathbf{Q}=(U/c,\mathbf{q})=q\mathbf{A}$

$\partial\cdot\mathbf{N}=0$: $\partial\cdot\mathbf{J}_{prob}=0$ Conservation of Particle # : Probabilty

$\partial\cdot\mathbf{J}=0$ Conservation of Charge

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
Quantum Principles

$\mathrm{Trace}[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM+EM Diagram: 4-Vectors, 4-Tensors Lorentz Scalars / Physical Constants

4-Vector SRQM Interpretation of QM

SciRealm.org
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http://scirealm.org/SRQM.pdf

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\eta^{\mu\nu}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$ Minkowski Metric

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4-Position $\mathbf{R}=(ct,\mathbf{r})$

4-Acceleration $\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$

4-Polarization: Spin $\mathbf{E}=(\varepsilon^0,\boldsymbol{\varepsilon})=(\boldsymbol{\varepsilon}\cdot\boldsymbol{\beta},\boldsymbol{\varepsilon})$ $\mathbf{S}=(s^0,\mathbf{s})=(\mathbf{s}\cdot\boldsymbol{\beta},\mathbf{s})$ Complex

$\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]=\gamma d/dt[..]$

$\mathbf{T}\cdot\Delta\mathbf{R}/c=\Delta\tau$ ProperTime

ProperTime Derivative

$\mathbf{U}\cdot\mathbf{A}=\mathbf{U}\cdot\mathbf{U}'=0$

$\mathbf{U}\cdot\mathbf{E}=0$ $\mathbf{U}\cdot\mathbf{S}=0$

4-UnitTemporal $\bar{\mathbf{T}}=\gamma(1,\boldsymbol{\beta})$

$\mathbf{T}\cdot\mathbf{S}=0$

4-UnitSpatial $\bar{\mathbf{S}}=\gamma_{\beta n}(\hat{\mathbf{n}}\cdot\boldsymbol{\beta},\hat{\mathbf{n}})_\perp$

c Speed of Light

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$

$-p_o$ $+$ $(\rho_{eo}+p_o)/c^2$ $\otimes$

SR Perfect Fluid $T^{\mu\nu}=((\rho_{eo}+p_o)/c^2)U^\mu U^\nu-(p_o)\eta^{\mu\nu}$ $T^{\mu\nu}=(\rho_{eo})V^{\mu\nu}+(-p_o)H^{\mu\nu}$ StressEnergy 4-Tensor

$-\partial\cdot T^{\mu\nu}=\mathbf{F}_{den}$ SR Conservation of StressEnergy if $\mathbf{F}_{den}=0^\mu$

$8\pi G/c^4$ Gravitational Const

Einstein GR $G^{\mu\nu}=R^{\mu\nu}-g^{\mu\nu}R/2$ 4-Tensor

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$\mathbf{K}\cdot\mathbf{R}=-\Phi_{phase}$

$\{\omega_o=0\}\leftrightarrow\{\mathbf{K}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{K}$ is null$\}$

4-WaveVector $\mathbf{K}=(\omega/c,\mathbf{k})$

$\sum_n[..]$

4-TotalWaveVector $\mathbf{K}_T=(\omega_T/c,\mathbf{k}_T)$

$..[\mathbf{K}_T\cdot\mathbf{R}]$ $..[-\Phi_{T\,phase}]$ (i)

Complex Plane-Waves $\mathbf{K}_T=-\partial[\Phi]$, $\mathbf{K}=i\partial$

Hamilton-Jacobi $\mathbf{P}_T=-\partial[S]$

$..[\mathbf{P}_T\cdot\mathbf{R}]$ $..\int[\mathbf{P}_T\cdot d\mathbf{R}]$ $..[-S_{action}]$

E_{To}/ω_{To} $(\hbar)$

ProperTime $\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$ Derivative

Wave Velocity ω_o/c^2 $v_{group}*v_{phase}=c^2$

$\eta_{\mu\nu}$ $\cdot$ Lorentz EM Force Eqn $\{\mathbf{U}\cdot F^{\alpha\beta}=(1/q)\mathbf{F}\}$

q

EM Faraday $F^{\alpha\beta}=\partial^\alpha A^\beta-\partial^\beta A^\alpha$ $=[\ 0\ ,\ -e^i/c]$ $[+e^i/c,-\varepsilon^{ij}{}_k b^k]$ 4-Tensor

$\eta_{\mu\nu}$ $\cdot$

$\varepsilon_o c^2$ $1/\mu_o$

E_o/ω_o $(\hbar)$ Einstein de Broglie $\mathbf{P}=\hbar\mathbf{K}$

m_o E_o/c^2 $E=mc^2$

n_o $\rho_{prob^o}=\chi^*\psi$ $=|\langle\chi|\psi\rangle|^2$ Born Rule

ρ_o φ_o/c^2 EM

4-Momentum $\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

$\{m_o=0\}\leftrightarrow\{\mathbf{P}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{P}$ is null$\}$

$\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]$

4-Force $\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$

H-J $\mathbf{P}_T\cdot\mathbf{R}$ $=\int\mathbf{P}_T\cdot d\mathbf{R}$ $=-S_{action}$

4-TotalMomentum $\mathbf{P}_T=(E_T/c,\mathbf{p}_T)=(H/c,\mathbf{p}_T)$

$\sum_n[..]$

Conservation 4-TotalMomentum

n_o

4-MassFlux 4-MomentumDensity $\mathbf{G}=(\rho_m c,\mathbf{g})=(\rho_e/c,\mathbf{g})$

$\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]$

4-ForceDensity $\mathbf{F}_{den}=\gamma(\dot{E}_{den}/c,\mathbf{f}_{den})$

4-MomentumField $\mathbf{P}_f=(E_f/c,\mathbf{p}_f)$ $=\mathbf{P}+\mathbf{Q}=\mathbf{P}+q\mathbf{A}$

Minimal Coupling $\mathbf{P}+\mathbf{Q}$

$\{\varphi_o=0\}\leftrightarrow\{\mathbf{A}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{A}$ is null$\}$

4-EMVectorPotential $\mathbf{A}=(\varphi/c,\mathbf{a})$

EM Charge

q

4-EMPotentialMomentum $\mathbf{Q}=(U/c,\mathbf{q})=q\mathbf{A}$

m_o E_o/c^2

4-NumberFlux $\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$

4-ProbCurrDensity 4-ProbabilityFlux $\mathbf{J}_{prob}=(\rho_{prob}c,\mathbf{j}_{prob})$ $=(i\hbar/2m_o)(\psi^*\partial[\psi]-\partial[\psi^*]\psi)+(q/m_o)(\psi^*\psi)\mathbf{A}$ Complex

q EM Charge

4-ChargeFlux 4-CurrentDensity $\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

$\varepsilon_o c^2$ $1/\mu_o$

$(\partial\cdot\partial)\mathbf{A}-\partial(\partial\cdot\mathbf{A})=\mu_o\mathbf{J}$ Maxwell EM Wave Eqn

$\partial\cdot\mathbf{A}=0$ Conservation of EM Field = Lorenz Gauge

Maxwell EM Eqns: Gauss-Ampère : Gauss-Faraday $\{\partial\cdot F^{\alpha\beta}=\mu_o\mathbf{J}\}:\{\partial_\alpha(\tfrac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta})=0^\beta\}$

$\partial\cdot\mathbf{N}=0$: $\partial\cdot\mathbf{J}_{prob}=0$ Conservation of Particle # : Probabilty

$\partial\cdot\mathbf{J}=0$ Conservation of Charge

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
Quantum Principles

$\mathrm{Trace}[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4

A Tensor Study of Physical 4-Vectors

SRQM+EM Diagram: 4-Vectors, 4-Tensors Lorentz Scalars / Physical Constants with Tensor Invariants

4-Vector SRQM Interpretation of QM

SciRealm.org
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R·R=(ct)²-r·r
ΔR·ΔR=(cΔt)²-Δr·Δr
dR·dR=(cdt)²-dr·dr
Invariant Interval

∂·R=4
SpaceTime Dimension

∂[R]=η^μν→Diag[1,-1,-1,-1]
Minkowski Metric
Tr[η^μν]= 4
Det[η^μν]= -1

4-Displacement
ΔR=(cΔt,Δr)
dR=(cdt,dr)

4-Position
R=(ct,r)

A·A= -(a₀)²

4-Acceleration
A=γ(cγ',γ'u+γa)

4-Polarization:Spin
E=(ε⁰,ε)=(ε·β,ε)
S=(s⁰,s)=(s·β,s)
Complex

E*·E= -1

U·∂[..]
d/dτ[..]=γd/dt[..]

T·ΔR/c=Δτ
ProperTime

ProperTime Derivative

U·A=U·U'=0

U·E=0
U·S=0

4-UnitTemporal
T=γ(1,β)

T·T= +1

T·S=0

c
Speed of Light

4-Velocity
U=γ(c,u)

U·U=c²

4-UnitSpatial
S=γβn(n̂·β,n)⊥

S·S= -1

n₀
ρprob₀ = χ*ψ
=|⟨χ|ψ⟩|²

Born Rule

ρ₀

φ₀/c²

EM

m₀
E₀/c²

4-NumberFlux
N=(nc,n)=n(c,u)
4-ProbCurrDensity
4-ProbabilityFlux
Jprob=(ρprobc, jprob)
=(iħ/2m₀)(ψ*∂[ψ]-∂[ψ*]ψ)+ (q/m₀)(ψ*ψ)A
Complex

q
EM Charge

4-ChargeFlux
4-CurrentDensity
J=(ρc,j)=ρ(c,u)

J·J=(ρ₀c)²

N·N=(n₀c)²
Jprob·Jprob=(ρprob₀c)²

∂·N=0 : ∂·Jprob=0
Conservation of Particle # : Probabilty

∂·J=0
Conservation of Charge

-p₀

+

(ρe₀+p₀)/c²

⊗

K·R= -Φphase

Tμν Tμν= ρe₀-3p₀
Det[Tμν]= -(ρe₀)(p₀)³
Tr[Tμν]= ρe₀-3p₀

SR Perfect Fluid
Tμν=((ρe₀+p₀)/c²)UμUν-(p₀)ημν
Tμν=(ρe₀)Vμν+(-p₀)Hμν
StressEnergy 4-Tensor

{ω₀=0} ↔ {K·U=0} ↔ {K is null}

4-WaveVector
K=(ω/c,k)

K·K=(ω₀/c)²
=(m₀c/ħ)²

Wave Velocity
Vgroup*Vphase=c²

ω₀/c²

ProperTime
U·∂=d/dτ=γd/dt
Derivative

E₀/ω₀
(ħ)

Einstein de Broglie
P =ħK

E=mc²

P·P=(m₀c)²=(E₀/c)²

4-Momentum
P=(mc,p)=(E/c,p)

{m₀=0} ↔ {P·U=0} ↔ {P is null}

{φ₀=0} ↔ {A·U=0} ↔ {A is null}

4-EMVectorPotential
A=(φ/c,a)

A·A=(φ₀/c)²

(∂·∂)A-∂(∂·A)=μ₀J
Maxwell EM Wave Eqn

∂·A=0
Conservation of EM Field
= Lorenz Gauge

-∂·Tμν=Fden
SR Conservation of StressEnergy if Fden=0μ

8πG/c⁴
Gravitational Const

Einstein GR
Gμν=Rμν-gμνR/2
4-Tensor

∂·Gμν=0μ
SR Conservation of Einstein Tensor

Tr[Gμν]= -R

d'Alembertian
∂·∂=(∂t/c)²-∇·∇
∂·∂= -(m₀c/ħ)²
Klein-Gordon

4-Gradient
∂=(∂t/c,-∇)

Complex Plane-Waves
KT=-∂[Φ],K=iA

KT·KT=(ωT₀/c)²

4-TotalWaveVector
KT=(ωT/c,kT)

..[KT·R]
..[-ΦTphase]
(i)

Hamilton-Jacobi
PT=-∂[S]

..[PT·R]
..∫[PT·dR]
..[-Saction]

ET₀/ωT₀
(ħ)

Tr[Fμν]=0
FμνFμν=2{(b·b)-(e·e/c²)}
Det[Fμν]={(e·b)/c}²

EM Faraday
Fαβ=∂αAβ-∂βAα
=[0 , -ei/c]
[+ei/c,-εijkbk]
4-Tensor

ημν
·

εoc²
1/μo

Lorentz EM Force Eqn
{ U·Fαβ =(1/q)F }

q

H-J

PT·R
=∫PT·dR
= -Saction

PT·PT=(H₀/c)²

4-TotalMomentum
PT=(ET/c,pT)=(H/c,pT)

∑n [..]

F·F=γ²[(Ė/c)² - f·f]

4-Force
F=γ(Ė/c,f=ṗ)

U·∂[..]
d/dτ[..]

n₀

G·G=(ρm₀c)²

4-MassFlux
4-MomentumDensity
G=(ρmc,g)=(ρe/c,g)

U·∂[..]
d/dτ[..]

4-ForceDensity
Fden=γ(Ėden/c,fden)

Fden·Fden=γ²[(Ėden/c)² - fden·fden]

Conservation 4-TotalMomentum

4-MomentumField
Pf=(Ef/c,pf)
=P+Q=P+qA

Q·Q=(U₀/c)²

4-EMPotentialMomentum
Q=(U/c,q)=qA

Minimal Coupling
P+Q

Maxwell EM Eqns: Gauss-Ampère : Gauss-Faraday
{ ∂·Fαβ =μ₀Jβ } : { ∂α(½εαβγδFγδ) = 0β }

SR 4-Tensor
(2,0)-Tensor Tμν
(1,1)-Tensor Tμν or Tμν
(0,2)-Tensor Tμν

SR 4-Vector
(1,0)-Tensor Vμ = **V** = (v⁰,**v**)
SR 4-CoVector:OneForm
(0,1)-Tensor Vμ = (v₀,-**v**)

SR 4-Scalar
(0,0)-Tensor S or S₀
Lorentz Scalar

Existing SR Rules
Quantum Principles

Trace[Tμν] = ημνTμν = Tμμ = T
V·V = Vμημν Vν = [(v⁰)² - **v·v**] = (v⁰₀)²
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

4-Vector SRQM Interpretation of QM

SRQM Diagram:
Physical Constants Emphasized

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$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

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$-\partial\cdot T^{\mu\nu}=\mathbf{F}_{den}$ SR Conservation of StressEnergy if $\mathbf{F}_{den}=0^{\mu}$

$\partial\cdot G^{\mu\nu}=0^{\mu}$ SR Conservation of Einstein Tensor

$-p_o$

$(\rho_{eo}+p_o)/c^2$

$8\pi G/c^4$

$\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]$ $\gamma d/dt[..]$

$\mathbf{U}\cdot\mathbf{E}=0$ $\mathbf{U}\cdot\mathbf{S}=0$

$\mathbf{K}\cdot\mathbf{R}=-\Phi_{phase}$

$..[\mathbf{K}\cdot\mathbf{R}]$ $..[-\Phi_{phase}]$ (i)

$\overline{\mathbf{T}}\cdot\Delta\mathbf{R}/c=\Delta\tau$ ProperTime

ω_o/c^2

$\mathbf{U}\cdot\mathbf{A}=\mathbf{U}\cdot\mathbf{U}'=0$

ProperTime $\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$ Derivative

E_{To}/ω_{To} $(\hbar)$

$..[\mathbf{P}_T\cdot\mathbf{R}]$ $..[-S_{action}]$

$\eta_{\mu\nu}$.

$\eta_{\mu\nu}$.

$\overline{\mathbf{T}}\cdot\overline{\mathbf{T}}=+1$

c

$\int\mathbf{P}_T\cdot d\mathbf{R}=-S_{action}$ $\mathbf{P}_T\cdot\mathbf{R}=-S_{action}$

q

$\varepsilon_o c^2$ $1/\mu_o$

$\overline{\mathbf{T}}\cdot\overline{\mathbf{S}}=0$

E_o/ω_o $(\hbar)$

m_o E_o/c^2

$\overline{\mathbf{S}}\cdot\overline{\mathbf{S}}=-1$

n_o $\rho_{prob^o}=\chi^*\psi$ $=|\langle\chi|\psi\rangle|^2$

ρ_o

φ_o/c^2

$\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]$

n_o

n_o

$\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]$

m_o E_o/c^2

q

q

$\varepsilon_o c^2$ $1/\mu_o$

$\partial\cdot\mathbf{A}=0$ Conservation of EM Field = Lorenz Gauge

$\partial\cdot\mathbf{N}=0$: $\partial\cdot\mathbf{J}_{prob}=0$ Conservation of Particle # : Probabilty

$\partial\cdot\mathbf{J}=0$ Conservation of Charge

Notice that all the main "Universal" or "Fundamental" Physical Constants are here: $G,c,\hbar,\varepsilon_o,\mu_o$.

Some depend on the actual particle type: q,m_o,ω_o
Some depend on regional conditions: $\tau,\rho_{eo},p_o,\rho_o,\varphi_o,\psi^*\psi$
Some depend on interaction: Φ_{phase},S_{action}
Some are mathematical: $0,4,\pi,i,\mathrm{Diag}[1,-1,-1,-1],d/d\tau$
Conservation Laws are also a type of "zero" constant in this regard.

The majority of the constants are Lorentz Scalars, but some are 4-Vector or 4-Tensor, and all are valid for all inertial observers.

Fundamental Physical Constants are SR Lorentz Scalars

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu}=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu}=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
Quantum Principles

The fact that these "tie together" a network of 4-Vectors is a good argument for why their values are constant. Changing even one would change the relationship properties among all of the 4-Vectors.

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram: Projection Tensors Temporal, Spatial, Null, SpaceTime

4-Vector SRQM Interpretation of QM

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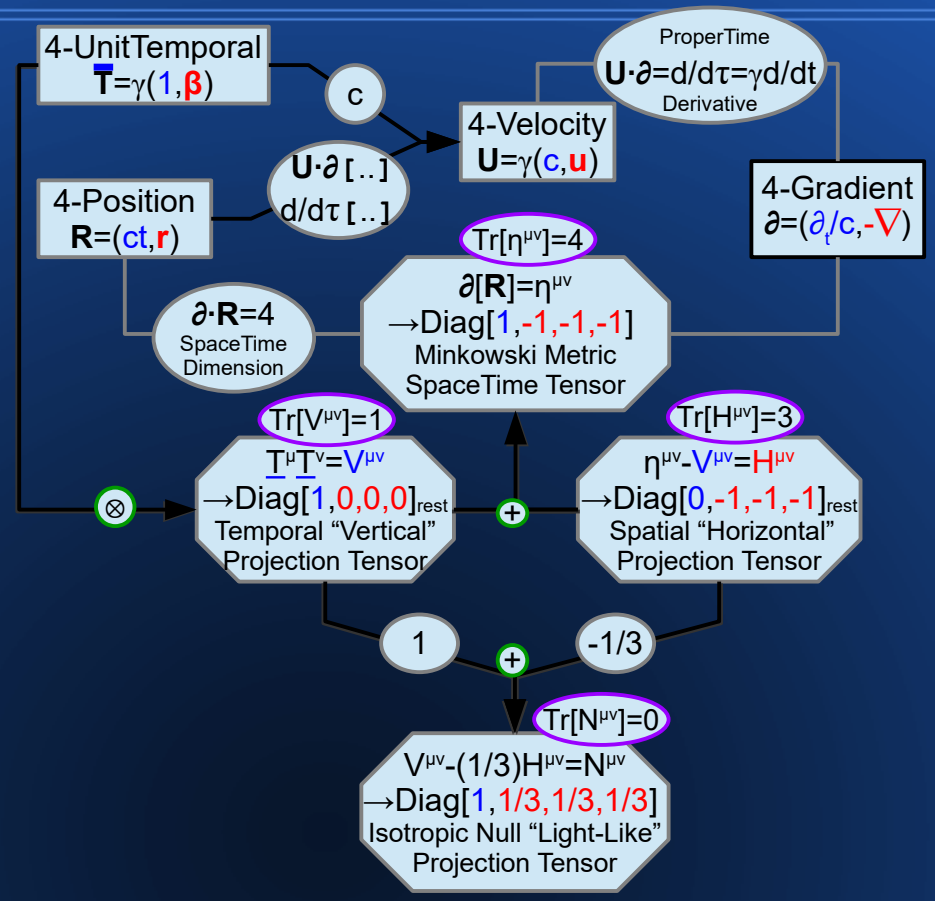


Projection Tensors act as follows:

Generic 4-Vector:

$A^\nu = (a^0,\mathbf{a}) = (a^0,a^1,a^2,a^3)$

Temporal Projection:

$V^\mu{}_\nu = \eta_{\omega\nu}V^{\mu\omega} \rightarrow \mathrm{Diag}[1,0,0,0]$

$V^\mu{}_\nu A^\nu = (a^0,0,0,0) = (a^0,\mathbf{0})$

Spatial Projection:

$H^\mu{}_\nu = \eta_{\omega\nu}H^{\mu\omega} \rightarrow \mathrm{Diag}[0,1,1,1]$

$H^\mu{}_\nu A^\nu = (0,a^1,a^2,a^3) = (0,\mathbf{a})$

SpaceTime Projection:

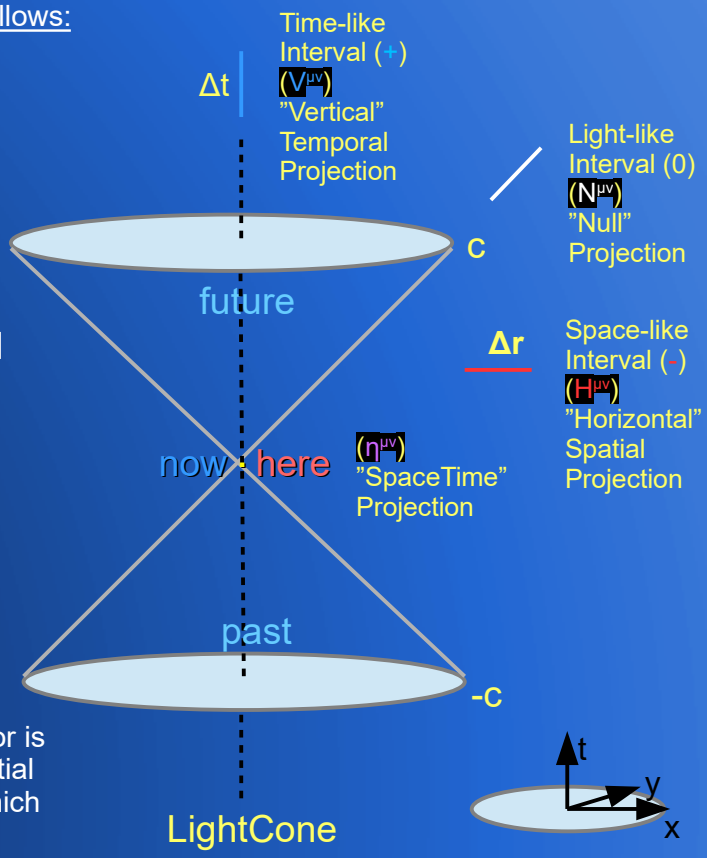
$V^\mu{}_\nu A^\nu + H^\mu{}_\nu A^\nu = \eta^\mu{}_\nu A^\nu$

$= \delta^\mu{}_\nu A^\nu = A^\mu = (a^0,\mathbf{a})$

$V^\mu{}_\nu + H^\mu{}_\nu = \eta^\mu{}_\nu = \delta^\mu{}_\nu$

$V^{\mu\nu} + H^{\mu\nu} = \eta^{\mu\nu}$

The Minkowski Metric Tensor is the Sum of Temporal & Spatial Projection Tensors, all of which are dimensionless.



SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$

$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$

= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4-Vector SRQM Interpretation of QM

SRQM Diagram: Projection Tensors & Perfect-Fluid Stress-Energy Tensor

A Tensor Study of Physical 4-Vectors

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John B. Wilson
SciRealm@aol.com
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4-UnitTemporal
$\bar{T}=\gamma(1,\beta)$

c

4-Velocity
$U=\gamma(c,u)$

ProperTime
$U\cdot\partial=d/d\tau=\gamma d/dt$
Derivative

$U\cdot\partial[..]$
$d/d\tau[..]$

4-Position
$R=(ct,r)$

4-Gradient
$\partial=(\partial_t/c,-\nabla)$

$\partial\cdot R=4$
SpaceTime Dimension

$Tr[\eta^{\mu\nu}]=4$
$\partial[R]=\eta^{\mu\nu}$
$\rightarrow Diag[1,-1,-1,-1]$
Minkowski Metric SpaceTime Tensor

$Tr[V^{\mu\nu}]=1$
$T^{\mu}T^{\nu}=V^{\mu\nu}$
$\rightarrow Diag[1,0,0,0]_{rest}$
Temporal "Vertical" Projection Tensor

$Tr[H^{\mu\nu}]=3$
$\eta^{\mu\nu}-V^{\mu\nu}=H^{\mu\nu}$
$\rightarrow Diag[0,-1,-1,-1]_{rest}$
Spatial "Horizontal" Projection Tensor

rest-energy-density ρ_{eo}

rest-pressure $-p_o$

$Tr[T^{\mu\nu}]=\rho_{eo}-3p_o$

Perfect-Fluid StressEnergy 4-Tensor
$T^{\mu\nu}_{rest}\rightarrow Diag[\rho_{eo},p_o,p_o,p_o]$
$T^{\mu\nu}=(\rho_{eo})V^{\mu\nu}+(-p_o)H^{\mu\nu}$

$\partial\cdot T^{\mu\nu}=0^{\nu}$
Conservation of StressEnergy

Perfect-Fluid StressEnergy 4-Tensor:

$T^{\mu\nu}=((\rho_{eo}+p_o)/c^2)U^{\mu}U^{\nu}-(p_o)\eta^{\mu\nu}$

can be written in much simpler form using Projection Tensors:

$T^{\mu\nu}=(\rho_{eo})V^{\mu\nu}-(p_o)H^{\mu\nu}$

Projection Tensors act as follows:

$A^{\nu}=(a^0,\mathbf{a})=(a^0,a^1,a^2,a^3)$

$V^{\mu}{}_{\nu}=\eta_{\omega\nu}V^{\mu\omega}\rightarrow Diag[1,0,0,0]$
$V^{\mu}{}_{\nu}A^{\nu}=(a^0,0,0,0)=(a^0,\mathbf{0})$

$H^{\mu}{}_{\nu}=\eta_{\omega\nu}H^{\mu\omega}\rightarrow Diag[0,1,1,1]$
$H^{\mu}{}_{\nu}A^{\nu}=(0,a^1,a^2,a^3)=(0,\mathbf{a})$

$V^{\mu}{}_{\nu}A^{\nu}+H^{\mu}{}_{\nu}A^{\nu}=\eta^{\mu}{}_{\nu}A^{\nu}$
$=\delta^{\mu}{}_{\nu}A^{\nu}=A^{\mu}=(a^0,\mathbf{a})$

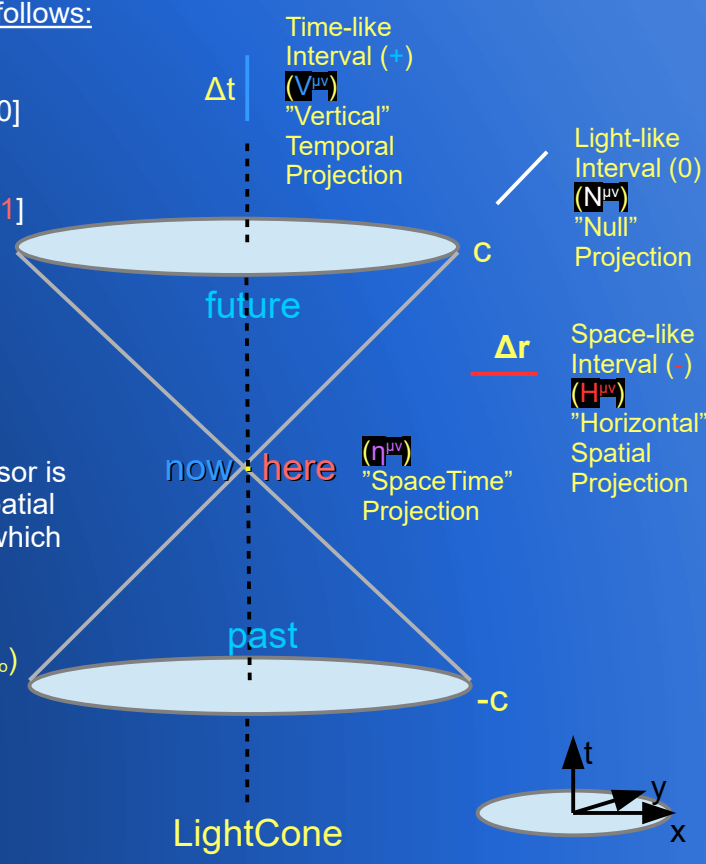
$V^{\mu}{}_{\nu}+H^{\mu}{}_{\nu}=\eta^{\mu}{}_{\nu}$
$V^{\mu\nu}+H^{\mu\nu}=\eta^{\mu\nu}$

The Minkowski Metric Tensor is the Sum of Temporal & Spatial Projection Tensors, all of which are dimensionless.

The rest-energy-density (ρ_{eo}) is the Temporal Projection.

The neg rest-pressure ($-p_o$) is the Spatial Projection.

$T^{\mu\nu}{}_{MCRF}\rightarrow Diag[\rho_{eo},p_o,p_o,p_o]$



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SR 4-Vector
(1,0)-Tensor $V^{\mu}=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu}=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$P^{\mu}N^{\nu}=(m_oU^{\mu})(n_oU^{\nu})=(m_on_o)(U^{\mu}U^{\nu})=(\rho_{mo})(U^{\mu}U^{\nu})$
$=(\rho_{mo})(c^2)(T^{\mu}T^{\nu})=(\rho_{eo})(T^{\mu}T^{\nu})=(\rho_{eo})(V^{\mu\nu})=\rho_{eo}V^{\mu\nu}$

$Trace[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^{\mu}{}_{\mu}=T$
$\mathbf{V}\cdot\mathbf{V}=V^{\mu}\eta_{\mu\nu}V^{\nu}=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM+EM Diagram: Projection Tensors & Stress-Energy Tensors: Special Cases

4-Vector SRQM Interpretation of QM

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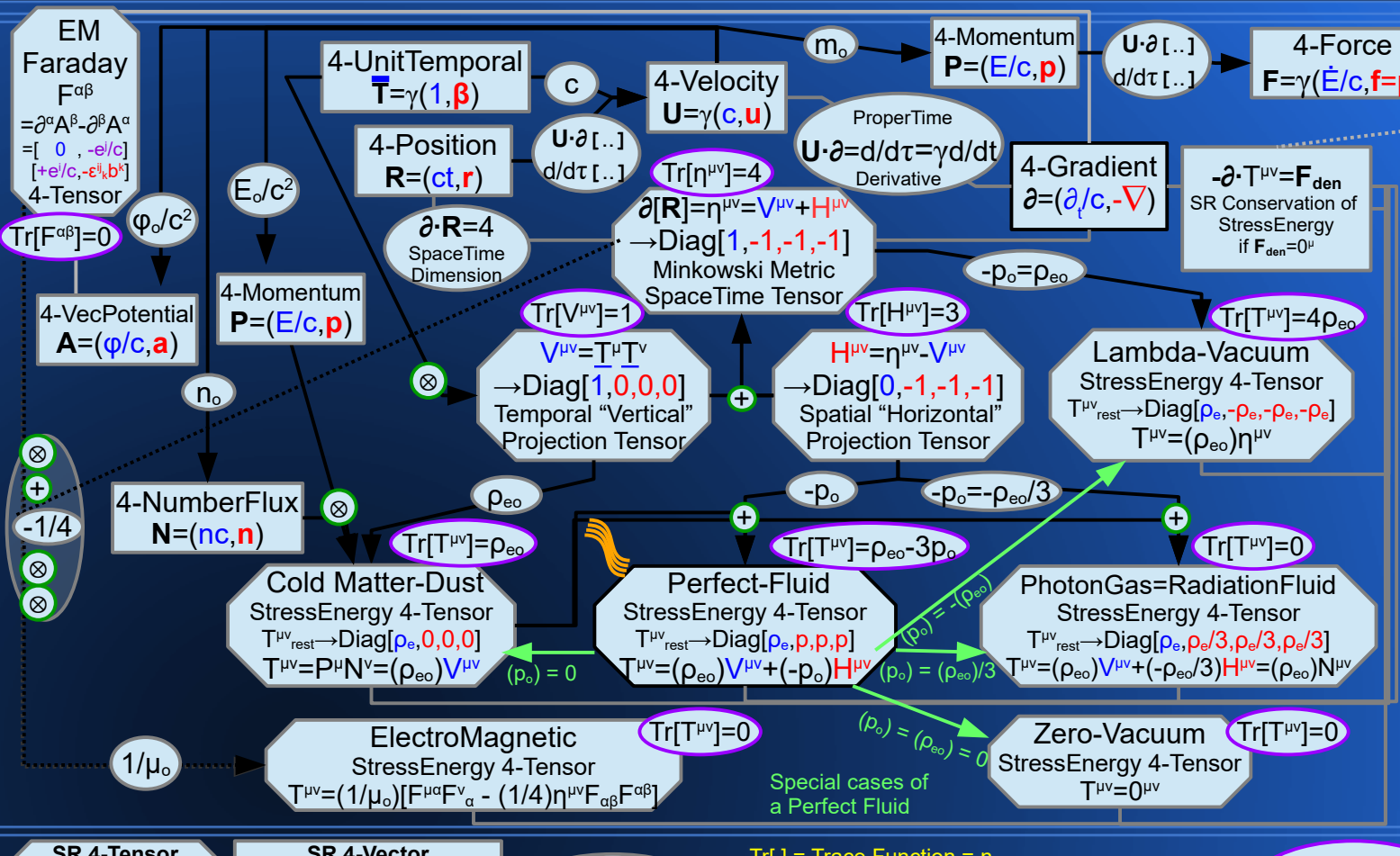

A few interesting special cases:

{for Perfect Fluid (no viscosity)}
$T^{\mu\nu}{}_{PerfectFluid} = (\rho_{eo})\ V^{\mu\nu} - (p_o)\ H^{\mu\nu}$
$T^{\mu\nu}{}_{PerfectFluid} = ((\rho_{eo}+p_o)/c^2)U^\mu U^\nu - (p_o)\eta^{\mu\nu}$
$Tr[T^{\mu\nu}] = 1(\rho_{eo}) - 3(p_o)$

$(p_o) = (\rho_{eo})/3$:
{PhotonGas = RadiationFluid}
$T^{\mu\nu}{}_{PhotonGas} = (\rho_{eo})\ V^{\mu\nu} - (\rho_{eo}/3)\ H^{\mu\nu} = (\rho_{eo})\ N^{\mu\nu}$
$T^{\mu\nu}{}_{PhotonGas} = (p_o)(4V^{\mu\nu} - \eta^{\mu\nu})$
$Tr[T^{\mu\nu}{}_{PhotonGas}] = 0$: Null (Light-Like) Projection

$(p_o) = 0$:
{Cold Matter Dust (pressureless) }
$T^{\mu\nu}{}_{MatterDust} = P^\mu N^\nu = (\rho_{mo})U^\mu U^\nu = (\rho_{eo})\ V^{\mu\nu}$
$Tr[T^{\mu\nu}{}_{MatterDust}] = (\rho_{eo})$: Temporal Projection

$(p_o) = -(\rho_{eo})$:
{LambdaVacuum Energy}
$T^{\mu\nu}{}_{VaccEnergy} = (\rho_{eo})\ V^{\mu\nu} + (\rho_{eo})\ H^{\mu\nu} = (\rho_{eo})\ \eta^{\mu\nu}$
$Tr[T^{\mu\nu}{}_{VaccEnergy}] = 4(\rho_{eo})$: SpaceTime Projection

$(p_o) = (\rho_{eo}) = 0$:
{ZeroVacuum Energy}
$T^{\mu\nu}{}_{VaccEnergy} = 0^{\mu\nu}$
$Tr[T^{\mu\nu}{}_{VaccEnergy}] = 0$: Zero Projection

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Tr[ ] = Trace Function = $\eta_{\mu\nu}$
$N^{\mu\nu} = V^{\mu\nu} - (1/3)\ H^{\mu\nu}$ = Null Projection Tensor
$N^{\mu\nu} \rightarrow Diag[1,1/3,1/3,1/3]$ with $Tr[N^{\mu\nu}] = 0$

Equation of State
$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$
4-Scalar

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

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4-Vector SRQM Interpretation of QM

SRQM Study: Physical 4-Tensors

Matter-Dust vs. Null-Dust

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Matter-Dust is a special case of a perfect fluid with time-like 4-Velocity **U**, energy density ρ, and zero pressure p, described by the energy-momentum tensor $T_{(dust)}^{\mu\nu}=\rho_o U^\mu U^\nu$. Because there is no pressure gradient, the fluid elements of the dust follow time-like geodesics.

Null-Dust corresponds to the limit in which the 4-Velocity **U** becomes null, and is described by $T_{(nulldust)}^{\mu\nu}=\rho_o K^\mu K^\nu$, $\mathbf{K}\cdot\mathbf{K}=0$, with ρ>0 and trace[T]=0. The elements of dust follow null geodesics.

Null-Dust is interpreted as a coherent zero-rest-mass field propagating at the speed of light (c) in the null direction **K**. A null-dust can describe propagating electromagnetic (EM) or gravitational waves.

Note:There is an unfortunate slight notational clash between:

4-(Dust)NumberFlux $\mathbf{N}=(nc,n\mathbf{u})=n_o\mathbf{U}$

4-"Unit"Null $\bar{\mathbf{N}}=(1,\pm\hat{\mathbf{n}})$

Use colored overbars on the "Unit" 4-Vectors
$\bar{\mathbf{T}}$ =Temporal : $\bar{\mathbf{N}}$ = Null : $\bar{\mathbf{S}}$=Spatial

4-Momentum
$\mathbf{P}=P^\mu=(E/c=mc,\mathbf{p}=m\mathbf{u})=m_o\mathbf{U}$

4-(Dust)NumberFlux
$\mathbf{N}=N^\mu=(nc,n\mathbf{u})=n_o\mathbf{U}$

E_o/c m_o n_o

4-Velocity
$\mathbf{U}=U^\mu=\gamma(c,\mathbf{u})=c\gamma(1,\boldsymbol{\beta})=c\bar{\mathbf{T}}$

$U^\mu U^\nu \rightarrow \mathrm{Diag}[c^2,0,0,0]_{\{MCRF\}}$

$n_o m_o$ ρ_{mo}

$n_o E_o$ ρ_{eo}

c

$V^{\mu\nu}=\bar{T}^\mu\bar{T}^\nu \rightarrow \mathrm{Diag}[1,0,0,0]_{\{MCRF\}}$
Temporal "Vertical" Projection Tensor

4-UnitTemporal
$\bar{\mathbf{T}}=\bar{T}^\mu=\gamma(1,\boldsymbol{\beta})=\gamma(1,\mathbf{u}/c)=\mathbf{U}/c$

4-"Unit"Null
$\bar{\mathbf{N}}=N^\mu=(1,\pm\hat{\mathbf{n}})$
$\sim \mathbf{K}_{photonic}=(|\mathbf{k}|,\mathbf{k})$

$N^{\mu\nu}=\bar{N}^\mu\bar{N}^\nu$
$\rightarrow_{\{MCRF\_z\text{-}dir\_no\ rest\ frame\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 1 & 0 & 0 & \pm1 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & \pm1 & 0 & 0 & 1 \end{array}$$

Null "Diagonal" Projection Tensor

ρ_{eo} Φ_o

(Cold) Matter-Dust

$T^{\mu\nu} \rightarrow P^\mu N^\nu=m_o U^\mu n_o U^\nu=(\rho_{mo})U^\mu U^\nu=$

$=(\rho_{mo}c^2)\bar{T}^\mu\bar{T}^\nu=(\rho_{eo})\bar{T}^\mu\bar{T}^\nu=(\rho_{eo})V^{\mu\nu} \rightarrow_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array}$$

$\rho_e=\rho_m c^2$ 0^{0j} 0^{i0} 0^{ij}

EoS[$T^{\mu\nu}$]=w=0

Tr[$T^{\mu\nu}$]=ρ_{eo}

Stress-Energy 4-Tensor
Symmetric, Spatial Isotropic, Pressureless

Null-Dust : "Gravitational Wave" Incoherent EM Mixture

$T^{\mu\nu} \rightarrow \Phi_o\bar{N}^\mu\bar{N}^\nu=\Phi_o N^{\mu\nu} \rightarrow_{\{MCRF\_z\text{-}dir\_no\ rest\ frame\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \Phi & 0 & 0 & \pm\Phi \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & \pm\Phi & 0 & 0 & \Phi \end{array}$$

Φ^{00} Φ^{0j} Φ^{i0} Φ^{ij}

Tr[$T^{\mu\nu}$]=0

Stress-Energy 4-Tensor
Symmetric, Null

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Equation of State
EoS[$T^{\mu\nu}$] = $w = p_o/\rho_{eo}$
4-Scalar

Trace[$T^{\mu\nu}$] = $\eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

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SRQM Study: Physical 4-Tensors

Vacuums vs. Fluids vs. Dusts

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Vacuums

Isotropy Group SO(1,3)=Full Lorentz Group
Segre Type {1,111}

Lambda Vacuum

$T^{\mu\nu} \rightarrow (\rho_{eo})\eta^{\mu\nu} = (\Lambda)\eta^{\mu\nu} \rightarrow_{\{MCRF\}}$

Dark Energy?

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & -\rho_e & 0 & 0 \\ y & 0 & 0 & -\rho_e & 0 \\ z & 0 & 0 & 0 & -\rho_e \end{array}$$

$$\begin{matrix} \rho_e=\rho_m c^2 & 0^{0j} \\ 0^{i0} & -\rho_e\delta^{ij} \end{matrix}$$

$EoS[T^{\mu\nu}]=w=-1$

$Tr[T^{\mu\nu}]=4\rho_{eo}$

Stress-Energy 4-Tensor
Symmetric, Spatial Isotropic

Zero:Nothing Vacuum

$T^{\mu\nu} \rightarrow 0^{\mu\nu} \rightarrow_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{matrix} 0 & 0^{0j} \\ 0^{i0} & 0^{ij} \end{matrix}$$

$EoS[T^{\mu\nu}]=w$ =undefined

$Tr[T^{\mu\nu}]=0$

Stress-Energy 4-Tensor
Symmetric, Isotropic

Fluids

Isotropy Group SO(3)=Ordinary Rotation Group
Segre Type {1,(111)}

Perfect Fluid Stress-Energy

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} \rightarrow_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & p & 0 & 0 \\ y & 0 & 0 & p & 0 \\ z & 0 & 0 & 0 & p \end{array}$$

$$\begin{matrix} \rho_e=\rho_m c^2 & 0^{0j} \\ 0^{i0} & p\delta^{ij} \end{matrix}$$

$EoS[T^{\mu\nu}]=w=p_o/\rho_{eo}$

$Tr[T^{\mu\nu}]=\rho_{eo}-3p_o$

4-Tensor
Symmetric,
Spatial Isotropic

PhotonGas=RadiationFluid

$T^{\mu\nu} \rightarrow (\rho_{eo})V^{\mu\nu} + (-\rho_{eo}/3)H^{\mu\nu} \rightarrow_{\{MCRF?\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & \rho_e/3 & 0 & 0 \\ y & 0 & 0 & \rho_e/3 & 0 \\ z & 0 & 0 & 0 & \rho_e/3 \end{array}$$

$$\begin{matrix} \rho_e=\rho_m c^2 & 0^{0j} \\ 0^{i0} & (\rho_e/3)\delta^{ij} \end{matrix}$$

$p_o=\rho_{eo}/3$

$Tr[T^{\mu\nu}]=0$

Stress-Energy 4-Tensor
Symmetric, Spatial Isotropic

Dusts

(Cold) Matter-Dust

$T^{\mu\nu} \rightarrow P^\mu N^\nu = m_o U^\mu n_o U^\nu = (\rho_{mo})U^\mu U^\nu =$

$= (\rho_{mo}c^2)\bar{T}^\mu\bar{T}^\nu = (\rho_{eo})\bar{T}^\mu\bar{T}^\nu = (\rho_{eo})V^{\mu\nu} \rightarrow_{\{MCRF\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \rho_e & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{matrix} \rho_e=\rho_m c^2 & 0^{0j} \\ 0^{i0} & 0^{ij} \end{matrix}$$

$EoS[T^{\mu\nu}]=w=0$

$p_o=0$

$Tr[T^{\mu\nu}]=\rho_{eo}$

Stress-Energy 4-Tensor
Symmetric, Spatial Isotropic, Pressureless

Isotropy Group 3D Lie Group E(2)=Euclidean Plane
Segre Type {2,11}

Null-Dust : "Gravitational Wave" Incoherent EM Mixture

$T^{\mu\nu} \rightarrow \Phi_o\bar{N}^\mu\bar{N}^\nu = \Phi_o N^{\mu\nu} \rightarrow_{\{MCRF\_z\text{-}dir\_no\ rest\ frame\}}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & \Phi & 0 & 0 & \pm\Phi \\ x & 0 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & \pm\Phi & 0 & 0 & \Phi \end{array}$$

$$\begin{matrix} \Phi^{00} & \Phi^{0j} \\ \Phi^{i0} & \Phi^{ij} \end{matrix}$$

$Tr[T^{\mu\nu}]=0$

Stress-Energy 4-Tensor
Symmetric, Null

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Lorentz Scalar

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$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$
4-Scalar

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM Diagram: 4-Tensors and 4-Scalars generated from 4-Vectors

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All SR 4-Tensors can be generated from SR 4-Vectors:

$F^{\mu\nu} = \partial \wedge \mathbf{A} = \partial^\mu A^\nu - \partial^\nu A^\mu$: Faraday EM 4-Tensor (from the 4-Gradient & 4-EMVectorPotential)

$M^{\mu\nu} = \mathbf{X} \wedge \mathbf{P} = X^\mu P^\nu - X^\nu P^\mu$: 4-AngularMomentum 4-Tensor (from the 4-Position & 4-Momentum)

$\eta^{\mu\nu} = \partial[\mathbf{R}] = \partial^\mu[R^\nu]$: SR:Minkowski Metric 4-Tensor (from the 4-Gradient & 4-Position)

$V^{\mu\nu} = \mathbf{T} \otimes \mathbf{T} = T^\mu T^\nu$: (V)ertical:Temporal Projection 4-Tensor (from the 4-UnitTemporal)

$H^{\mu\nu} = \eta^{\mu\nu} - V^{\mu\nu} = \eta^{\mu\nu} - T^\mu T^\nu$: (H)orizontal:Spatial Projection 4-Tensor (from previously made 4-Tensors above)

$T_{Cold\_Dust}{}^{\mu\nu} = \mathbf{P} \otimes \mathbf{N} = P^\mu N^\nu$: (Cold)Dust Stress-Energy 4-Tensor (from the 4-Momentum & 4-DustNumberFlux)

$(\rho_{eo}) = T_{Cold\_Dust}{}^{\mu\nu} V_{\mu\nu}$: MCRF EnergyDensity 4-Scalar (from previously made 4-Tensors above)

$T_{Lambda\_Vacuum}{}^{\mu\nu} = (\rho_{eo})\eta^{\mu\nu}$: LambdaVacuum (Dark Energy) Stress-Energy 4-Tensor (from previously made 4-Tensors above)

$(p_o) = (k)(1/3)T_{Lambda\_Vacuum}{}^{\mu\nu} H_{\mu\nu}$: MCRF Pressure 4-Scalar (from previously made 4-Tensors above)

with the pressure initially set to the EnergyDensity

and (k) an arbitrary constant which sets pressure level

$T_{Perfect\_Fluid}{}^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu}$: PerfectFluid Stress-Energy 4-Tensor (from previously made 4-Tensors above)

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(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Equation of State
$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$

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$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM Study: 4D Stokes':Gauss' Theorem

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John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

4D Generalized Stokes':Gauss' Theorem in Special Relativity

$\int_\Omega d^4\mathbf{X}\,(\partial_\mu V^\mu) = \oint_{\partial\Omega} dS\,(V^\mu N_\mu)$

$\int_\Omega d^4\mathbf{X}\,(\boldsymbol{\partial}\cdot\mathbf{V}) = \oint_{\partial\Omega} dS\,(\mathbf{V}\cdot\mathbf{N})$

4-Gradient

$\boldsymbol{\partial}=\partial_R=\partial_X=\partial^\mu=(\partial_t/c,-\nabla)$

$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

4D Stokes' Theorem

Integration of {4D Div = 4D Surface Flow}

[{V}·m³] $\int_\Omega d^4\mathbf{X}(\partial_\mu V^\mu) = \int_\Omega d^4\mathbf{X}(\boldsymbol{\partial}\cdot\mathbf{V})$ = [{V}·m³] $\oint_{\partial\Omega} dS(V^\mu N_\mu) = \oint_{\partial\Omega} dS(\mathbf{V}\cdot\mathbf{N})$

Ω = 4D Minkowski Region, ∂Ω = it's 3D boundary
$d^4\mathbf{X}$ = 4D Volume Element, $\mathbf{V} = V^\mu$ = Arbitrary 4-Vector Field
dS = 3D Surface Element, $\mathbf{N} = N^\mu$ = Surface Normal

with:

$\mathbf{V} = V^\mu$ is a 4-Vector field defined in 4D Minkowski Region Ω

$(\boldsymbol{\partial}\cdot\mathbf{V}) = (\partial_\mu V^\mu)$ is the 4-Divergence of $\mathbf{V}$

$(\mathbf{V}\cdot\mathbf{N}) = (V^\mu N_\mu)$ is the component of $\mathbf{V}$ along the boundary normal $\mathbf{N}$-direction

Ω is a 4D simply-connected region of Minkowski SpaceTime

∂Ω = S is its 3D boundary with its own 3D Volume element dS and outward pointing normal $\mathbf{N}$.

$\mathbf{N} = N^\mu$ is the outward-pointing normal of the boundary

$d^4\mathbf{X} = (c\,dt)(d^3\mathbf{x}) = (c\,dt)(dx\,dy\,dz)$ is the 4D differential volume element

Ω = 4D Minkowski Region, ∂Ω = it's 3D boundary
$d^4\mathbf{X}$ = 4D Volume Element, $\mathbf{V} = V^\mu$ = Arbitrary 4-Vector Field
dS = 3D Surface Element, $\mathbf{N} = N^\mu$ = Surface Normal

In vector calculus, the divergence theorem, also known as Gauss's theorem or Ostrogradsky's theorem, is a result that relates the flow (that is, flux) of a vector field through a surface to the behavior of the vector field inside the surface. More precisely, the divergence theorem states that the outward flux of a vector field through a closed surface is equal to the volume integral of the divergence over the region inside the surface. Intuitively, it states that the sum of all sources minus the sum of all sinks gives the net flow out of a region. In vector calculus, and more generally in differential geometry, the generalized Stokes' theorem is a statement about the integration of differential forms on manifolds, which both simplifies and generalizes several theorems from vector calculus.

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Minimal Coupling = (EM)Potential Interaction
Conservation of 4-TotalMomentum

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

$\mathbf{P} = (E/c=mc,\mathbf{p})$: 4-Momentum

$\mathbf{Q} = (V/c,\mathbf{q}) = q\mathbf{A} = q(\varphi/c,\mathbf{a})$: 4-PotentialMomentum

$\mathbf{A} = (\varphi/c,\mathbf{a})$: 4-VectorPotential

$\mathbf{P}_f = (E_f/c=(E+q\varphi)/c,\mathbf{p}_f)$: 4-MomentumIncPotentialField

$\mathbf{P}_T = (E_T/c,\mathbf{p}_T) = (H/c,\mathbf{p}_T) = \Sigma_n [\mathbf{P}_{fn}]$: 4-TotalMomentum

$\mathbf{P} = \mathbf{P}_f - q\mathbf{A} = (E_f/c - q\varphi/c,\mathbf{p}_f - q\mathbf{a})$: Minimal Coupling Relation

$\mathbf{P}_f = \mathbf{P} + \mathbf{Q} = \mathbf{P} + q\mathbf{A}$: Conservation of 4-MomentumIncPotentialField

$\mathbf{P}_f = \mathbf{P} + \mathbf{Q}$

$\mathbf{P}_f = \mathbf{P} + q\mathbf{A}$

$\mathbf{P}_f = (m_o)\mathbf{U} + (q\varphi_o/c^2)\mathbf{U}$

$\mathbf{P}_f = (E_o/c^2)\mathbf{U} + (q\varphi_o/c^2)\mathbf{U}$

$\mathbf{P}_f = ([E_o+q\varphi_o]/c^2)\mathbf{U}$

$\mathbf{P}_f = ([E_o+q\varphi_o]/c^2)\gamma(c,\mathbf{u})$

$\mathbf{P}_f = ([E+q\varphi]/c^2)(c,\mathbf{u})$

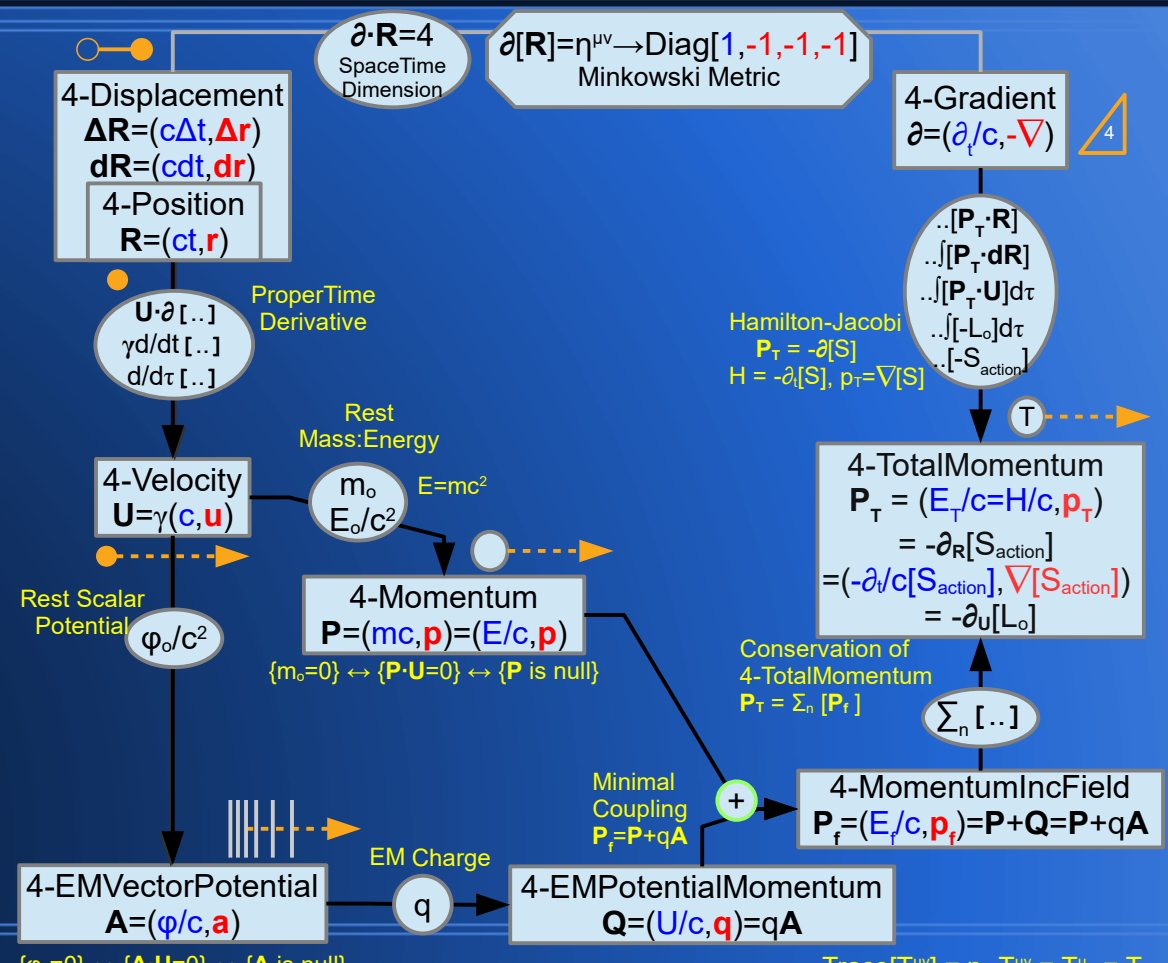
$\mathbf{P}_f = ([E+q\varphi]/c,[E+q\varphi]\mathbf{u}/c^2)$

$\mathbf{P}_f = ([E+q\varphi]/c,E\mathbf{u}/c^2 + q\varphi\mathbf{u}/c^2)$

$\mathbf{P}_f = ((E+q\varphi)/c,\mathbf{p}+q\mathbf{a})$

4-MomentumIncPotentialField has a contribution from:
a Mass "charge" (m_o) Lorentz Scalar
an EM charge (q) Lorentz Scalar interacting with a potential (φ_o)

$\mathbf{P}_T = \Sigma_n [\mathbf{P}_f]$: Conservation of 4-TotalMomentum
4-TotalMomentum is the Sum over all such 4-Momenta



SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$

$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$

= Lorentz Scalar Invariant

SRQM Study:

SRQM Hamiltonian:Lagrangian Connection

$H + L = (\mathbf{p}_T\cdot\mathbf{u}) = \gamma(\mathbf{P}_T\cdot\mathbf{U}) + -(\mathbf{P}_T\cdot\mathbf{U})/\gamma$

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John B. Wilson
SciRealm@aol.com
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4-Momentum $\mathbf{P} = m_o\mathbf{U} = (E_o/c^2)\mathbf{U}$
4-VectorPotential $\mathbf{A} = (\varphi_o/c^2)\mathbf{U}$
4-TotalMomentumIncField $\mathbf{P}_f = (\mathbf{P} + q\mathbf{A}) = (E_f/c=(E+q\varphi)/c, \mathbf{p}_f=\mathbf{p}+q\mathbf{a})$
4-TotalMomentum $\mathbf{P}_T = (E_T/c=H/c, \mathbf{p}_T) = \Sigma_n [\mathbf{P}_{fn}]$:

$\mathbf{P}\cdot\mathbf{U} = \gamma(E - \mathbf{p}\cdot\mathbf{u}) = E_o = m_oc^2$: $\mathbf{A}\cdot\mathbf{U} = \gamma(\varphi - \mathbf{a}\cdot\mathbf{u}) = \varphi_o$
$\mathbf{P}_T\cdot\mathbf{U} = (\mathbf{P}\cdot\mathbf{U} + q\mathbf{A}\cdot\mathbf{U}) = E_o + q\varphi_o = m_oc^2 + q\varphi_o$

$\gamma = 1/\mathrm{Sqrt}[1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}]$: Relativistic Gamma Identity
$(\gamma - 1/\gamma) = (\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})$: Manipulate into this form… still an identity
$(\gamma - 1/\gamma)(\mathbf{P}_T\cdot\mathbf{U}) = (\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\mathbf{P}_T\cdot\mathbf{U})$: Still covariant with Lorentz Scalar
$\gamma(\mathbf{P}_T\cdot\mathbf{U}) + -(\mathbf{P}_T\cdot\mathbf{U})/\gamma = (\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\mathbf{P}_T\cdot\mathbf{U})$
$\gamma(\mathbf{P}_T\cdot\mathbf{U}) + -(\mathbf{P}_T\cdot\mathbf{U})/\gamma = (\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})(E_o + q\varphi_o)$
$\gamma(\mathbf{P}_T\cdot\mathbf{U}) + -(\mathbf{P}_T\cdot\mathbf{U})/\gamma = (\gamma\mathbf{u}\cdot\mathbf{u})(E_o + q\varphi_o)/c^2$
$\gamma(\mathbf{P}_T\cdot\mathbf{U}) + -(\mathbf{P}_T\cdot\mathbf{U})/\gamma = [\gamma(E_o/c^2 + q\varphi_o/c^2)\mathbf{u}\cdot\mathbf{u}]$
$\gamma(\mathbf{P}_T\cdot\mathbf{U}) + -(\mathbf{P}_T\cdot\mathbf{U})/\gamma = [(\gamma E_o\mathbf{u}/c^2 + \gamma q\varphi_o\mathbf{u}/c^2)\cdot\mathbf{u}]$
$\gamma(\mathbf{P}_T\cdot\mathbf{U}) + -(\mathbf{P}_T\cdot\mathbf{U})/\gamma = [(E\mathbf{u}/c^2 + q\varphi\mathbf{u}/c^2)\cdot\mathbf{u}]$
$\gamma(\mathbf{P}_T\cdot\mathbf{U}) + -(\mathbf{P}_T\cdot\mathbf{U})/\gamma = [(\mathbf{p}+q\mathbf{a})\cdot\mathbf{u}]$
$\gamma(\mathbf{P}_T\cdot\mathbf{U}) + -(\mathbf{P}_T\cdot\mathbf{U})/\gamma = (\mathbf{p}_T\cdot\mathbf{u})$
$\{\ H\ \} + \{\ L\ \} = (\mathbf{p}_T\cdot\mathbf{u})$: The {Hamiltonian:Lagrangian} Connection

Lorentz Gamma Identity
$\gamma = 1/\sqrt{[1 - \beta^2]}$
$\gamma^2 = 1/(1 - \beta^2)$
$(1 - \beta^2)\gamma^2 = 1$
$(\gamma^2 - \gamma^2\beta^2) = 1$
$(\gamma^2 - 1) = \gamma^2\beta^2$
$(\gamma - 1/\gamma) = (\gamma\beta^2)$

$H = \gamma(\mathbf{P}_T\cdot\mathbf{U}) = \gamma((\mathbf{P}+q\mathbf{A})\cdot\mathbf{U})$ = The Hamiltonian with minimal coupling
$L = -(\mathbf{P}_T\cdot\mathbf{U})/\gamma = -((\mathbf{P}+q\mathbf{A})\cdot\mathbf{U})/\gamma$ = The Lagrangian with minimal coupling

H:L Connection in Density Format
$H + L = (\mathbf{p}_T\cdot\mathbf{u})$
$nH + nL = n(\mathbf{p}_T\cdot\mathbf{u})$, with number density $n = \gamma n_o$
$\mathcal{H} + \mathcal{L} = (\mathbf{g}_T\cdot\mathbf{u})$, with
momentum density $\{\mathbf{g}_T = n\mathbf{p}_T\}$
Hamiltonian Density $\{\mathcal{H} = nH\}$
Lagrangian Density $\{\mathcal{L} = nL = (\gamma n_o)(L_o/\gamma) = n_oL_o\}$
Lagrangian Density is Lorentz Scalar

for an EM field (photonic):
$\mathcal{H} = (1/2)\{\varepsilon_o\mathbf{e}\cdot\mathbf{e} + \mathbf{b}\cdot\mathbf{b}/\mu_o\}$
$\mathcal{L} = (1/2)\{\varepsilon_o\mathbf{e}\cdot\mathbf{e} - \mathbf{b}\cdot\mathbf{b}/\mu_o\} = (-1/4\mu_o)F_{\mu\nu}F^{\mu\nu}$
$\mathcal{H} + \mathcal{L} = \varepsilon_o\mathbf{e}\cdot\mathbf{e} = (\mathbf{g}_T\cdot\mathbf{u})$
$|\mathbf{u}| = c$
$|\mathbf{g}_T| = \varepsilon_o\mathbf{e}\cdot\mathbf{e}/c$
Poynting Vector $|\mathbf{s}| = |\mathbf{g}|c^2 \rightarrow c\varepsilon_o\mathbf{e}\cdot\mathbf{e}$

$H_o + L_o = 0$ Calculating the Rest Values

$H_o = (\mathbf{P}_T\cdot\mathbf{U})$ $\quad H = \gamma H_o$
$L_o = -(\mathbf{P}_T\cdot\mathbf{U})$ $\quad L = L_o/\gamma$

4-Vector notation gives a very nice way to find the Hamiltonian/Lagrangian connection:
$(H) + (L) = (\mathbf{p}_T\cdot\mathbf{u})$, where $H = \gamma(\mathbf{P}_T\cdot\mathbf{U})$ & $L = -(\mathbf{P}_T\cdot\mathbf{U})/\gamma$

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

4-Vector SRQM Interpretation of QM

SRQM Study:

SRQM Hamiltonian:Lagrangian Connection

$H + L = (\mathbf{p_T}\cdot\mathbf{u}) = \gamma(\mathbf{P_T}\cdot\mathbf{U}) + -(\mathbf{P_T}\cdot\mathbf{U})/\gamma$

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$\gamma = 1/\sqrt{[1 - \beta^2]}$:	Lorentz Relativistic Factor
$(\gamma - 1/\gamma) = (\gamma\beta^2)$:	Relativistic Identity, rearranged
$(\gamma - 1/\gamma)(\mathbf{P_T}\cdot\mathbf{U}) = (\gamma\beta^2)(\mathbf{P_T}\cdot\mathbf{U})$:	Identity * Lorentz Scalar
$\gamma(\mathbf{P_T}\cdot\mathbf{U}) + -(\mathbf{P_T}\cdot\mathbf{U})/\gamma = (\gamma\beta^2)(\mathbf{P_T}\cdot\mathbf{U})$	
$\gamma(\mathbf{P_T}\cdot\mathbf{U}) + -(\mathbf{P_T}\cdot\mathbf{U})/\gamma = (\mathbf{p_T}\cdot\mathbf{u})$	
$(H) + (L) = (\mathbf{p_T}\cdot\mathbf{u})$:	Identity, but not Lorentz Scalars

Binomial Approximation
$(1+x)^n \sim 1+nx+O(x^2)$ for $|x|<<1$

The non-relativistic Hamiltonian H is an approximation of the relativistic H:

$H = \gamma(m_oc^2 + q\Phi_o)$
$H = (1/\sqrt{[1-(v/c)^2]})(m_oc^2 + q\Phi_o)$
$H \sim ([1+(v/c)^2/2])(m_oc^2 + q\Phi_o) = (m_oc^2 + q\Phi_o)+(1/2)(m_oc^2v^2/c^2 + q\Phi_ov^2/c^2)$
$H \sim (m_oc^2 + q\Phi_o) + (1/2)(m_ov^2 + 0)$
$H \sim (1/2)(m_ov^2) + (m_oc^2 + q\Phi_o)$
H ~ (Kinetic) + (Rest+Potential) = T + V {for |v| << c}

The non-relativistic Lagrangian L is an approximation of the relativistic L:

$L = -(m_oc^2 + q\Phi_o)/\gamma$
$-L = (m_oc^2 + q\Phi_o)/\gamma = \sqrt{[1-(v/c)^2]}(m_oc^2 + q\Phi_o)$
$-L \sim (m_oc^2 + q\Phi_o) - (1/2)(m_oc^2v^2/c^2 + q\Phi_ov^2/c^2)$
$-L \sim (m_oc^2 + q\Phi_o) - (1/2)(m_ov^2 + \sim 0)$
$L \sim (1/2)(m_ov^2) - (m_oc^2 + q\Phi_o)$
L ~ (Kinetic) - (Rest+Potential) = T - V {for |v| << c}

Thus, (H ~ T + V) and (L ~ T - V) only in the non-relativistic limit (|v| << c)
$H + L \sim (T + V) + (T - V) = 2T = 2\,(1/2\, m_o\mathbf{u}\cdot\mathbf{u}) = \mathbf{p}\cdot\mathbf{u}$
Thus, (H) + (L) = ($\mathbf{p}\cdot\mathbf{u}$) in the non-relativistic case.

$H = \pm m_oc^2\sqrt{[1 + (\mathbf{p_T} - q\mathbf{a})^2/(m_o^2c^2)]} + q\varphi$
$H \sim \pm m_oc^2[1 + (\mathbf{p_T} - q\mathbf{a})^2/(2m_o^2c^2)] + q\varphi$ for $|(\mathbf{p_T} - q\mathbf{a})^2/(m_oc)^2| << 1$
$H \sim \pm [m_oc^2 + (\mathbf{p_T} - q\mathbf{a})^2/(2m_o)] + q\varphi$ for $|(\mathbf{p_T} - q\mathbf{a})^2/(m_oc)^2| << 1$ {non-relativistic limit}

Relativistic Hamiltonian $H = \gamma(\mathbf{P_T}\cdot\mathbf{U})$	Relativistic Lagrangian $L = -(\mathbf{P_T}\cdot\mathbf{U})/\gamma$	$\mathbf{p_T}\cdot\mathbf{u} = (\gamma\beta^2)(\mathbf{P_T}\cdot\mathbf{U}) =$ $H + L = \mathbf{p_T}\cdot\mathbf{u} = \gamma(\mathbf{P_T}\cdot\mathbf{U}) - (\mathbf{P_T}\cdot\mathbf{U})/\gamma$
$H = \gamma(\mathbf{P_T}\cdot\mathbf{U})$ $H = \gamma((\mathbf{P} + \mathbf{Q})\cdot\mathbf{U})$ $H = \gamma(\mathbf{P}\cdot\mathbf{U} + \mathbf{Q}\cdot\mathbf{U})$ $H = \gamma\mathbf{P}\cdot\mathbf{U} + \gamma\mathbf{Q}\cdot\mathbf{U}$ $H = \gamma m_o\mathbf{U}\cdot\mathbf{U} + \gamma q\mathbf{A}\cdot\mathbf{U}$ $H = \gamma m_oc^2 + q\gamma\varphi_o$ $H = \gamma(m_oc^2 + q\varphi_o)$ $H = mc^2 + q\varphi$ $H = (\gamma\beta^2 + 1/\gamma)m_oc^2 + q\varphi$ $H = (\gamma m_o\beta^2c^2 + m_oc^2/\gamma) + q\varphi$ $H = (\gamma m_ou^2 + m_oc^2/\gamma) + q\varphi$ $H = \mathbf{p}\cdot\mathbf{u} + m_oc^2/\gamma + q\varphi$ $H = E + q\varphi$ $H = \pm c\sqrt{[m_o^2c^2+p^2]} + q\varphi$ $H = \pm c\sqrt{[m_o^2c^2+(\mathbf{p_T} - q\mathbf{a})^2]} + q\varphi$ $H = \pm m_oc^2\sqrt{[1+(\mathbf{p_T} - q\mathbf{a})^2/(m_o^2c^2)]}+q\varphi$	$L = -(\mathbf{P_T}\cdot\mathbf{U})/\gamma$ $L = -((\mathbf{P} + \mathbf{Q})\cdot\mathbf{U})/\gamma$ $L = -(\mathbf{P}\cdot\mathbf{U} + \mathbf{Q}\cdot\mathbf{U})/\gamma$ $L = -\mathbf{P}\cdot\mathbf{U}/\gamma - \mathbf{Q}\cdot\mathbf{U}/\gamma$ $L = -m_o\mathbf{U}\cdot\mathbf{U}/\gamma - q\mathbf{A}\cdot\mathbf{U}/\gamma$ $L = -m_oc^2/\gamma - q\mathbf{A}\cdot\mathbf{U}/\gamma$ $L = -m_oc^2/\gamma - q(\varphi/c,\mathbf{a})\cdot\gamma(c,\mathbf{u})/\gamma$ $L = -m_oc^2/\gamma - q(\varphi/c,\mathbf{a})\cdot(c,\mathbf{u})$ $L = -m_oc^2/\gamma - q(\varphi - \mathbf{a}\cdot\mathbf{u})$ $L = -m_oc^2/\gamma - q\varphi + q\mathbf{a}\cdot\mathbf{u}$ $L = -m_oc^2/\gamma - q\varphi_o/\gamma$ $L = -(m_oc^2 + q\varphi_o)/\gamma$ $L = -(m_oc^2 + V_o)/\gamma$	$H + L = \gamma(\mathbf{P_T}\cdot\mathbf{U}) - (\mathbf{P_T}\cdot\mathbf{U})/\gamma$ $H + L = (\gamma - 1/\gamma)(\mathbf{P_T}\cdot\mathbf{U})$ $H + L = (\gamma\beta^2)(\mathbf{P_T}\cdot\mathbf{U})$ $H + L = (\gamma\beta^2)((\mathbf{P} + \mathbf{Q})\cdot\mathbf{U})$ $H + L = (\gamma\beta^2)(\mathbf{P}\cdot\mathbf{U} + \mathbf{Q}\cdot\mathbf{U})$ $H + L = (\gamma\beta^2)(m_oc^2 + q\varphi_o)$ $H + L = (\gamma m_o\beta^2c^2 + q\gamma\varphi_o\beta^2)$ $H + L = (\gamma m_o\mathbf{u}\cdot\mathbf{u}c^2/c^2 + q\varphi_o\gamma\mathbf{u}\cdot\mathbf{u}/c^2)$ $H + L = (\gamma m_o\mathbf{u}\cdot\mathbf{u} + q\mathbf{a}\cdot\mathbf{u})$ $H + L = (\mathbf{p}\cdot\mathbf{u} + q\mathbf{a}\cdot\mathbf{u})$ $H + L = (\mathbf{p}+q\mathbf{a})\cdot\mathbf{u}$ $H + L = \mathbf{p_T}\cdot\mathbf{u}$
Rest Hamiltonian, Invariant $H_o = (\mathbf{P_T}\cdot\mathbf{U}) = H/\gamma$	Rest Lagrangian, Invariant $L_o = -(\mathbf{P_T}\cdot\mathbf{U}) = \gamma L$	Rest Factor, Invariant $H_o + L_o = 0$

$\mathbf{P_T} = -\partial_U[L_o] = -\partial_U[\gamma L] = -\partial_U[-(\mathbf{P_T}\cdot\mathbf{U})] = \mathbf{P_T}$
$(E_T/c,\mathbf{p_T}) = -(\partial_{\gamma c},-\partial_{\gamma u})[\gamma L] = (-\partial_c,\partial_u)[L]$
$\mathbf{p_T} = \partial_u[L] = (\partial/\partial\mathbf{u})[L]$

Rules to manipulate the equations

$\mathbf{P_T} = \mathbf{P} + \mathbf{Q}$
$\mathbf{P} = m_o\mathbf{U} = (E_o/c^2)\mathbf{U}$: $\mathbf{Q} = q\mathbf{A} = q(\varphi_o/c^2)\mathbf{U}$
$\mathbf{P_T}\cdot\mathbf{U} = H_o$
$\mathbf{U}\cdot\mathbf{U} = c^2$
$\mathbf{P}\cdot\mathbf{U} = m_oc^2 = E_o$: $\mathbf{A}\cdot\mathbf{U} = \varphi_o$
$m = \gamma m_o$: $E = \gamma E_o$: $\varphi = \gamma\varphi_o$

4-Vector notation gives a very nice way to find the Hamiltonian/Lagrangian connection:
(H) + (L) = ($\mathbf{p_T}\cdot\mathbf{u}$), where $H = \gamma(\mathbf{P_T}\cdot\mathbf{U})$ & $L = -(\mathbf{P_T}\cdot\mathbf{U})/\gamma$

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study:
SR Lagrangian, Lagrangian Density, and Relativistic Action (S)

4-Vector SRQM Interpretation of QM

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Relativistic Action (S) is Lorentz Scalar Invariant

$S = \int L dt = \int (L_o/\gamma)(dt) = \int (L_o/\gamma)(\gamma d\tau) = \int (L_o)(d\tau)$

$S = \int L dt = \int (\mathcal{L}/n) dt = \int [\mathcal{L}/(n)] dt = \int \mathcal{L}(d^3x) dt = \int (\mathcal{L}/c)(d^3x)(cdt) = \int (\mathcal{L}/c)(d^4x)$

Explicitly-Covariant Relativistic Action (S)

Particle Form	Density Form {= n_o*Particle}
$S = \int L_o d\tau = -\int H_o d\tau$	$S = (1/c)\int (n_o L_o)(d^4x) = -(1/c)\int (n_o H_o)(d^4x)$
$S = -\int (\mathbf{P_T}\cdot\mathbf{U}) d\tau$	$S = (1/c)\int (\mathcal{L})(d^4x)$
$S = -\int (\mathbf{P_T}\cdot\mathbf{dR}/d\tau) d\tau$	
$S = -\int (\mathbf{P_T}\cdot\mathbf{dR})$	$S = \int (\mathcal{L}/c)(d^4x)$
$S = -\int (\mathbf{P_T}\cdot\mathbf{U}) d\tau$	$S = -(1/c)\int n_o (\mathbf{P_T}\cdot\mathbf{U})(d^4x)$
$S = -\int ((\mathbf{P} + q\mathbf{A})\cdot\mathbf{U}) d\tau$	$S = -(1/c)\int n_o ((\mathbf{P} + q\mathbf{A})\cdot\mathbf{U})(d^4x)$
$S = -\int (\mathbf{P}\cdot\mathbf{U} + q\mathbf{A}\cdot\mathbf{U}) d\tau$	$S = -(1/c)\int (n_o\mathbf{P}\cdot\mathbf{U} + n_o q\mathbf{A}\cdot\mathbf{U})(d^4x)$
$S = -\int (E_o + q\mathbf{U}\cdot\mathbf{A}) d\tau$	$S = -(1/c)\int (n_o E_o + n_o q\mathbf{U}\cdot\mathbf{A})(d^4x)$
$S = -\int (E_o + q\varphi_o) d\tau$	$S = -(1/c)\int (\rho_{E^o} + \mathbf{J}\cdot\mathbf{A})(d^4x)$
$S = -\int (E_o + V_o) d\tau$	
$S = -\int (m_o c^2 + V_o) d\tau$	$S = (1/c)\int (\mathcal{L})(d^4x)$
$S = -\int (m_o c^2) d\tau - q(\mathbf{A}\cdot d\mathbf{X})$	$S = (1/c)\int ((1/2)\{\varepsilon_o \mathbf{e}\cdot\mathbf{e} - \mathbf{b}\cdot\mathbf{b}/\mu_o\})(d^4x)$
	$S = (1/c)\int ((-1/4\mu_o) F_{\mu\nu}F^{\mu\nu})(d^4x)$
with $V_o = q\varphi_o$	for an EM field = no rest frame

thus $S = \int L dt = -\int [(m_o c^2 + V_o)/\gamma] dt$ w/ $L = -[(m_o c^2 + V_o)/\gamma]$

$S = (\mathbf{R}\cdot\partial_U)[L_o]$

Lagrangian { $L = (\mathbf{p_T}\cdot\mathbf{u}) - H$ } is *not* Lorentz Scalar Invariant

Rest Lagrangian { $L_o = \gamma L = -(\mathbf{P_T}\cdot\mathbf{U})$ } is Lorentz Scalar Invariant

Lagrangian Density { $\mathcal{L} = nL = (\gamma n_o)(L_o/\gamma) = n_o L_o$ } is Lorentz Scalar Invariant

$n = \gamma n_o = \#/d^3x = \#/(dx)(dy)(dz)$ = number density

$dt = \gamma d\tau$

$cd\tau = n_o(cdt)(dx)(dy)(dz) = n_o(d^4x)$

$d\tau = (n_o/c)(d^4x)$

H:L Connection in Density Format for Photonic System (no rest-frame)

$H + L = (\mathbf{p_T}\cdot\mathbf{u})$

$nH + nL = n(\mathbf{p_T}\cdot\mathbf{u})$, with number density $n = \gamma n_o$

$\mathcal{H} + \mathcal{L} = (\mathbf{g_T}\cdot\mathbf{u})$, with

momentum density $\{\mathbf{g_T} = n\mathbf{p_T}\}$

Hamiltonian density $\{\mathcal{H} = nH\}$

Lagrangian Density $\{\mathcal{L} = nL = (\gamma n_o)(L_o/\gamma) = n_o L_o\}$

Lagrangian Density is Lorentz Scalar

for an EM field (photonic):

$\mathcal{H} = (1/2)\{\varepsilon_o \mathbf{e}\cdot\mathbf{e} + \mathbf{b}\cdot\mathbf{b}/\mu_o\} = n_o E_o = \rho_{E^o}$ = EM Field Energy Density

$\mathcal{L} = (1/2)\{\varepsilon_o \mathbf{e}\cdot\mathbf{e} - \mathbf{b}\cdot\mathbf{b}/\mu_o\} = (-1/4\mu_o) F_{\mu\nu}F^{\mu\nu} = (-1/4\mu_o)$*Faraday EM Tensor Inner Product

$\mathcal{H} + \mathcal{L} = \varepsilon_o \mathbf{e}\cdot\mathbf{e} = (\mathbf{g_T}\cdot\mathbf{u})$

$|\mathbf{u}| = c$

$|\mathbf{g_T}| = \varepsilon_o \mathbf{e}\cdot\mathbf{e}/c$

Poynting Vector $|\mathbf{s}| = |\mathbf{g}|c^2 \rightarrow c\varepsilon_o \mathbf{e}\cdot\mathbf{e}$

$\varepsilon_o\mu_o = 1/c^2$:Electric:Magnetic Constant Eqn

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

The Relativistic Action Equation is seen in many different formats

SRQM Study: SR Hamilton-Jacobi Equation and Relativistic Action (S)

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Lagrangian { $L = (\mathbf{p}_T\cdot\mathbf{u}) - H = -(\mathbf{P}_T\cdot\mathbf{U})/\gamma$ } is *not* a Lorentz Scalar
Rest Lagrangian { $L_o = \gamma L = -(\mathbf{P}_T\cdot\mathbf{U})$ } is a Lorentz Scalar

Relativistic Action (S) is Lorentz Scalar
$S = \int L dt$
$S = \int (L_o/\gamma)(\gamma d\tau)$
$S = \int (L_o)(d\tau)$

Explicitly Covariant Relativistic Action (S)
$S = \int L_o d\tau = -\int H_o d\tau$
$S = -\int (\mathbf{P}_T\cdot\mathbf{U}) d\tau$
$S = -\int (\mathbf{P}_T\cdot d\mathbf{R}/d\tau) d\tau$
$S = -\int (\mathbf{P}_T\cdot d\mathbf{R})$
$S = -\int (\mathbf{P}_T\cdot\mathbf{U}) d\tau$
$S = -\int ((\mathbf{P} + q\mathbf{A})\cdot\mathbf{U}) d\tau$
$S = -\int (\mathbf{P}\cdot\mathbf{U} + q\mathbf{A}\cdot\mathbf{U}) d\tau$
$S = -\int (E_o + q\varphi_o) d\tau$
$S = -\int (E_o + V_o) d\tau$ with $V_o = q\varphi_o$
$S = -\int (m_o c^2 + V_o) d\tau$
$S = -\int (H_o) d\tau$

Rest Lagrangian
$L_o = -(\mathbf{P}_T\cdot\mathbf{U})$

Proper Time
$\mathbf{R}\cdot\partial_U = (\int d\mathbf{R})\cdot\partial_U = (\int \mathbf{U} d\tau)\cdot\partial_U = \int [..] d\tau$
Integral

4-Scalars
Relativistic Action Equation
Integral Format

4-Vectors
Relativistic Hamilton-Jacobi Equation
Differential Format

$$
\begin{aligned}
S_{action} &= -\int [\mathbf{P}_T\cdot d\mathbf{R}] \\
&= -\int [\mathbf{P}_T\cdot\mathbf{U}] d\tau \\
&= -\int [(H/c, \mathbf{p}_T)\cdot\gamma(c,\mathbf{u})] d\tau \\
&= -\int [\gamma(H - \mathbf{p}_T\cdot\mathbf{u}] d\tau \\
&= -\int [H_o] d\tau \\
&= \int [L_o] d\tau \\
&= (\mathbf{R}\cdot\partial_U)[L_o] \\
&= (\mathbf{R}\cdot\partial_U)[-(\mathbf{P}_T\cdot\mathbf{U})]
\end{aligned}
$$

Inverse

T

4-TotalMomentum
$$
\begin{aligned}
\mathbf{P}_T &= (E_T/c = H/c, \mathbf{p}_T) \\
&= -\partial_R[S_{action}] \\
&= (-\partial_t/c[S_{action}], \nabla[S_{action}]) \\
&= -\partial_U[L_o] = \partial_U[H_o]
\end{aligned}
$$

$\mathbf{P}_T = -\partial_U[L_o] = -\partial_U[\gamma L] = -\partial_U[-(\mathbf{P}_T\cdot\mathbf{U})] = \mathbf{P}_T$
$(E_T/c, \mathbf{p}_T) = -(\partial_{\gamma c}, -\partial_{\gamma u})[\gamma L] = (-\partial_c, \partial_u)[L]$
$\mathbf{p}_T = \partial_u[L] = (\partial/\partial\mathbf{u})[L]$

Hamilton-Jacobi Equation
$\partial[-S] = -\partial[S] = \mathbf{P}_T$

$S = -\int (E_o + q\varphi_o) d\tau$
$S = -(E_o + q\varphi_o)\int d\tau$
$S = -(E_o + q\varphi_o)(\tau + const)$

$-S = (E_o + q\varphi_o)(\tau + const)$
$\partial[-S] = (E_o + q\varphi_o)\partial[(\tau + const)]$
$\partial[-S] = (E_o + q\varphi_o)\partial[\tau]$
$\partial[-S] = (E_o + q\varphi_o)\partial[\mathbf{R}\cdot\mathbf{U}/c^2]$
$\partial[-S] = ((E_o + q\varphi_o)/c^2)\partial[\mathbf{R}\cdot\mathbf{U}]$
$\partial[-S] = (E_o/c^2 + q\varphi_o/c^2)\mathbf{U}$
$\partial[-S] = (m_o + q\varphi_o/c^2)\mathbf{U}$
$\partial[-S] = m_o\mathbf{U} + q(\varphi_o/c^2)\mathbf{U}$
$\partial[-S] = \mathbf{P} + q\mathbf{A}$
$\partial[-S] = \mathbf{P}_T$
Verified!

$\mathbf{R}\cdot\mathbf{U} = c^2\tau : \tau = \mathbf{R}\cdot\mathbf{U}/c^2$

The Hamilton-Jacobi Equation is incredibly simple in 4-Vector form
$-\partial[S_{action}] = \mathbf{P}_T$: gives temporal ($-\partial_t[S_{action}] = H = E_T$) & spatial ($\nabla[S_{action}] = \mathbf{p}_T$)

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(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study:
Relativistic Action (S), Rest Lagrangian (L_o) Path to 4-TotalMomentum

4-Vector SRQM Interpretation of QM

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4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})=(\mathbf{U}\cdot\partial_R)\mathbf{R}$
$=U^\alpha=(d/d\tau)\mathbf{R}$

Proper Time Derivative
$\mathbf{U}\cdot\partial_R=d\mathbf{R}/d\tau\cdot\partial_R=d/d\tau=\gamma d/dt$

$\mathbf{P_T} = -\partial_U[L_o] = -\partial_U[\gamma L] = -\partial_U[-(\mathbf{P_T}\cdot\mathbf{U})] = \mathbf{P_T}$
$(E_T/c,\mathbf{p_T}) = -(\partial_{\gamma c},-\partial_{\gamma u})[\gamma L] = (-\partial_c,\partial_u)[L]$
$\mathbf{p_T} = \partial_u[L] = (\partial/\partial\mathbf{u})[L]$

Rest Hamiltonian
$H_o= (\mathbf{P_T}\cdot\mathbf{U})$

-1

S_{action}
$= -\int[\mathbf{P_T}\cdot d\mathbf{R}]$
$= -\int[\mathbf{P_T}\cdot\mathbf{U}]d\tau$
$= -\int[(H/c,\mathbf{p_T})\cdot\gamma(c,\mathbf{u})]d\tau$
$= -\int[\gamma(H-\mathbf{p_T}\cdot\mathbf{u}]d\tau$
$= -\int[H_o]d\tau$
$= \int[L_o]d\tau$
$=(\mathbf{R}\cdot\partial_U)[L_o]$
$=(\mathbf{R}\cdot\partial_U)[-(\mathbf{P_T}\cdot\mathbf{U})]$

-1

4-PositionGradient:4-Gradient
$\partial_R{}^\beta=\partial_R=\partial/\partial R_\beta=\partial=(\partial_t/c,-\nabla)$
$\rightarrow(\partial/\partial ct,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

4-TotalMomentum
$\mathbf{P_T} = (E_T/c=H/c,\mathbf{p_T})$
$= -\partial_R[S_{action}]$
$=(-\partial_t/c[S_{action}],\nabla[S_{action}])$
$= -\partial_U[L_o] = \partial_U[H_o]$

4-VelocityGradient
$\partial_U{}^\beta=\partial_U=\partial/\partial U_\beta=(\partial_{U_t}/c,-\nabla_U)$
$\rightarrow(\partial/\partial\gamma c,-\partial/\partial\gamma u_x,-\partial/\partial\gamma u_y,-\partial/\partial\gamma u_z)$

-1

Rest Lagrangian
$L_o= -(\mathbf{P_T}\cdot\mathbf{U})$
$=(\mathbf{U}\cdot\partial_R)[S_{action}]$

Proper Time Integral
$\mathbf{R}\cdot\partial_U=(\int d\mathbf{R})\cdot\partial_U=(\int\mathbf{U}d\tau)\cdot\partial_U=\int[..]d\tau$

4-Position
$\mathbf{R}=(ct,\mathbf{r})=(\mathbf{R}\cdot\partial_U)\mathbf{U}$
$=R^\alpha=(\int d\tau)\mathbf{U}=(\int d\tau)d\mathbf{R}/d\tau=\int d\mathbf{R}$

$\mathbf{U} = c^2\partial[\tau]$
$\mathbf{P} = m_o\mathbf{U} = -\partial[S_{action}]$

$m_o\mathbf{U} = m_oc^2\partial[\tau] = -\partial[S_{action}]$
$E_o\partial[\tau] = -\partial[S_{action}]$
$\partial[E_o\tau] = \partial[-S_{action}]$

$E_o\tau = -S_{action}$
$S_{action} = -E_o\tau$
$S_{action} = -E_o\int d\tau = -m_oc^2\int d\tau$

$(\mathbf{U}\cdot\partial_R) = d/d\tau = \gamma d/dt$

$(\mathbf{U}\cdot\partial_R)[\tau] = d\tau/d\tau = 1$
$(\mathbf{U}\cdot\partial_R)[t] = \gamma dt/dt = \gamma$
$(\mathbf{U}\cdot\partial_R)[\mathbf{R}] = \mathbf{U}$
$(\mathbf{U}\cdot\partial_R)[\mathbf{U}] = \mathbf{A}$
$(\mathbf{U}\cdot\partial_R)[\mathbf{P}] = \mathbf{F}$
$(\mathbf{U}\cdot\partial_R)[\mathbf{R}\wedge\mathbf{P}] = \mathbf{R}\wedge\mathbf{F}$

$(\mathbf{U}\cdot\partial_R)[S_{action}] = (\mathbf{U}\cdot\partial_R)[-\mathbf{P_T}\cdot\mathbf{R}]$
$= -\mathbf{P_T}\cdot\mathbf{U} = L_o$

$(\mathbf{R}\cdot\partial_U) = \int[..]d\tau = \int[..](1/\gamma)dt$

$(\mathbf{R}\cdot\partial_U)[1] =?= \int[1]d\tau = \tau$
$(\mathbf{R}\cdot\partial_U)[\gamma] = \int[\gamma](1/\gamma)dt = t$
$(\mathbf{R}\cdot\partial_U)[\mathbf{U}] = \mathbf{R}$
$(\mathbf{R}\cdot\partial_U)[\mathbf{A}] = \mathbf{U}$
$(\mathbf{R}\cdot\partial_U)[\mathbf{F}] = \mathbf{P}$
$(\mathbf{R}\cdot\partial_U)[\mathbf{R}\wedge\mathbf{F}] = \mathbf{R}\wedge\mathbf{P}$

$(\mathbf{R}\cdot\partial_U)[L_o] = (\mathbf{R}\cdot\partial_U)[-\mathbf{P_T}\cdot\mathbf{U}]$
$= [-\mathbf{P_T}\cdot\mathbf{R}]= S_{action}$

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Relativistic Hamilton-Jacobi Equation ($P_T = -\partial[S]$) Differential Format : 4-Vectors

4-Vector SRQM Interpretation of QM

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$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\eta^{\mu\nu}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$ Minkowski Metric

Relativistic Action (S) is Lorentz Scalar Invariant
$S = \int L dt = \int(L_o/\gamma)(\gamma d\tau) = \int(L_o)(d\tau) = \int L_o d\tau$

Explicitly-Covariant Relativistic Action (S): $d\tau = (1/c)\sqrt{[\mathbf{dR}\cdot\mathbf{dR}]}$

$S = \int L_o d\tau = -\int H_o d\tau$
$S = -\int(\mathbf{P_T}\cdot\mathbf{U})d\tau$
$S = -\int(\mathbf{P_T}\cdot\mathbf{dR/d\tau})d\tau$
$S = -\int(\mathbf{P_T}\cdot\mathbf{dR})$
$S = -\int(\mathbf{P_T}\cdot\mathbf{U})d\tau$
$S = -\int((\mathbf{P} + q\mathbf{A})\cdot\mathbf{U})d\tau$
$S = -\int(\mathbf{P}\cdot\mathbf{U} + q\mathbf{A}\cdot\mathbf{U})d\tau$
$S = -\int(E_o + q\varphi_o)d\tau$
$S = -\int(E_o + V_o)d\tau$ with $V_o = q\varphi_o$
$S = -\int(m_oc^2 + V_o)d\tau$

4-Displacement
$\mathbf{\Delta R}=(c\Delta t,\mathbf{\Delta r})$
$\mathbf{dR}=(cdt,\mathbf{dr})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{dR}\cdot\mathbf{dR}=(cdt)^2-\mathbf{dr}\cdot\mathbf{dr}=(cd\tau)^2$
Invariant Interval

$(1/c)\sqrt{[..]}$

Proper Time
$d\tau = (1/c)\sqrt{[\mathbf{dR}\cdot\mathbf{dR}]}$
Differential

$\mathbf{U}\cdot\mathbf{R}=c^2\tau$

ProperTime Derivative

$\mathbf{U}\cdot\partial_R[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

Proper Time
$\mathbf{U}\cdot\partial = d/d\tau = \gamma d/dt$
Derivative

Proper Time
$d\tau$
Differential

$\mathbf{P_T}\cdot\mathbf{U} = \gamma(H-\mathbf{p_T}\cdot\mathbf{u}) = H_o = -L_o$

Invariant Rest Hamiltonian
= -Invariant Rest Lagrangian

$\mathbf{P_T}\cdot\mathbf{dR} = -L_o d\tau$

4-PositionGradient
4-Gradient
$\partial_R=\partial=(\partial_t/c,-\nabla)$
$\rightarrow(\partial/\partial ct,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

$S = -\int(\mathbf{P_T}\cdot\mathbf{dR})$
$S = \int L_o d\tau$
SR Action

$..[\mathbf{P_T}\cdot\mathbf{R}]$
$..\int[\mathbf{P_T}\cdot\mathbf{dR}]$
$..\int[\mathbf{P_T}\cdot\mathbf{U}]d\tau$
$..\int[-L_o]d\tau$
$..[-S_{action}]$

$\mathbf{P_T} = -\partial[S]$
$H = -\partial_t[S]$, $\mathbf{p_T}=\nabla[S]$
Hamilton-Jacobi Equation

4-TotalMomentum
$\mathbf{P_T}=(E_T/c,\mathbf{p_T})=(H/c,\mathbf{p_T})$

Conservation
4-TotalMomentum

$\sum_n[..]$

$\mathbf{U}\cdot\partial[..]$
$d/d\tau[..]$

4-Force
$\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$

n_o

ρ_o

EM

φ_o/c^2

m_o
E_o/c^2

$E=mc^2$

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

4-MomentumIncField
$\mathbf{P_f}=(E_f/c,\mathbf{p_f})=((E+U)/c,\mathbf{p}+q\mathbf{a})$

Minimal Coupling

4-NumberFlux
$\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$

q

4-ChargeFlux
4-CurrentDensity
$\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

4-EMVectorPotential
$\mathbf{A}=(\varphi/c,\mathbf{a})$

q

4-EMPotentialMomentum
$\mathbf{Q}=(U/c,\mathbf{q})=q\mathbf{A}$

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SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Relativistic Action Equation
(S = -∫(P_T·dR)) Integral Format : 4-Scalars

4-Vector SRQM Interpretation of QM

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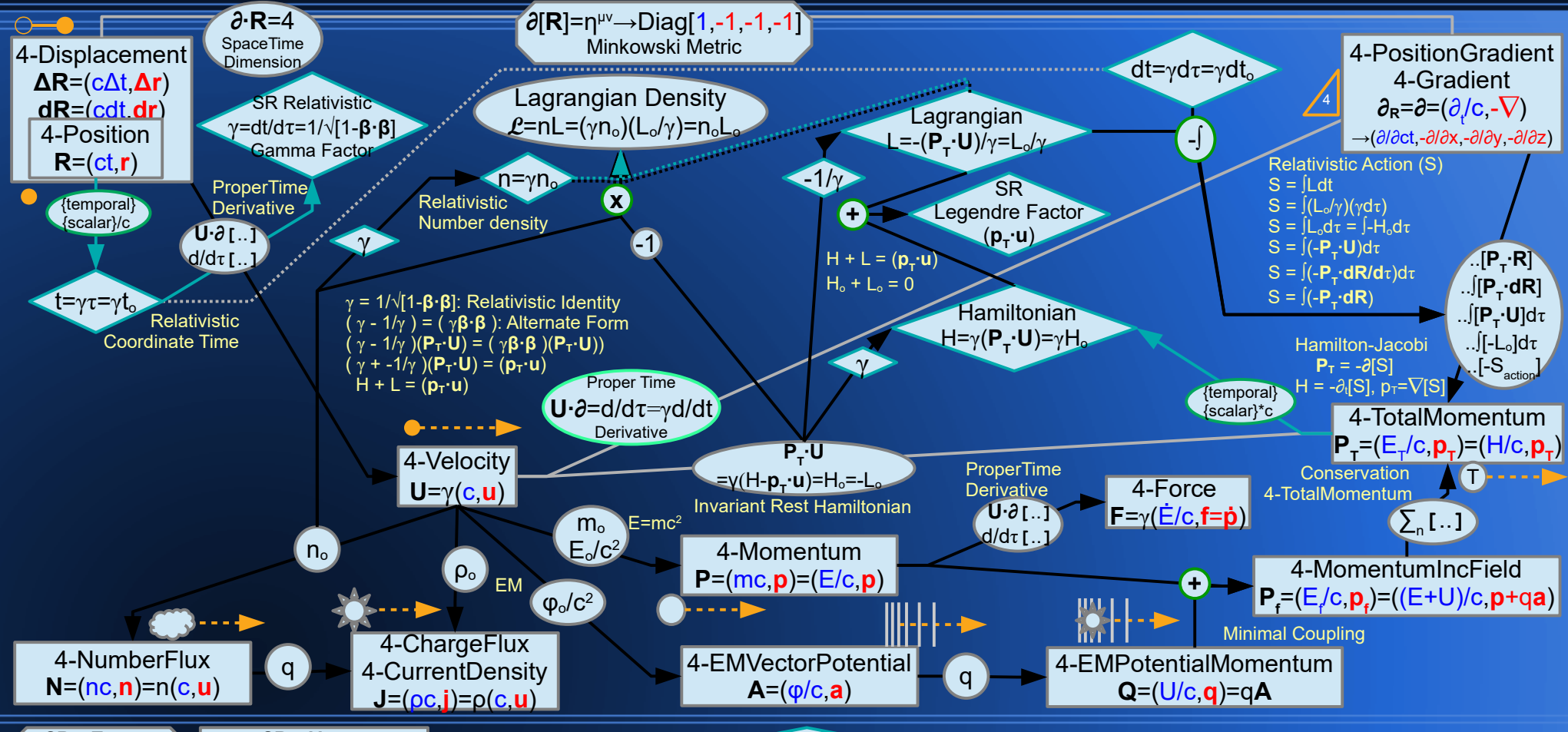

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor V^{μ} = **V** = $(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_0
Lorentz Scalar

SR Relativistic Scalar
(not Lorentz Invariant)

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_0)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram: Relativistic Factors
Hamiltonian & Lagrangian
Relativistic Euler-Lagrange Equation

4-Vector SRQM Interpretation of QM

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4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial[\mathbf{R}]=\eta^{\mu\nu}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$ Minkowski Metric

SR Relativistic $\gamma=dt/d\tau=1/\sqrt{[1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$ Gamma Factor

{temporal} {scalar}/c

ProperTime Derivative $\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]$

$t=\gamma\tau=\gamma t_o$ Relativistic Coordinate Time

Lagrangian Density $\mathcal{L}=nL=(\gamma n_o)(L_o/\gamma)=n_oL_o$

$n=\gamma n_o$ Relativistic Number density

γ

$\gamma = 1/\sqrt{[1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}]}$: Relativistic Identity
$(\gamma - 1/\gamma) = (\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})$: Alternate Form
$(\gamma - 1/\gamma)(\mathbf{P_T}\cdot\mathbf{U}) = (\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\mathbf{P_T}\cdot\mathbf{U})$
$(\gamma + -1/\gamma)(\mathbf{P_T}\cdot\mathbf{U}) = (\mathbf{p_T}\cdot\mathbf{u})$
$H + L = (\mathbf{p_T}\cdot\mathbf{u})$

Proper Time $\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$ Derivative

Particle Dynamics $\mathbf{U}=(d/d\tau)\mathbf{R}$

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{P_T}\cdot\mathbf{U}=\gamma(H-\mathbf{p_T}\cdot\mathbf{u})=H_o=-L_o$ Invariant Rest Hamiltonian

$dt=\gamma d\tau=\gamma dt_o$

Lagrangian $L=-(\mathbf{P_T}\cdot\mathbf{U})/\gamma=L_o/\gamma$

Lagrangian Action $S=\int Ldt$

$-1/\gamma$

SR Legendre Factor $(\mathbf{p_T}\cdot\mathbf{u})$

$H + L = (\mathbf{p_T}\cdot\mathbf{u})$
$H_o + L_o = 0$

Hamiltonian $H=\gamma(\mathbf{P_T}\cdot\mathbf{U})=\gamma H_o$

$\partial_U[\mathbf{U}]=\eta^{\alpha\beta}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$ Minkowski Metric

4-PositionGradient
4-Gradient
$\partial_R=\partial=(\partial_t/c,-\nabla)$
$\rightarrow(\partial/\partial ct,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Relativistic Euler-Lagrange Eqn
$\partial_R = (d/d\tau)\partial_U$

$\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]$

4-VelocityGradient
$\partial_U=(\partial_{U_t}/c,-\nabla_U)$
$\rightarrow(\partial/\partial\gamma c,-\partial/\partial\gamma u_x,-\partial/\partial\gamma u_y,-\partial/\partial\gamma u_z)$

$..[\mathbf{P_T}\cdot\mathbf{R}]$
$..\int[\mathbf{P_T}\cdot d\mathbf{R}]$
$..[-S_{action}]$

Hamilton-Jacobi
$\mathbf{P_T} = -\partial[S]$
$H = -\partial_t[S]$, $p_T=\nabla[S]$

{temporal} {scalar}*c

4-TotalMomentum
$\mathbf{P_T}=(E_T/c,\mathbf{p_T})=(H/c,\mathbf{p_T})$

Conservation 4-TotalMomentum

$\sum_n[..]$

4-Force
$\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$

ProperTime Derivative $\mathbf{U}\cdot\partial[..]$ $d/d\tau[..]$

m_o E_o/c^2 $E=mc^2$

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

4-MomentumIncField
$\mathbf{P_f}=(E_f/c,\mathbf{p_f})=((E+U)/c,\mathbf{p}+q\mathbf{a})$

Minimal Coupling

n_o

ρ_o EM φ_o/c^2

4-NumberFlux
$\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$

q

4-ChargeFlux
4-CurrentDensity
$\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

4-EMVectorPotential
$\mathbf{A}=(\varphi/c,\mathbf{a})$

q

4-EMPotentialMomentum
$\mathbf{Q}=(U/c,\mathbf{q})=q\mathbf{A}$

Note Similarity:
4-Velocity is ProperTime Derivative of 4-Position
$\mathbf{U} = (d/d\tau)\mathbf{R}$ [m/s] = [1/s]*[m]

Relativistic Euler-Lagrange Eqn
$\partial_R = (d/d\tau)\partial_U$ [1/m] = [1/s]*[s/m]

The differential form just inverses the dimensional units

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SR Relativistic Scalar
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$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Relativistic Euler-Lagrange Equation
The Easy Derivation ($\mathbf{U}=(d/d\tau)\mathbf{R}\rightarrow(\partial_R=(d/d\tau)\partial_U)$)

4-Vector SRQM Interpretation of QM

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Note Similarity:

4-Velocity is ProperTime Derivative of 4-Position
$\mathbf{U} = (d/d\tau)\mathbf{R}$ [m/s] = [1/s]*[m]

Relativistic Euler-Lagrange Eqn
$\partial_R = (d/d\tau)\partial_U$ [1/m] = [1/s]*[s/m]

The differential form just inverses the dimensional units, so the placement of the **R** and **U** switch.

That is it: so simple! Much, much easier than how I was taught in Grad School.

To complete the process and create the Equations of Motion, one just applies the base form to a Lagrangian.

This can be:
a classical Lagrangian
a relativistic Lagrangian
a Lorentz scalar Lagrangian
a quantum Lagrangian

Relativistic Particle Dynamics Eqn (4-Vector)
$\mathbf{U} = (d/d\tau)\mathbf{R}$
Classical limit, spatial component (3-vector)
$\mathbf{u} = (d/dt)\mathbf{r}$

[m]

4-Position
$\mathbf{R}=(ct,\mathbf{r})=(\mathbf{R}\cdot\partial_U)\mathbf{U}$
$=R^\alpha=(\int d\tau)\mathbf{U}$

$\mathbf{U}\cdot\partial_R$ [..]
$\gamma d/dt$ [..]
$d/d\tau$ [..]

[1/s]

[m/s]

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})=(\mathbf{U}\cdot\partial_R)\mathbf{R}$
$=U^\alpha=(d/d\tau)\mathbf{R}$

Natural 4-Vector
= 4D (1,0)-Tensor

$\partial_R\cdot\mathbf{R}=4$
SpaceTime Dimension

[1]

$\partial_U[\mathbf{U}]=\eta^{\alpha\beta}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$
Minkowski Metric

[1]

[1/s]

[s]

Proper Time
$\mathbf{R}\cdot\partial_U=\int d\tau$
Integral

Proper Time
$\mathbf{U}\cdot\partial_R=d/d\tau=\gamma d/dt$
Derivative

$\mathbf{U}\cdot\partial_R = d/d\tau = \gamma d/dt$
Interestingly, this has it's own similar inverse relations.
$d/d\tau = \gamma d/dt$
$dt = \gamma d\tau$

$\partial_U\cdot\mathbf{U}=4$
SpaceTime Dimension

[1]

[1]

$\partial_R[\mathbf{R}]=\eta^{\alpha\beta}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$
Minkowski Metric

4

4-VelocityGradient
$\partial_U{}^\beta=\partial_U=\partial/\partial U_\beta=(\partial_{U_t}/c,-\nabla_U)$
$\rightarrow(\partial/\partial\gamma c,-\partial/\partial\gamma u_x,-\partial/\partial\gamma u_y,-\partial/\partial\gamma u_z)$

$\mathbf{U}\cdot\partial_R$ [..]
$\gamma d/dt$ [..]
$d/d\tau$ [..]

[1/s]

4-PositionGradient:4-Gradient
$\partial_R{}^\beta=\partial_R=\partial/\partial R_\beta=\partial=(\partial_t/c,-\nabla)$
$\rightarrow(\partial/\partial ct,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

4-Vector
{ Index-raised-from One-Form }
= 4D (1,0)-Tensor

[s/m]

[1/m]

$\eta^{\alpha\beta}$
Raise index

Relativistic 4D Euler-Lagrange Eqn
$\partial_R = (d/d\tau)\partial_U$
$\partial/\partial\underline{\mathbf{R}} = (d/d\tau)\partial/\partial\underline{\mathbf{U}}$
$\partial[L]/\partial\underline{\mathbf{R}} = (d/d\tau)\partial[L]/\partial\underline{\mathbf{U}}$
Classical limit, spatial component
$\partial[L]/\partial\mathbf{r} = (d/dt)\partial[L]/\partial\mathbf{u}$
$\partial[L]/\partial\mathbf{x} = (d/dt)\partial[L]/\partial\mathbf{u}$

$\eta^{\alpha\beta}$
Raise index

VelocityGradient One-Form
$\partial_{U^\alpha}=\partial/\partial U^\alpha=(\partial_{U_t}/c,\nabla_U)$
$\rightarrow(\partial/\partial\gamma c,\partial/\partial\gamma u_x,\partial/\partial\gamma u_y,\partial/\partial\gamma u_z)$

PositionGradient One-Form
Gradient One-Form
$\partial_{R^\alpha}=\partial/\partial R^\alpha=(\partial_t/c,\nabla)$
$\rightarrow(\partial/\partial ct,\partial/\partial x,\partial/\partial y,\partial/\partial z)$

One-Form
= 4D (0,1)-Tensor

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathbf{P_T} = -\partial_U[L_o] = -\partial_U[\gamma L] = -\partial_U[-(\mathbf{P_T}\cdot\mathbf{U})] = \mathbf{P_T}$
$(E_T/c,\mathbf{p_T}) = -(\partial_{\gamma c},-\partial_{\gamma u})[\gamma L] = (-\partial_c,\partial_u)[L]$
$\mathbf{p_T} = \partial_u[L] = (\partial/\partial\mathbf{u})[L]$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Relativistic Euler-Lagrange Equation
The Easy Derivation ($\mathbf{U}=(d/d\tau)\mathbf{R}\rightarrow(\partial_R=(d/d\tau)\partial_U)$)

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Relativistic 4D Euler-Lagrange Eqn

$\partial_R = (d/d\tau)\partial_U$

$\partial/\partial\underline{\mathbf{R}} = (d/d\tau)\partial/\partial\underline{\mathbf{U}}$

$\partial[L]/\partial\underline{\mathbf{R}} = (d/d\tau)\partial[L]/\partial\underline{\mathbf{U}}$

Classical limit, spatial component

$\partial[L]/\partial\mathbf{r} = (d/dt)\partial[L]/\partial\mathbf{u}$

$\partial[L]/\partial\mathbf{x} = (d/dt)\partial[L]/\partial\mathbf{u}$

$L = -[(m_oc^2 + V_o)/\gamma]$

$\partial[L]/\partial\mathbf{u} = -(m_oc^2 + V_o)\partial[1/\gamma]/\partial\mathbf{u}$

$\partial[L]/\partial\mathbf{u} = (m_oc^2 + V_o)(\gamma\mathbf{u}/c^2)$

$(d/dt)\partial[L]/\partial\mathbf{u} = (d/dt)[(m_oc^2 + V_o)(\gamma\mathbf{u}/c^2)]$

$\partial[L]/\partial\mathbf{x} = -(1/\gamma)\partial[V_o]/\partial\mathbf{x}$

$(d/dt)\partial[L]/\partial\mathbf{u} = \partial[L]/\partial\mathbf{x}$

$(d/dt)[(m_oc^2 + V_o)(\gamma\mathbf{u}/c^2)] = -(1/\gamma)\partial[V_o]/\partial\mathbf{x}$

$(d/d\tau)\mathbf{P}_T = -\partial[V_o]$

Relativistic Particle Dynamics Eqn (4-Vector)

$\mathbf{U} = (d/d\tau)\mathbf{R}$

Classical limit, spatial component (3-vector)

$\mathbf{u} = (d/dt)\mathbf{r}$

[m] 4-Position
$\mathbf{R}=(ct,\mathbf{r})=(\mathbf{R}\cdot\partial_U)\mathbf{U}$
$=R^\alpha=(\int d\tau)\mathbf{U}$

$\mathbf{U}\cdot\partial_R[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

[1/s]

[m/s] 4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})=(\mathbf{U}\cdot\partial_R)\mathbf{R}$
$=U^\alpha=(d/d\tau)\mathbf{R}$

$\partial_R\cdot\mathbf{R}=4$ SpaceTime Dimension [1]

$\partial_U[\mathbf{U}]=\eta^{\alpha\beta}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$ Minkowski Metric [1]

[s] Proper Time $\mathbf{R}\cdot\partial_U=\int d\tau$ Integral

[1/s] Proper Time $\mathbf{U}\cdot\partial_R=d/d\tau=\gamma d/dt$ Derivative

$\partial_U\cdot\mathbf{U}=4$ SpaceTime Dimension [1]

$\partial_R[\mathbf{R}]=\eta^{\alpha\beta}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$ Minkowski Metric [1]

[s/m] 4-VelocityGradient
$\partial_U{}^\beta=\partial_U=\partial/\partial U_\beta=(\partial_{U_t}/c,-\nabla_U)$
$\rightarrow(\partial/\partial\gamma c,-\partial/\partial\gamma u_x,-\partial/\partial\gamma u_y,-\partial/\partial\gamma u_z)$

$\mathbf{U}\cdot\partial_R[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

[1/s]

[1/m] 4-PositionGradient:4-Gradient
$\partial_R{}^\beta=\partial_R=\partial/\partial R_\beta=\partial=(\partial_t/c,-\nabla)$
$\rightarrow(\partial/\partial ct,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Relativistic 4D Euler-Lagrange Eqn

$\partial_R = (d/d\tau)\partial_U$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathbf{P}_T = -\partial_U[L_o] = -\partial_U[\gamma L] = -\partial_U[-(\mathbf{P}_T\cdot\mathbf{U})] = \mathbf{P}_T$
$(E_T/c,\mathbf{p}_T) = -(\partial_{\gamma c},-\partial_{\gamma u})[\gamma L] = (-\partial_c,\partial_u)[L]$
$\mathbf{p}_T = \partial_u[L] = (\partial/\partial\mathbf{u})[L]$

4-TotalMomentum
$\mathbf{P}_T = (E_T/c=H/c,\mathbf{p}_T)$
$= -\partial_R[S_{action}]$
$=(-\partial_t/c[S_{action}],\nabla[S_{action}])$
$= -\partial_U[L_o] = \partial_U[H_o]$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM Diagram:
Relativistic Euler-Lagrange Equation Alternate Forms: Particle vs. Density

SciRealm.org
John B. Wilson
SciRealm@aol.com
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4-Velocity **U** is ProperTime Derivative of 4-Position **R**. The Euler-Lagrange Eqn can be generated by taking the differential form of the same equation.

<u>Relativistic 4-Vector Kinematical Eqn</u>

$\mathbf{U} = (d/d\tau)[\mathbf{R}]$

$\mathbf{U}\cdot\mathbf{K} = (d/d\tau)[\mathbf{R}]\cdot\mathbf{K}$

<u>Relativistic Euler-Lagrange Eqns</u>

{uses gradient-type 4-Vectors}

$\partial_R = (d/d\tau)\partial_U$: {particle format}

$\partial_{R\cdot K} = (d/d\tau)\,\partial_{U\cdot K}$

$\partial_{(-\Phi)} = (d/d\tau)\,\partial_{U\cdot K}$

$\partial_{(-\Phi)} = (\mathbf{U}\cdot\partial_R)\,\partial_{U\cdot K}$

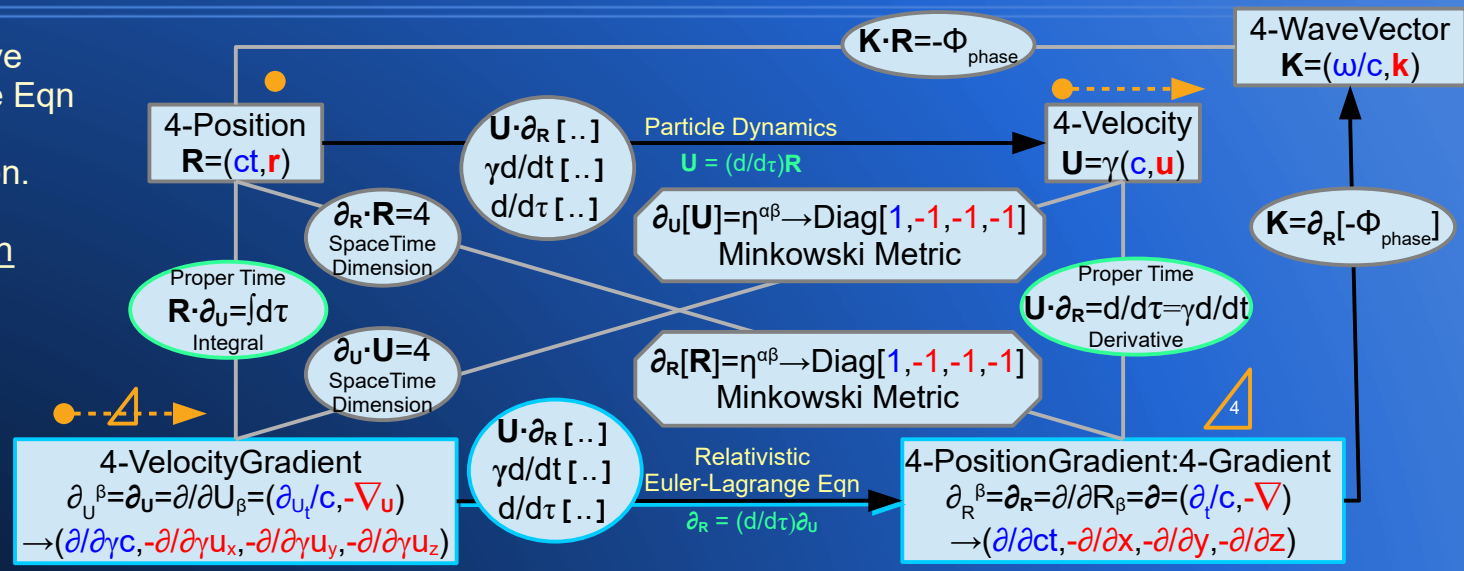
$\partial/\partial(-\Phi) = (\mathbf{U}\cdot\partial_R)\,\partial/\partial[\mathbf{U}\cdot\mathbf{K}]$

$\partial/\partial(-\Phi) = (\partial_R)\,\partial/\partial[\mathbf{K}]$

$\partial/\partial(-\Phi) = (\partial_R)\,\partial/\partial[\partial_R(-\Phi)]$

$\partial/\partial(\Phi) = (\partial_R)\,\partial/\partial[\partial_R(\Phi)]$

$\partial_{[\Phi]} = (\partial_R)\,\partial_{[\partial_R(\Phi)]}$: {density format}



$\mathcal{L} = (1/2)\{\,\partial_R[\Phi]\cdot\partial_R[\Phi] - (m_o c/\hbar)^2\,\Phi^2\,\}$: KG Lagrangian Density

$\partial_{[\Phi]}\,\mathcal{L} = (\partial_R)\,\partial_{[\partial_R(\Phi)]}\,\mathcal{L}$: Euler-Lagrange Eqn {density format}

$-(m_o c/\hbar)^2\,\Phi = (\partial_R)\cdot\partial_R[\Phi]$

$(\partial_R\cdot\partial_R)[\Phi] = -\,(m_o c/\hbar)^2\,\Phi$

$(\partial\cdot\partial) = -\,(m_o c/\hbar)^2$: KG Eqn of Motion

Klein-Gordon Relativistic Quantum Wave Eqn

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$

$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$

= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Relativistic Euler-Lagrange Equation
Equation of Motion (EoM) for EM particle

4-Vector SRQM Interpretation of QM

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4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\partial_R\cdot\mathbf{R}=4$
SpaceTime Dimension

$\partial_R[\mathbf{R}]=\eta^{\mu\nu}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$
Minkowski Metric

$L_o = -(\mathbf{P}_T\cdot\mathbf{U})$
$\partial_U[L_o] = -\mathbf{P}_T = -(\mathbf{P}+q\mathbf{A})$
$(d/d\tau)[\partial_U[L_o]] = (d/d\tau)[-\mathbf{P}_T] = -(d/d\tau)[\mathbf{P}+q\mathbf{A}] = -(\mathbf{F}+q(d/d\tau)[\mathbf{A}]) = -(\mathbf{F}+q\mathbf{U}\cdot\partial[\mathbf{A}]) = -(\mathbf{F}+qU_\nu\partial^\nu[\mathbf{A}])$
$\partial_R[L_o] = \partial_R[-\mathbf{P}_T\cdot\mathbf{U}] = -\partial_R[(\mathbf{P}+q\mathbf{A})\cdot\mathbf{U}] = (\mathbf{0}) + -q\partial_R[\mathbf{A}\cdot\mathbf{U}] = -q\partial_R[U_\nu A^\nu] = -qU_\nu\partial_R[A^\nu]$
assuming the 4-Gradient ∂_R of the 4-Velocity $\mathbf{U}$ is zero.

$L_o = -\mathbf{P}_T\cdot\mathbf{U}$
$= -(\mathbf{P}+q\mathbf{A})\cdot\mathbf{U}$

Euler-Lagrange Eqn: $(d/d\tau)\partial_U = \partial_R$
$-(\mathbf{F}+qU_\nu\partial^\nu[\mathbf{A}]) = -qU_\nu\partial_R[A^\nu]$
$\mathbf{F} + qU_\nu\partial^\nu[\mathbf{A}] = qU_\nu\partial_R[A^\nu]$
$\mathbf{F} = qU_\nu\partial_R[A^\nu] - qU_\nu\partial^\nu[\mathbf{A}]$
$\mathbf{F} = qU_\nu(\partial_R[A^\nu] - \partial^\nu[\mathbf{A}])$
$F^\mu = qU_\nu(\partial^\mu[A^\nu] - \partial^\nu[A^\mu])$
$F^\mu = qU_\nu(F^{\mu\nu}) = (dP^\mu/d\tau)$: EoM for EM particle
Lorentz Force Equation

4-PositionGradient
4-Gradient
$\partial_R=\partial=(\partial_t/c,-\nabla)$
$\rightarrow(\partial/\partial ct,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Relativistic Euler-Lagrange Eqn
$\partial_R = (d/d\tau)\partial_U$

$\mathbf{U}\cdot\partial[..]$
$d/d\tau[..]$

ProperTime Derivative
$\mathbf{U}\cdot\partial[..]$
$d/d\tau[..]$

-1

Proper Time
$\mathbf{U}\cdot\partial_R=d/d\tau=\gamma d/dt$
Derivative

4-VelocityGradient
$\partial_U=(\partial_{U_t}/c,-\nabla_U)$
$\rightarrow(\partial/\partial\gamma c,-\partial/\partial\gamma u_x,-\partial/\partial\gamma u_y,-\partial/\partial\gamma u_z)$

$..[\mathbf{P}_T\cdot\mathbf{R}]$
$..\int[\mathbf{P}_T\cdot d\mathbf{R}]$
$..[-S_{action}]$

Hamilton-Jacobi
$\mathbf{P}_T = -\partial[S]$
$H = -\partial_t[S]$, $p_T=\nabla[S]$

$\partial_U[\mathbf{U}]=\eta^{\alpha\beta}\rightarrow\mathrm{Diag}[1,-1,-1,-1]$
Minkowski Metric

$\partial_U\cdot\mathbf{U}=4$
SpaceTime Dimension

Note Similarity:
4-Velocity is ProperTime Derivative of 4-Position
$\mathbf{U} = (d/d\tau)\mathbf{R}$ [m/s] = [1/s]*[m]

Relativistic Euler-Lagrange Eqn
$\partial_R = (d/d\tau)\partial_U$ [1/m] = [1/s]*[s/m]

The differential form just inverses the dimensional units

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{P}_T\cdot\mathbf{U} = (\mathbf{P}+q\mathbf{A})\cdot\mathbf{U}$
$=\gamma(H-\mathbf{p}_T\cdot\mathbf{u})=H_o=-L_o$

$H + L = (\mathbf{p}_T\cdot\mathbf{u})$
$H_o + L_o = 0$

Invariant Rest Hamiltonian

4-TotalMomentum
$\mathbf{P}_T=(E_T/c,\mathbf{p}_T)=(H/c,\mathbf{p}_T)$

Conservation 4-TotalMomentum

T

$\sum_n[..]$

4-EMPotentialMomentum
$\mathbf{Q}=(U/c,\mathbf{q})=q\mathbf{A}$

n_o

ρ_o
EM

φ_o/c^2

4-EMVectorPotential
$\mathbf{A}=(\varphi/c,\mathbf{a})$

q

+
Minimal Coupling

4-MomentumIncField
$\mathbf{P}_f=(E_f/c,\mathbf{p}_f)=((E+U)/c,\mathbf{p}+q\mathbf{a})$

4-NumberFlux
$\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$

q

4-ChargeFlux
4-CurrentDensity
$\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

m_o
E_o/c^2
$E=mc^2$

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

$\mathbf{U}\cdot\partial[..]$
$d/d\tau[..]$

4-Force
$\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$

x
index lower

q

EM Faraday
$F^{\alpha\beta}=\partial^\alpha A^\beta-\partial^\beta A^\alpha$
$=[\ 0\ ,\ -e^i/c]$
$[+e^i/c,-\varepsilon^{ij}{}_k b^k]$
4-Tensor

$(dP^\mu/d\tau) = F^\mu = qU_\nu(F^{\mu\nu})$
Equation of motion for charged particle

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:

Relativistic Euler-Lagrange Equation using L_o Equation of Motion (EoM) for EM particle

4-Vector SRQM Interpretation of QM

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$\gamma = 1/\mathrm{Sqrt}[1-\mathbf{\beta}\cdot\mathbf{\beta}]$: Relativistic Gamma Identity
$(\gamma - 1/\gamma) = (\gamma\mathbf{\beta}\cdot\mathbf{\beta})$: Manipulate into this form… still an identity
$\gamma(\mathbf{P_T}\cdot\mathbf{U}) + -(\mathbf{P_T}\cdot\mathbf{U})/\gamma = (\gamma\mathbf{\beta}\cdot\mathbf{\beta})(\mathbf{P_T}\cdot\mathbf{U})$
$\gamma(\mathbf{P_T}\cdot\mathbf{U}) + -(\mathbf{P_T}\cdot\mathbf{U})/\gamma = (\mathbf{p_T}\cdot\mathbf{u})$
$\{\ H\ \} + \{\ L\ \} = (\mathbf{p_T}\cdot\mathbf{u})$: The Hamiltonian/Lagrangian connection

$H = \gamma H_o = \gamma(\mathbf{P_T}\cdot\mathbf{U}) = \gamma((\mathbf{P}+q\mathbf{A})\cdot\mathbf{U})$ = The Hamiltonian with minimal coupling
$L = L_o/\gamma = -(\mathbf{P_T}\cdot\mathbf{U})/\gamma = -((\mathbf{P}+q\mathbf{A})\cdot\mathbf{U})/\gamma$ = The Lagrangian with minimal coupling

$H_o = (\mathbf{P_T}\cdot\mathbf{U}) = -L_o = (\mathbf{U}\cdot\mathbf{P_T})$: Rest Hamiltonian = Total RestEnergy
$L_o = -(\mathbf{P_T}\cdot\mathbf{U}) = -H_o$

$(d/d\tau)\partial_U[L_o] = \partial_R[L_o]$

4-Velocity is ProperTime Derivative of 4-Position
$\mathbf{U} = (d/d\tau)\mathbf{R}$ [m/s] = [1/s]*[m]

Relativistic Euler-Lagrange Eqn
$\partial_R = (d/d\tau)\partial_U$ [1/m] = [1/s]*[s/m]

$\partial/\partial\underline{\mathbf{R}} = (d/d\tau)\partial/\partial\underline{\mathbf{U}}$
$\partial[L]/\partial\underline{\mathbf{R}} = (d/d\tau)\partial[L]/\partial\underline{\mathbf{U}}$

Classical limit, spatial component
$\partial[L]/\partial\mathbf{r} = (d/dt)\partial[L]/\partial\mathbf{u}$
$\partial[L]/\partial\mathbf{x} = (d/dt)\partial[L]/\partial\mathbf{u}$

$\mathbf{F_{EM}} = \gamma q\{(\mathbf{u}\cdot\mathbf{e})/c, (\mathbf{e}) + (\mathbf{u}\times\mathbf{b})\}$
$\mathbf{e} = (-\nabla\varphi - \partial_t\mathbf{a})$ and $\mathbf{b} = [\nabla\times\mathbf{a}]$

If $\mathbf{a}\sim 0$, then $\mathbf{f} = -q\nabla\varphi = -\nabla U$, the force is neg grad of a potential

4-TotalMomentum
$\mathbf{P_T} = (E_T/c, \mathbf{p_T}) = (H/c, \mathbf{p_T}) = \mathbf{P}+q\mathbf{A}$

Rest Lagrangian L_o
$= -(\mathbf{P_T}\cdot\mathbf{U})$
$= -(\mathbf{P}+q\mathbf{A})\cdot\mathbf{U}$
$= -\mathbf{P}\cdot\mathbf{U} - q\mathbf{A}\cdot\mathbf{U}$

4-Velocity
$\mathbf{U} = \gamma(c, \mathbf{u})$

ProperTime
$\mathbf{U}\cdot\partial = d/d\tau = \gamma d/dt$
Derivative

4-VelocityGradient
$\partial_U = (\partial_{U_t}/c, -\nabla_U)$
$\rightarrow(\partial/\partial\gamma c, -\partial/\partial\gamma u_x, -\partial/\partial\gamma u_y, -\partial/\partial\gamma u_z)$

4-(Position)Gradient
$\partial_R = \partial = (\partial_t/c, -\nabla_R)$
$\rightarrow(\partial/\partial ct, -\partial/\partial x, -\partial/\partial y, -\partial/\partial z)$

x

4-VelocityGradient part
$(d/d\tau)\partial_U[L_o] = (d/d\tau)\partial/\partial\underline{\mathbf{U}}[L_o]$
$(d/d\tau)\partial_U{}^\alpha[L_o] = (d/d\tau)\partial/\partial U_\alpha[L_o]$
$= (d/d\tau)\partial_U[-\mathbf{P_T}\cdot\mathbf{U}]$
$= (d/d\tau)[-\mathbf{P_T}]$
$= -(d/d\tau)[\mathbf{P}+q\mathbf{A}]$
$= -(\mathbf{F}+q(d/d\tau)[\mathbf{A}])$
$= -(\mathbf{F}+q(\mathbf{U}\cdot\partial)[\mathbf{A}])$
$= -(F^\alpha + qU_\beta\partial^\beta[A^\alpha])$

Relativistic Rest Lagrangian Euler-Lagrange Equations of Motion

$(d/d\tau)\partial_U[L_o] = \partial_R[L_o]$

4-PositionGradient part
$\partial_R[L_o] = \partial/\partial\underline{\mathbf{R}}[L_o]$
$\partial_R{}^\alpha[L_o] = \partial/\partial R_\alpha[L_o]$
$= \partial_R[-\mathbf{P_T}\cdot\mathbf{U}]$
$= -\partial_R[(\mathbf{P}+q\mathbf{A})\cdot\mathbf{U}]$
$= (\mathbf{0}) + -q\partial_R[\mathbf{A}\cdot\mathbf{U}]$
$= -q\partial_R[U_\beta A^\beta]$
$= -qU_\beta\partial^\alpha[A^\beta]$

$\mathbf{P_T} = -\partial_U[L_o] = -\partial_U[\gamma L] = -\partial_U[-(\mathbf{P_T}\cdot\mathbf{U})] = \mathbf{P_T}$
$(E_T/c, \mathbf{p_T}) = -(\partial_{\gamma c}, -\partial_{\gamma u})[\gamma L] = (-\partial_c, \partial_u)[L]$
$\mathbf{p_T} = \partial_u[L] = (\partial/\partial\mathbf{u})[L]$

$-(F^\alpha + qU_\beta\partial^\beta[A^\alpha]) = -qU_\beta\partial^\alpha[A^\beta]$
$(F^\alpha + qU_\beta\partial^\beta[A^\alpha]) = qU_\beta\partial^\alpha[A^\beta]$
$F^\alpha = qU_\beta\partial^\alpha[A^\beta] - qU_\beta\partial^\beta[A^\alpha]$
$F^\alpha = qU_\beta(\partial^\alpha[A^\beta] - \partial^\beta[A^\alpha])$
$(dP^\alpha/d\tau) = F^\alpha = qU_\beta(F^{\alpha\beta})$
Lorentz Force Equation

Assumes:
$\partial[\mathbf{U}] = \partial_R[\mathbf{U}] = 0^{\mu\nu}$
$\partial[\mathbf{P}] = \partial_R[\mathbf{P}] = 0^{\mu\nu}$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
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= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:

Relativistic Euler-Lagrange Equation using L_o' Equation of Motion (EoM) for EM particle

4-Vector SRQM Interpretation of QM

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$L_o' = aL_o+b$: Lagrangian-with-affine-transform L_o' gives same physics, when using the Euler-Lagrange Equation

Let (a = const = -1) and (b = const = $-\frac{1}{2}m_oc^2 = -\frac{1}{2}m_o\mathbf{U}\cdot\mathbf{U}$)
Then $L_o' = -L_o - \frac{1}{2}m_oc^2 = \frac{1}{2}m_o\mathbf{U}\cdot\mathbf{U} + q\mathbf{A}\cdot\mathbf{U}$

Note, however, that some other defintions do *not* work with L_o'
The 4-TotalMomentum relation no longer works.

$(d/d\tau)\partial_U[L_o] = \partial_R[L_o]$

4-Velocity is ProperTime Derivative of 4-Position
$\mathbf{U} = (d/d\tau)\mathbf{R}$ [m/s] = [1/s]*[m]

Relativistic Euler-Lagrange Eqn
$\partial_R = (d/d\tau)\partial_U$ [1/m] = [1/s]*[s/m]

$\partial/\partial\underline{\mathbf{R}} = (d/d\tau)\partial/\partial\underline{\mathbf{U}}$
$\partial[L]/\partial\underline{\mathbf{R}} = (d/d\tau)\partial[L]/\partial\underline{\mathbf{U}}$

Classical limit, spatial component
$\partial[L]/\partial\mathbf{r} = (d/dt)\partial[L]/\partial\mathbf{u}$
$\partial[L]/\partial\mathbf{x} = (d/dt)\partial[L]/\partial\mathbf{u}$

$\mathbf{F}_{EM} = \gamma q\{ (\mathbf{u}\cdot\mathbf{e})/c, (\mathbf{e}) + (\mathbf{u}\times\mathbf{b}) \}$
$\mathbf{e} = (-\nabla\varphi - \partial_t\mathbf{a})$ and $\mathbf{b} = [\nabla\times \mathbf{a}]$

If $\mathbf{a}\sim 0$, then $\mathbf{f} = -q\nabla\varphi = -\nabla U$, the force is neg grad of a potential

4-TotalMomentum
$\mathbf{P}_T=(E_T/c,\mathbf{p}_T)=(H/c,\mathbf{p}_T)$
$=\mathbf{P}+q\mathbf{A}$

Rest Lagrangian L_o'
$= -a(\mathbf{P}_T\cdot\mathbf{U})+b$
$= \mathbf{P}\cdot\mathbf{U}+q\mathbf{A}\cdot\mathbf{U}-(\frac{1}{2})m_o\mathbf{U}\cdot\mathbf{U}$
$= m_o\mathbf{U}\cdot\mathbf{U}+q\mathbf{A}\cdot\mathbf{U}-(\frac{1}{2})m_o\mathbf{U}\cdot\mathbf{U}$
$= (\frac{1}{2})m_o\mathbf{U}\cdot\mathbf{U}+q\mathbf{A}\cdot\mathbf{U}$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

ProperTime Derivative
$\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$

4-VelocityGradient
$\partial_U=(\partial_{U_t}/c,-\nabla_U)$
$\rightarrow(\partial/\partial\gamma c,-\partial/\partial\gamma u_x,-\partial/\partial\gamma u_y,-\partial/\partial\gamma u_z)$

4-(Position)Gradient
$\partial_R=\partial=(\partial_t/c,-\nabla_R)$
$\rightarrow(\partial/\partial ct,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

x

4-VelocityGradient part
$(d/d\tau)\partial_U[L_o'] = (d/d\tau)\partial/\partial\underline{\mathbf{U}}[L_o']$
$(d/d\tau)\partial_U{}^\alpha[L_o'] = (d/d\tau)\partial/\partial U_\alpha[L_o']$
$= (d/d\tau)\partial_U[(\frac{1}{2})m_o\mathbf{U}\cdot\mathbf{U}+q\mathbf{A}\cdot\mathbf{U}]$
$= (d/d\tau)[m_o\mathbf{U}+q\mathbf{A}]$
$= (d/d\tau)[\mathbf{P}+q\mathbf{A}]$
$= (\mathbf{F}+q(d/d\tau)[\mathbf{A}])$
$= (\mathbf{F}+q(\mathbf{U}\cdot\partial)[\mathbf{A}])$
$= (F^\alpha+qU_\beta\partial^\beta[A^\alpha])$

Relativistic Rest Lagrangian Euler-Lagrange Equations of Motion

$(d/d\tau)\partial_U[L_o'] = \partial_R[L_o']$

4-PositionGradient part
$\partial_R[L_o'] = \partial/\partial\underline{\mathbf{R}}[L_o']$
$\partial_R{}^\alpha[L_o'] = \partial/\partial R_\alpha[L_o']$
$= \partial_R[(\frac{1}{2})m_o\mathbf{U}\cdot\mathbf{U}+q\mathbf{A}\cdot\mathbf{U}]$
$= (\mathbf{0}) + q\partial_R[\mathbf{A}\cdot\mathbf{U}]$
$= q\partial_R[U_\beta A^\beta]$
$= qU_\beta\partial^\alpha[A^\beta]$

$\mathbf{P}_T = -\partial_U[L_o] = -\partial_U[\gamma L] = -\partial_U[-(\mathbf{P}_T\cdot\mathbf{U})] = \mathbf{P}_T$
$(E_T/c,\mathbf{p}_T) = -(\partial_{\gamma c},-\partial_{\gamma u})[\gamma L] = (-\partial_c,\partial_u)[L]$
$\mathbf{p}_T = \partial_u[L] = (\partial/\partial\mathbf{u})[L]$
$\mathbf{P}_T \neq -\partial_U[L_o']$

$(F^\alpha+qU_\beta\partial^\beta[A^\alpha]) = qU_\beta\partial^\alpha[A^\beta]$
$F^\alpha = qU_\beta\partial^\alpha[A^\beta] - qU_\beta\partial^\beta[A^\alpha]$
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$\gamma(\mathbf{P_T}\cdot\mathbf{U}) + -(\mathbf{P_T}\cdot\mathbf{U})/\gamma = (\gamma\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\mathbf{P_T}\cdot\mathbf{U})$
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$\{\ H\ \} + \{\ L\ \} = (\mathbf{p_T}\cdot\mathbf{u})$: The Hamiltonian/Lagrangian connection

$H = \gamma H_o = \gamma(\mathbf{P_T}\cdot\mathbf{U}) = \gamma((\mathbf{P}+q\mathbf{A})\cdot\mathbf{U})$ = The Hamiltonian with minimal coupling
$L = L_o/\gamma = -(\mathbf{P_T}\cdot\mathbf{U})/\gamma = -((\mathbf{P}+q\mathbf{A})\cdot\mathbf{U})/\gamma$ = The Lagrangian with minimal coupling

$H_o = (\mathbf{P_T}\cdot\mathbf{U}) = -L_o = (\mathbf{U}\cdot\mathbf{P_T})$: Rest Hamiltonian = Total RestEnergy
$L_o = -(\mathbf{P_T}\cdot\mathbf{U}) = -H_o$

$\partial_{P_T}[H_o] = \partial_{P_T}[\mathbf{U}\cdot\mathbf{P_T}] = \partial_{P_T}[\mathbf{U}]\cdot\mathbf{P_T} + \mathbf{U}\cdot\partial_{P_T}[\mathbf{P_T}] = 0 + \mathbf{U}\cdot\partial_{P_T}[\mathbf{P_T}] = \mathbf{U} = d/d\tau[\mathbf{X}]$
Thus: $(d/d\tau)[\mathbf{X}] = \partial_{P_T}[H_o] = (\partial/\partial\underline{\mathbf{P_T}})[H_o]$
$\partial_X[H_o] = \partial_X[\mathbf{U}\cdot\mathbf{P_T}] = \partial_X[\mathbf{U}]\cdot\mathbf{P_T} + \mathbf{U}\cdot\partial_X[\mathbf{P_T}] = 0 + \mathbf{U}\cdot\partial_X[\mathbf{P_T}] = d/d\tau[\mathbf{P_T}]$
Thus: $(d/d\tau)[\mathbf{P_T}] = \partial_X[H_o] = (\partial/\partial\underline{\mathbf{X}})[H_o]$

Relativistic Hamilton's Equations (4-Vector):
$(d/d\tau)[\mathbf{X}] = (\partial/\partial\underline{\mathbf{P_T}})[H_o]$
$(d/d\tau)[\mathbf{P_T}] = (\partial/\partial\underline{\mathbf{X}})[H_o]$

$(d/d\tau)[\mathbf{X}] = \gamma(d/dt)[\mathbf{X}] = (\partial/\partial\underline{\mathbf{P_T}})[H_o] = (\partial/\partial\underline{\mathbf{P_T}})[(\mathbf{P_T}\cdot\mathbf{U})] = \mathbf{U}$
$(d/d\tau)[\mathbf{P_T}] = \gamma(d/dt)[\mathbf{P_T}] = (\partial/\partial\underline{\mathbf{X}})[H_o] = (\partial/\partial\underline{\mathbf{X}})[(\mathbf{P_T}\cdot\mathbf{U})] = \partial[(\mathbf{P_T}\cdot\mathbf{U})]$

Taking just the spatial components:
$\gamma(d/dt)[\mathbf{x}] = (\partial/\partial\underline{\mathbf{p_T}})[H_o] = (\partial/\partial\mathbf{p_T})(H/\gamma)$
$\gamma(d/dt)[\mathbf{p_T}] = (\partial/\partial\underline{\mathbf{x}})[H_o] = (\partial/\partial\underline{\mathbf{x}})(H/\gamma)$

Take the Classical limit $\{\gamma\to 1\}$

Classical Hamilton's Equations (3-vector):
$(d/dt)[\mathbf{x}] = (+\partial/\partial\mathbf{p_T})[H]$
$(d/dt)[\mathbf{p_T}] = (-\partial/\partial\mathbf{x})[H]$

4-TotalMomentum
$\mathbf{P_T} = (E_T/c = H/c, \mathbf{p_T})$
$= -\partial_R[S_{action}]$
$= (-\partial_t/c[S_{action}], \nabla[S_{action}])$
$= -\partial_U[L_o] = \partial_U[H_o]$

Rest Hamiltonian H_o
$= (\mathbf{P_T}\cdot\mathbf{U}) = \gamma(H-\mathbf{p_T}\cdot\mathbf{u})$
$= (\mathbf{P}+q\mathbf{A})\cdot\mathbf{U}$
$= \mathbf{P}\cdot\mathbf{U}+q\mathbf{A}\cdot\mathbf{U}$

4-Velocity
$\mathbf{U} = \gamma(c,\mathbf{u})$

4-Position
$\mathbf{X} = (ct,\mathbf{x})$

$(d/d\tau)[\mathbf{X}]$
$= \mathbf{U} = \gamma(c,\mathbf{u})$
= 4-Velocity
$= \mathbf{P}/m_o$
$= (\mathbf{P_T}-q\mathbf{A})/m_o$

Relativistic Rest Hamiltonian Hamilton's Equations of Motion

$(d/d\tau)[\mathbf{X}] = (\partial/\partial\underline{\mathbf{P_T}})[H_o]$

$(d/d\tau)[\mathbf{P_T}] = (\partial/\partial\underline{\mathbf{X}})[H_o]$

$(\partial_{P_T})[H_o] = (\partial/\partial\underline{\mathbf{P_T}})[H_o]$
$= (\partial/\partial\underline{\mathbf{P_T}})[\mathbf{P_T}\cdot\mathbf{U}]$
$= (\partial/\partial\underline{\mathbf{P_T}})[\underline{\mathbf{P_T}}\mathbf{U}]$
$= (\partial\underline{\mathbf{P_T}}/\partial\underline{\mathbf{P_T}})\mathbf{U}$
$= \mathbf{U} = \gamma(c,\mathbf{u})$
= 4-Velocity
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$(d/d\tau)[\mathbf{P_T}]$
$= (d/d\tau)[\mathbf{P}+q\mathbf{A}]$
$= [\mathbf{F} + q(d/d\tau)\mathbf{A}]$
$= [\mathbf{F} + q(\mathbf{U}\cdot\partial)\mathbf{A}]$
$= [F^\alpha + q(U_\beta\partial^\beta)A^\alpha]$
$= F^\alpha + qU_\beta\partial^\beta[A^\alpha]$

$F^\alpha + qU_\beta\partial^\beta[A^\alpha] = q\partial^\alpha[A^\beta]U_\beta$
$F^\alpha = q\partial^\alpha[A^\beta]U_\beta - qU_\beta\partial^\beta[A^\alpha]$
$F^\alpha = qU_\beta(\partial^\alpha[A^\beta] - \partial^\beta[A^\alpha])$
$(dP^\alpha/d\tau) = F^\alpha = qU_\beta(F^{\alpha\beta})$
Lorentz Force Equation

$(\partial_X)[H_o] = (\partial/\partial\underline{\mathbf{X}})[H_o]$
$= (\partial/\partial\underline{\mathbf{X}})[\mathbf{P}\cdot\mathbf{U}+q\mathbf{A}\cdot\mathbf{U}]$
$= [0 + q(\partial\mathbf{A}/\partial\underline{\mathbf{X}})\cdot\mathbf{U}]$
$= [q\partial[\mathbf{A}]\cdot\mathbf{U}]$
$= q\partial[\mathbf{A}]\cdot\mathbf{U}$
$= q\partial^\alpha[A^\beta]U_\beta$
$= q\partial[\mathbf{A}]\cdot(\mathbf{P_T}-q\mathbf{A})/m_o$

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$(d/d\tau)[X^\alpha]$
$= U^\alpha = \gamma(c,\mathbf{u})$
$=$ 4-Velocity
$= P^\alpha/m_o$
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Relativistic
Hamilton's Equations of Motion
using Rest Hamiltonian

$(d/d\tau)[X^\alpha] = (\partial_{Pt}{}^\alpha)[H_o] = (\partial/\partial P_{T\alpha})[H_o]$

$(d/d\tau)[P_T{}^\alpha] = (\partial_X{}^\alpha)[H_o] = (\partial/\partial X_\alpha)[H_o]$

$(\{\partial_{P_T}\}^\alpha)[H_o] = (\partial/\partial\{P_T\}_\alpha)[H_o]$
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$= (\partial/\partial\{P_T\}_\alpha)[\{P_T\}_\beta U^\beta]$
$= (\partial\{P_T\}_\beta/\partial\{P_T\}_\alpha)[U^\beta]$
$= \delta^\alpha{}_\beta U^\beta$
$= U^\alpha = \gamma(c,\mathbf{u})$
$=$ 4-Velocity
$= P^\alpha/m_o$
$= (P_T{}^\alpha - qA^\alpha)/m_o$

$(d/d\tau)[P_T{}^\alpha]$
$= (d/d\tau)[P^\alpha + qA^\alpha]$
$= [F^\alpha + q(d/d\tau)A^\alpha]$
$= F^\alpha + q(\mathbf{U}\cdot\partial)A^\alpha$
$= F^\alpha + q(U_\beta\partial^\beta)A^\alpha$

$\partial^\alpha[H_o] = (\partial/\partial X_\alpha)[H_o]$
$= (\partial/\partial X_\alpha)[\mathbf{P}\cdot\mathbf{U}+q\mathbf{A}\cdot\mathbf{U}]$
$= [0 + (\partial/\partial X_\alpha)qA^\beta U_\beta]$
$= q(\partial A^\beta/\partial X_\alpha)U_\beta$
$= q\partial^\alpha[A^\beta]U_\beta$
$= qU_\beta\partial^\alpha[A^\beta]$

$F^\alpha + qU_\beta\partial^\beta[A^\alpha] = q\partial^\alpha[A^\beta]U_\beta$
$F^\alpha = q\partial^\alpha[A^\beta]U_\beta - qU_\beta\partial^\beta[A^\alpha]$
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SRQM Study:
EM Lorentz Force Eqn→
Classical Force = - Grad[Potential] = -∇[U]

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Lorentz EM Force Equation:
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$dP^\alpha/d\tau = F^\alpha = q(\partial^\alpha A^\beta - \partial^\beta A^\alpha)U_\beta$

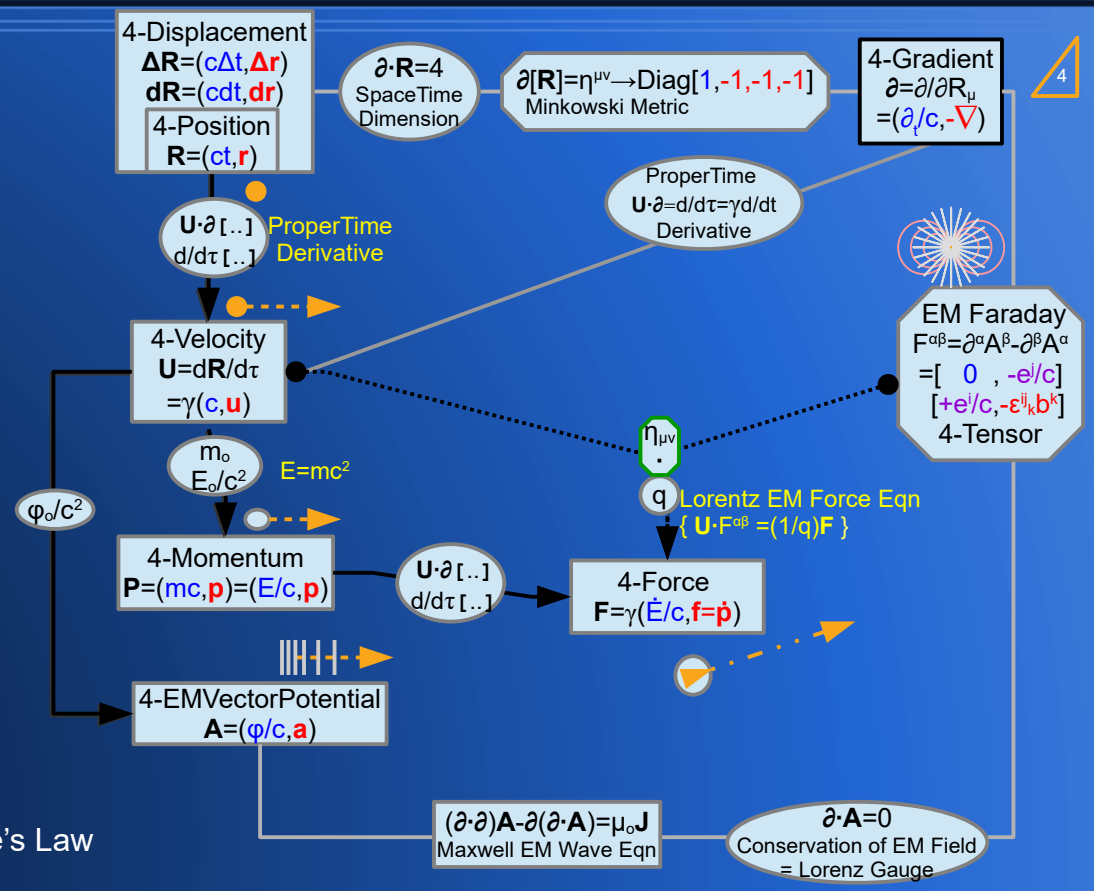
Examine just the spatial components F^i of 4-Force $\mathbf{F} = F^\alpha$:
$F^i = q(\partial^i A^\beta - \partial^\beta A^i)U_\beta$
$F^i = q(\partial^i A^0 - \partial^0 A^i)U_0 + q(\partial^i A^j - \partial^j A^i)U_j$
$\gamma\mathbf{f} = q(-\nabla[\varphi/c] - (\partial^t/c)\mathbf{a})(\gamma c) + q(-\nabla[\mathbf{a}\cdot\mathbf{u}] - -\mathbf{u}\cdot\nabla[\mathbf{a}])\gamma$
$\mathbf{f} = q(-\nabla[\varphi/c] - (\partial^t/c)\mathbf{a})(c) + q(\mathbf{u}\cdot\nabla[\mathbf{a}]-\nabla[\mathbf{a}\cdot\mathbf{u}])$
$\mathbf{f} = q(-\nabla[\varphi] - \partial^t\mathbf{a} + \mathbf{u}\cdot\nabla[\mathbf{a}] - \nabla[\mathbf{a}\cdot\mathbf{u}])$
$\mathbf{f} = q(-\nabla[\varphi] - \partial^t\mathbf{a} + \mathbf{u} \times \mathbf{b})$
$\mathbf{f} = q(\mathbf{e} + \mathbf{u} \times \mathbf{b})$

Take the limit of $\{ |\nabla[\varphi]| >> |\partial^t\mathbf{a} - \mathbf{u} \times \mathbf{b}| \}$
$\mathbf{f} \sim q(-\nabla[\varphi]) = -\nabla[q\varphi] = -\nabla[U] = -\text{Grad}[\text{Potential}]$

The Classical Force = -Grad[Potential]
when $\{ |\nabla[\varphi]| >> |\partial^t\mathbf{a} - \mathbf{u} \times \mathbf{b}| \}$ or when $\{ \mathbf{a} = \mathbf{0}$, non-magnetic$\}$

The majority of non-gravitational, non-nuclear potentials dealt with in CM are those mediated by the EM potential.

ex. Spring Potential $\{ U = kx^2/2 \}$, then $\{ \mathbf{f} = -\nabla[kx^2/2] = -k\mathbf{x} \}$ Hooke's Law



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SRQM Diagram:
Relativistic Hamilton's Equations
Equation of Motion (EoM) for Harmonic Oscillator

SciRealm.org
John B. Wilson
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$\mathbf{A}\cdot\mathbf{U} = \varphi_o$: RestScalar-Potential
$q\mathbf{A}\cdot\mathbf{U} = q\varphi_o = V_o$: RestVoltage = Electrical PotentialEnergy

Let { $q\mathbf{A}\cdot\mathbf{U} = V_o = -k\mathbf{X}\cdot\mathbf{X}/2$ } then { $\mathbf{A}\cdot\mathbf{U} = \varphi_o = -(k/q)\mathbf{X}\cdot\mathbf{X}/2$ }

RestHamiltonian $H_o = (\mathbf{P}_T\cdot\mathbf{U}) = \mathbf{P}\cdot\mathbf{U} + q\mathbf{A}\cdot\mathbf{U} = \mathbf{P}\cdot\mathbf{U} - k\mathbf{X}\cdot\mathbf{X}/2$

$\partial(\mathbf{A}\cdot\mathbf{U}) = \partial(\mathbf{A})\cdot\mathbf{U} + \mathbf{A}\cdot\partial(\mathbf{U})$
$\partial^\alpha(A^\beta U_\beta) = \partial^\alpha[A^\beta]U_\beta + A^\beta\partial^\alpha[U_\beta]$
$\partial^\alpha(A^\beta U_\beta) = \partial^\alpha[A^\beta]U_\beta + 0^\alpha$: assuming conservative field $\partial^\alpha[U_\beta]=0^\alpha{}_\beta$
$\partial^\alpha(A^\beta U_\beta) = \partial^\alpha[A^\beta]U_\beta$

$\partial[-(k/q)\mathbf{X}\cdot\mathbf{X}/2] = -(k/q)\mathbf{X}$

$\partial^\alpha(A^\beta U_\beta) = \partial^\alpha[A^\beta]U_\beta = -(k/q)X^\alpha$

$F^\alpha = qU_\beta(\partial^\alpha[A^\beta] - \partial^\beta[A^\alpha])$: Lorentz Force Eqn.
$F^\alpha = qU_\beta\partial^\alpha[A^\beta] - qU_\beta\partial^\beta[A^\alpha]$
$= -(k)X^\alpha - qU_\beta\partial^\beta[A^\alpha]$
$= -(k)X^\alpha - q(d/d\tau)[A^\alpha])$

Take spatial part: $\gamma\mathbf{f} = -k\mathbf{x} - q(\gamma d/dt[\mathbf{a}])$

Classical limit:
$\gamma \to 1 : |\mathbf{v}| << c$
$d/dt[\mathbf{a}] \to \mathbf{0}$: 3-vector-potential changes very slowly

{ $\mathbf{A}\cdot\mathbf{U} = \varphi_o = -(k/q)\mathbf{X}\cdot\mathbf{X}/2 = -(k/2q)\{ (ct)^2 - \mathbf{x}\cdot\mathbf{x} \}$

For t = 0
$\varphi_o = -(k/2q)\{-\mathbf{x}\cdot\mathbf{x}\} = (k/2q)\{\mathbf{x}\cdot\mathbf{x}\}$
$V_o = -(k/2)\{-\mathbf{x}\cdot\mathbf{x}\} = (k/2)\{\mathbf{x}\cdot\mathbf{x}\}$: PE of classical harmonic oscillator

Spring Potential { $U = kx^2/2$ }, then { $\mathbf{f} = -\nabla[kx^2/2] = -k\mathbf{x}$ } Hooke's Law
$\mathbf{f} = -k\mathbf{x}$: Spring Force acting on classical harmonic oscillator

RestHamiltonian
$H_o = (\mathbf{P}_T\cdot\mathbf{U}) = \mathbf{P}\cdot\mathbf{U} + q\mathbf{A}\cdot\mathbf{U} = \mathbf{P}\cdot\mathbf{U} - k\mathbf{X}\cdot\mathbf{X}/2$
$H_o = \mathbf{P}\cdot\mathbf{U} - k\mathbf{X}\cdot\mathbf{X}/2$: Covariant Relativistic version
$H_o = E_o + (k/2)\{\mathbf{x}\cdot\mathbf{x}\}$: Classical
Tot = Rest + Potential
$(d/d\tau)[X^\alpha] = (\partial/\partial P_{T\alpha})[H_o]$ gives $\mathbf{U} = \mathbf{U}$
$(d/d\tau)[P_T{}^\alpha] = (\partial/\partial X_\alpha)[H_o]$ gives $(d/d\tau)[P_T{}^\alpha] = -k\mathbf{X}\cdot\mathbf{U}$

RestLagrangian
$L_o = -(\mathbf{P}_T\cdot\mathbf{U}) = -\mathbf{P}\cdot\mathbf{U} + -q\mathbf{A}\cdot\mathbf{U} = -\mathbf{P}\cdot\mathbf{U} + k\mathbf{X}\cdot\mathbf{X}/2$
$(d/d\tau)\partial_U[L_o] = \partial_X[L_o]$: Relativistic Euler-Lagrange Eqn
$(d/d\tau)\partial_U[-\mathbf{P}\cdot\mathbf{U} + k\mathbf{X}\cdot\mathbf{X}/2] = \partial_X[-\mathbf{P}\cdot\mathbf{U} + k\mathbf{X}\cdot\mathbf{X}/2]$
$(d/d\tau)\partial_U[-\mathbf{P}\cdot\mathbf{U}] = \partial_X[k\mathbf{X}\cdot\mathbf{X}/2]$
$(d/d\tau)[-\mathbf{P}] = k\mathbf{X}$
$[-\mathbf{F}] = k\mathbf{X}$
$\mathbf{F} = -k\mathbf{X}$

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A Tensor Study of Physical 4-Vectors

SRQM: The Speed-of-Light (c)

c^2 Invariant Relations (part 1)

4-Vector SRQM Interpretation of QM

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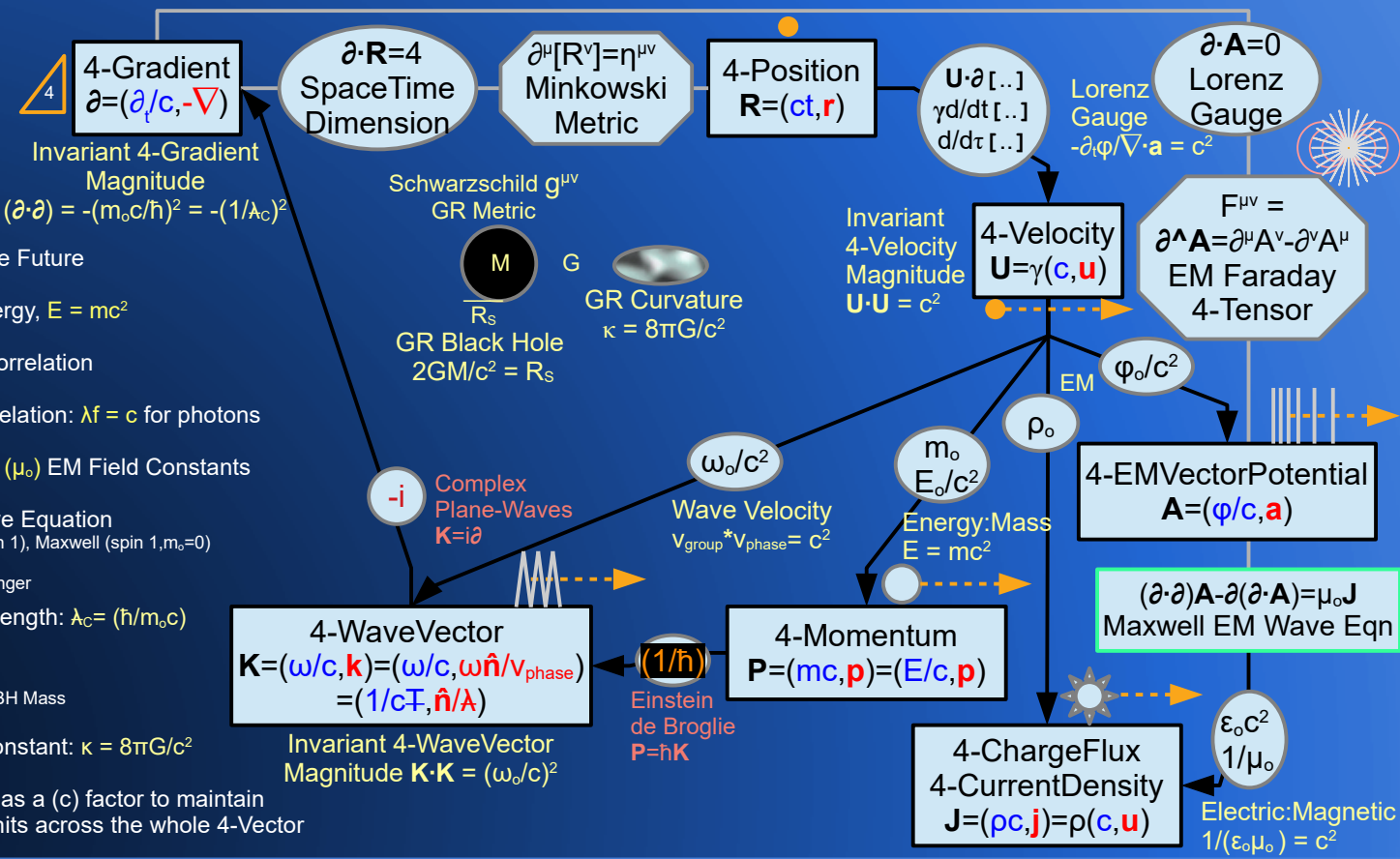
The Speed-of-Light (c) is THE connection between Time and Space: d**R** = (cdt,dr)

This physical constant appears in several seemingly unrelated places. You don't notice these cool relations when you set c→1. Also notice that the set of all these relations definitely rules out a variable speed-of-light. (c) is an Invariant Lorentz Scalar constant.

$\mathbf{U}\cdot\mathbf{U} = \gamma^2(c^2-\mathbf{u}\cdot\mathbf{u}) = c^2$	Speed of all things into the Future
$(E_o/m_o)=(\gamma E_o/\gamma m_o)=(E/m) = c^2$	Mass is concentrated Energy, $E = mc^2$
$\lvert u * v_{phase}\rvert=\lvert v_{group} * v_{phase}\rvert = c^2$	Particle-Wave "Duality" Correlation
$\lambda\!\!\!^{-2}(\omega^2-\omega_o^2) = \lambda^2(f^2-f_o^2) = c^2$	Wavelength-Frequency Relation: $\lambda f = c$ for photons
$(1/\varepsilon_o\mu_o) = c^2$	Electric (ε_o) and Magnetic (μ_o) EM Field Constants
$-(\hbar/m_o)^2(\partial\cdot\partial) = c^2$	Relativistic Quantum Wave Equation Klein-Gordon (spin 0), Proca (spin 1), Maxwell (spin 1,m_o=0) Factors to Dirac (spin ½) Classical-limit ($\lvert\mathbf{v}\rvert$<<c) to Schrödinger
$(\hbar/\lambda\!\!\!^{-}_C m_o)^2 = c^2$	Reduced Compton Wavelength: $\lambda\!\!\!^{-}_C = (\hbar/m_oc)$
$2GM/R_S = c^2$	GR Black Hole Equation R_S = Schwarzschild Radius G = GR GravitationalConst, M = BH Mass
$8\pi G/\kappa = c^2$	GR Einstein Curvature Constant: $\kappa = 8\pi G/c^2$
($c^{\pm1}$ * scalar, **3-vector**) = **4-Vector**	Every Physical 4-Vector has a (c) factor to maintain equivalent dimensional units across the whole 4-Vector



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$\partial^\mu[R^\nu]=\eta^{\mu\nu}$ Minkowski Metric — $\eta_{\mu\nu}$ · 4-Vector Scalar Product

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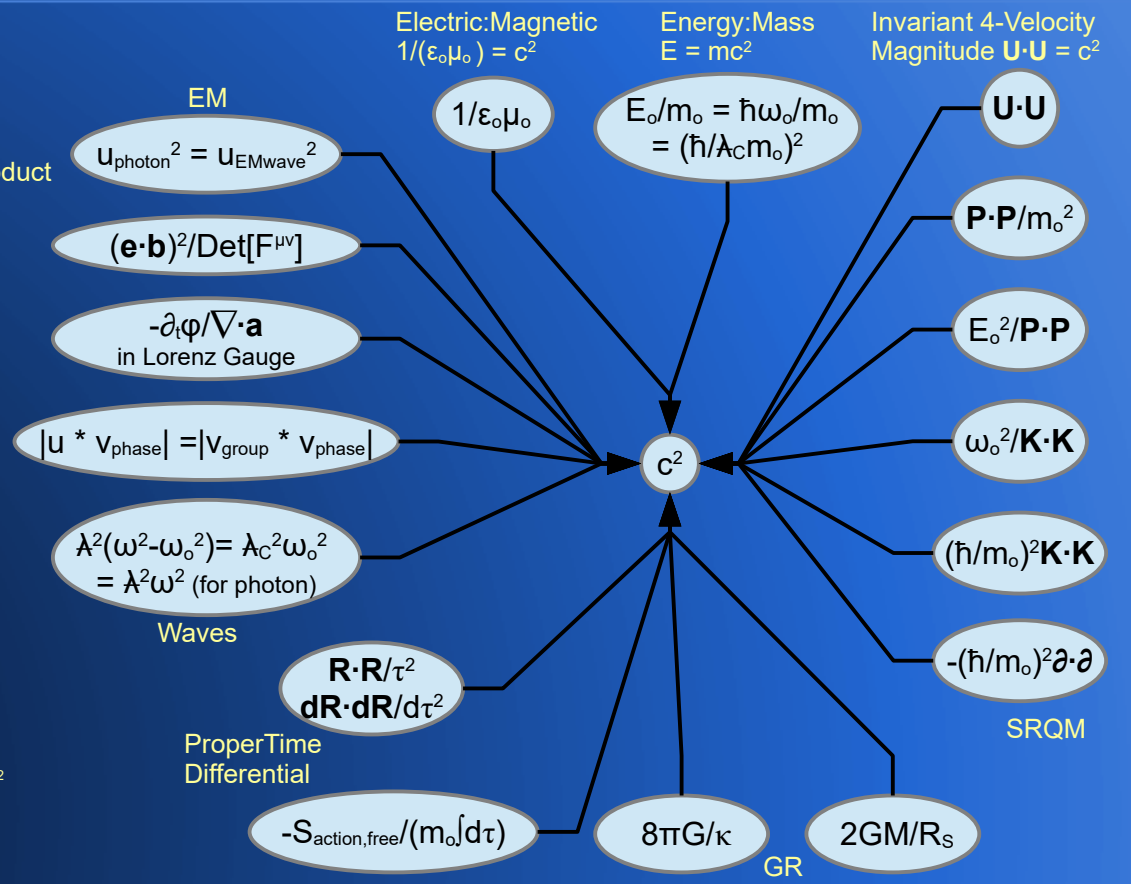

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A Tensor Study of Physical 4-Vectors

SRQM 4-Vector Study: 4-ThermalVector Relativistic Thermodynamics

4-Vector SRQM Interpretation of QM

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The 4-ThermalVector **Θ** is used in Relativistic Thermodynamics.

My prime motivation for the form of this 4-Vector is that the probability distributions calculated by statistical mechanics ought to be covariant functions since they are based on counting arguments.

F(state) ~ e^-(E/k_BT) = e^-(βE), with this β = 1/k_BT, (not v/c)

A covariant way to get this is the Lorentz Scalar Product of the 4-Momentum **P** with the 4-ThermalVector **Θ**.
F(state) ~ e^-(**P·Θ**) = e^-(E_o/$k_B T_o$)

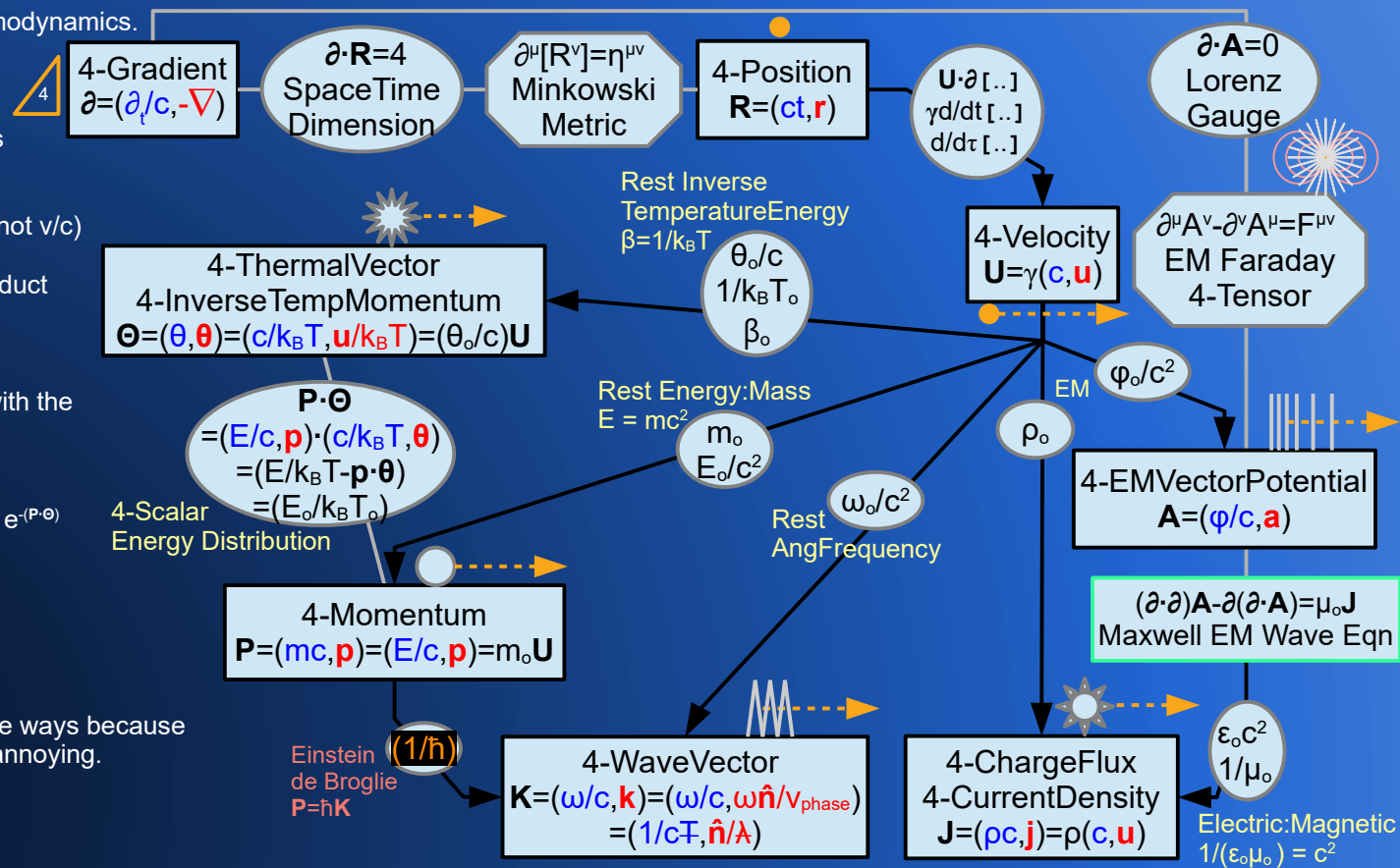
This also gets Boltzmann's constant (k_B) out there with the other Lorentz Scalars like (c) and (ħ)

see (Relativistic) Maxwell-Jüttner distribution

$f[\mathbf{P}] = N_o/(2c(m_oc)^d K_{[(d+1)/2]}[m_oc\Theta_o])*(m_oc\Theta_o/2\pi)^{(d-1)/2} * e^{-(\mathbf{P}\cdot\Theta)}$

$f[\mathbf{P}] = N_o/(2c(m_oc)^3 K_{[2]}[m_oc\Theta_o])*(m_oc\Theta_o/2\pi) * e^{-(\mathbf{P}\cdot\Theta)}$
$f[\mathbf{P}] = (\Theta_o)N_o/(4\pi c(m_oc)^2 K_{[2]}[m_oc\Theta_o]) * e^{-(\mathbf{P}\cdot\Theta)}$
$f[\mathbf{P}] = cN_o/(4\pi k_B T_o(m_oc)^2 K_{[2]}[m_oc\Theta_o])* e^{-(\mathbf{P}\cdot\Theta)}$
$f[\mathbf{P}] = N_o/(4\pi k_B T_o m_o^2 c K_{[2]}[m_oc^2/k_B T_o])* e^{-(\mathbf{P}\cdot\Theta)}$

It is possible to find this distribution written in multiple ways because many authors don't show constants, which is quite annoying.
Show the damn constants people!
(k_B),(c),(ħ) deserve at least that much respect.
See also: Cooper-Frye Distribution



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Be careful not to confuse (unfortunate symbol clash):
Thermal β =1/k_BT
Relatvisitic β = v/c
These are totally separate uses of (β)

Trace[$T^{\mu\nu}$] = $\eta_{\mu\nu}T^{\mu\nu}$ = $T^{\mu}{}_{\mu}$ = T
V·V = $V^{\mu}\eta_{\mu\nu}V^{\nu}$ = [$(v^0)^2$ - **v·v**] = $(v^0{}_o)^2$
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The 4-ThermalVector is used in Relativistic Thermodynamics.
It can be used in a manifestly-invariant tensor derivation of the Ideal Gas Law
$(P_o)\mathbf{\Theta} = \mathbf{N}$

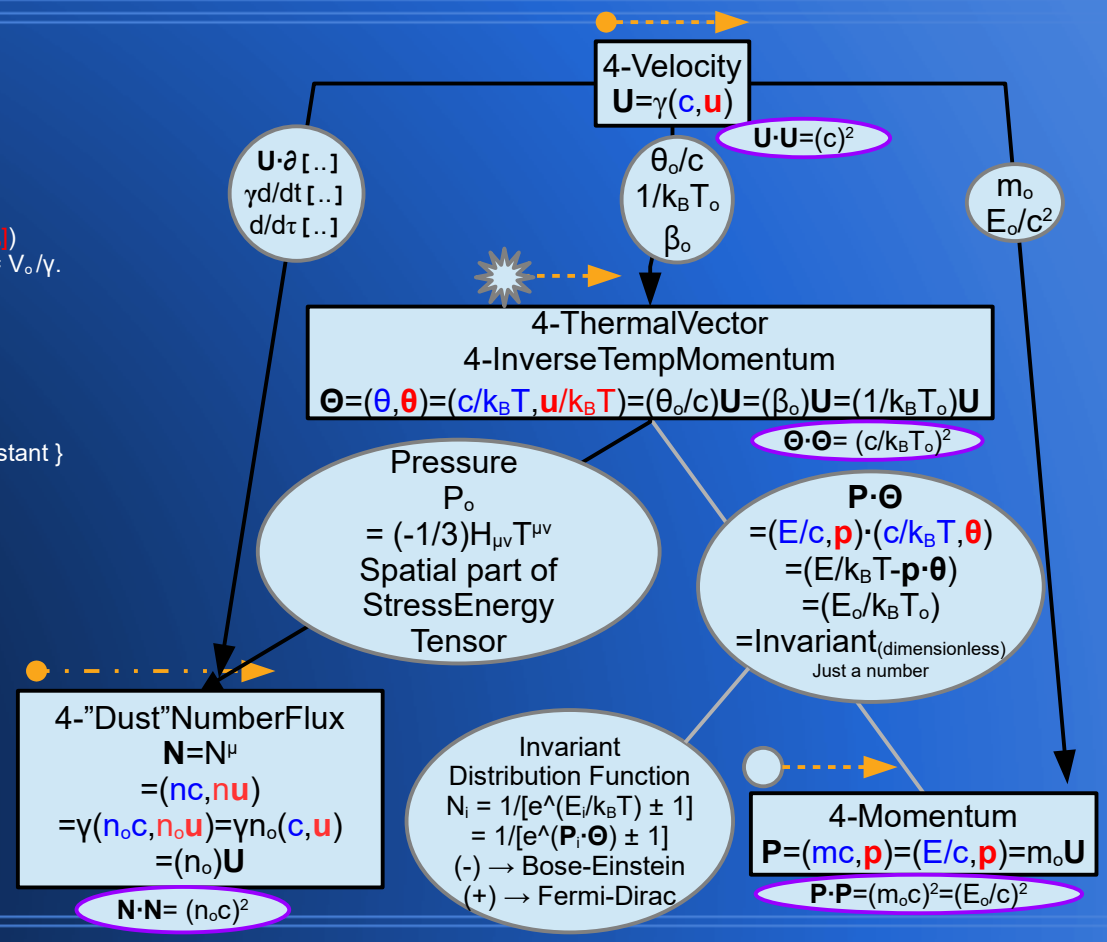
The definition of the 4-ThermalVector matches the original definition of the Planck-Einstein temperature transformation rule.

4-ThermalVector $\mathbf{\Theta} = (c/[k_BT], \mathbf{u}/[k_BT]) = (1/[k_BT_o])\mathbf{U} = (1/[k_BT_o])\gamma(c,\mathbf{u}) = \gamma(c/[k_BT_o], \mathbf{u}/[k_BT_o])$
which gives $(1/T) = \gamma(1/T_o)$ or $T = T_o/\gamma$. The temperature transforms like a volume, $V = V_o/\gamma$.

Examine the Ideal Gas Law.
$P\,V = N\,k_B\,T$
$(P)(V) = (N)(k_B)(T)$
$(P_o)(V_o/\gamma) = (N_o)(k_B)(T_o/\gamma)$
because Lorentz Scalars: { $P = P_o$: $N = N_o$: (k_B) is a physical constant }
$P_oV_o/\gamma = N_ok_BT_o/\gamma$
$P_oV_o = N_ok_BT_o$

Rearrange: { $P/k_BT = N/V = n$} and write in Manifestly-Invariant-Tensor-Form:
$(P_o)\mathbf{\Theta} = \mathbf{N}$
$(P_o)(1/[k_BT_o])\mathbf{U} = (n_o)\mathbf{U}$
$(P_o/[k_BT_o]) = (n_o)$
$(P_o/[k_BT_o]) = (N_o/V_o)$
$P_oV_o = N_ok_BT_o$

Pressure $(P_o) = (-1/3)H_{\mu\nu}T^{\mu\nu}$
$H_{\mu\nu}$ = Spatial-Projection 4-Tensor
$T^{\mu\nu}$ = PerfectFluid StressEnergy 4-Tensor



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Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.

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SRQM 4-Vector Study: 4-ThermalVector Unruh-Hawking Radiation

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The 4-ThermalVector is used in Relativistic Thermodynamics.
It can be used in a partial derivation of Unruh-Hawking Radiation (up to a numerical constant).

Let a "Unruh-DeWitt thermal detector" be in the Momentarily-Comoving-Rest-Frame (MCRF) of a constant spatial acceleration (**a**), in which $|\mathbf{u}|\to 0$, $\gamma\to 1$, $\gamma'\to 0$.

4-Acceleration$_{MCRF}$ = $\mathbf{A}_{MCRF} = A_{MCRF}{}^{\mu} = (0,\mathbf{a})_{MCRF}$

Take the Lorentz Scalar Product with the 4-ThermalVector
$\mathbf{A}_{MCRF}\cdot\mathbf{\Theta} = (0,\mathbf{a})_{MCRF}\cdot(c/k_BT,\mathbf{u}/k_BT) = (-\mathbf{a}\cdot\mathbf{u}/k_BT)$ = Lorentz Scalar Invariant

The (**u**) here is part of the 4-ThermalVector: the 3-velocity of the thermal radiation. (not from $\mathbf{A}_{MCRF}$)
Let the thermal radiation be photonic:EM in nature, so $|\mathbf{u}| = c$, and in a direction opposing the acceleration of the "thermal detector", which removes the minus sign.

$\mathbf{A}_{MCRF}\cdot\mathbf{\Theta}_{radiation} = (ac/k_BT)$ = Invariant Lorentz Scalar

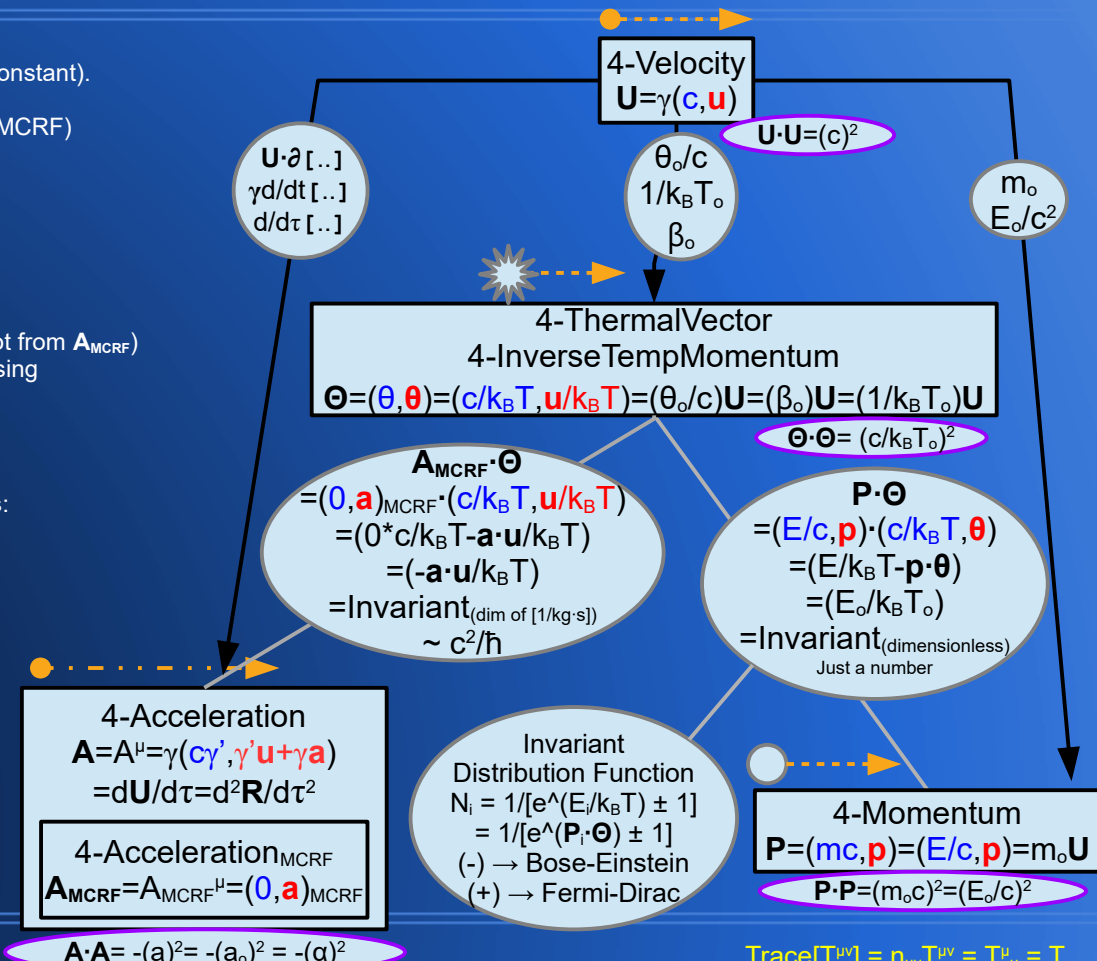
Use Dimensional Analysis to find appropriate Lorentz Scalar Invariant with same units:
[Invariant Units] = $[m/s^2]\cdot[m/s] / [kg\cdot m^2/s^2] = [1/kg\cdot s] \sim c^2/\hbar = [m/s]^2 / [kg\cdot m^2/s]$

$\mathbf{A}_{MCRF}\cdot\mathbf{\Theta}_{radiation} = (ac/k_BT)$ = Invariant $\sim c^2/\hbar$

Temperature $T \sim \hbar a/k_Bc$, {from EM radiation, only from the dir. of acceleration}

Further methods give the constant of proportionality ($1/2\pi$):
See (Imaginary Time, Euclideanization, Wick Rotation, Matsubara Frequency)
See (Thermal QFT, Bogoliubov transformation)

$T_{Unruh} = \hbar a/2\pi k_Bc$ {due to constant Minkowski-hyperbolic acceleration}
$T_{Hawking} = \hbar g/2\pi k_Bc$ {due to gravitational acceleration a=g}
$T_{Schwarzschild\ BH} = \hbar c^3/8\pi GMk_B$ {Temp at BH Event Horizon, $g=GM/R_S^2$, $R_S=2GM/c^2$}
$T_{SR} = -\hbar(\mathbf{a}\cdot\mathbf{u})/2\pi k_Bc^2$ {correct version from 4-Vector derivation $\mathbf{A}_{MCRF}\cdot\mathbf{\Theta}_{radiation} = 2\pi c^2/\hbar$}



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A Tensor Study of Physical 4-Vectors

SRQM 4-Vector Study: 4-ThermalVector Unruh-Hawking Radiation

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Temperature $T \sim \hbar a/k_B c$, {from EM radiation, only from the dir. of acceleration}

Further methods give the constant of proportionality ($1/2\pi$):
See (Imaginary Time, Euclideanization, Wick Rotation, Matsubara Frequency)
See (Thermal QFT, Bogoliubov transformation)

$T_{Unruh} = \hbar a/2\pi k_B c$ {due to constant Minkowski-hyperbolic acceleration}
$T_{Hawking} = \hbar g/2\pi k_B c$ {due to gravitational acceleration a=g}
$T_{Schwarzschild\ BH} = \hbar c^3/8\pi G M k_B$ {Temp at BH Event Horizon, $g=GM/R_S^2$, $R_S=2GM/c^2$}
$T_{SR} = -\hbar(\mathbf{a}\cdot\mathbf{u})/2\pi k_B c^2$ {correct version from 4-Vector derivation $\mathbf{A}_{MCRF}\cdot\mathbf{\Theta}_{radiation} = 2\pi c^2/\hbar$}

Alternate forms:

$\mathbf{A}_{MCRF}\cdot\mathbf{\Theta}_{radiation} = 2\pi c^2/\hbar$

$(1/kT_o)\mathbf{A}_{MCRF}\cdot\mathbf{U} = 2\pi c^2/\hbar$

$(1/kT_o)\mathbf{A}_{MCRF}\cdot\mathbf{U} = 2\pi\omega_o c^2/\hbar\omega_o$

$\mathbf{A}_{MCRF}\cdot\mathbf{U} = 2\pi\omega_o c^2$

$\mathbf{A}_{MCRF}\cdot\mathbf{U} = 2\pi(\mathbf{K}\cdot\mathbf{U})c^2$

$\mathbf{A}_{MCRF} = 2\pi(\mathbf{K})c^2$

$\mathbf{A}_{MCRF} = (2\pi c^2)\mathbf{K} = (2\pi c^2/\hbar)\mathbf{P}$

$(d\mathbf{P}/d\tau)_{MCRF}\cdot\mathbf{\Theta}_{radiation} = 2\pi\omega_o$

$\mathbf{F}_{MCRF}\cdot\mathbf{\Theta}_{radiation} = 2\pi\omega_o$: { for m_o = constant }

The 2π factor is interesting

There are cases when the dimensional units must match.
see 4-Momentum related to 4-WaveVector:
$\mathbf{P} = \hbar\mathbf{K}$ → [J·s/m] = [J·s/rad][rad/m]
$\hbar = h/2\pi$ → [J·s/rad] = [J·s/cyc]*[1 cyc/2π rad]

And other where the 2π factor has implicit [rad] units.
see Circles & Spheres:
$C = 2\pi r$ → [m] = [2π][m]
$A = \pi r^2$ → [m²] = [π][m]²
$V = (4/3)\pi r^3$ → [m³] = [(4/3)π][m]³

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$\mathbf{U}\cdot\mathbf{U}=(c)^2$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

θ_o/c
$1/k_B T_o$
β_o

m_o
E_o/c^2

4-ThermalVector
4-InverseTempMomentum
$\mathbf{\Theta}=(\theta,\boldsymbol{\theta})=(c/k_BT,\mathbf{u}/k_BT)=(\theta_o/c)\mathbf{U}=(\beta_o)\mathbf{U}=(1/k_BT_o)\mathbf{U}$
$\mathbf{\Theta}\cdot\mathbf{\Theta}= (c/k_BT_o)^2$

$\mathbf{A}_{MCRF}\cdot\mathbf{\Theta}$
$=(0,\mathbf{a})_{MCRF}\cdot(c/k_BT,\mathbf{u}/k_BT)$
$=(0*c/k_BT-\mathbf{a}\cdot\mathbf{u}/k_BT)$
$=(-\mathbf{a}\cdot\mathbf{u}/k_BT)$
$=$Invariant$_{(\text{dim of } [1/kg\cdot s])}$
$\sim c^2/\hbar$

$\mathbf{P}\cdot\mathbf{\Theta}$
$=(E/c,\mathbf{p})\cdot(c/k_BT,\boldsymbol{\theta})$
$=(E/k_BT-\mathbf{p}\cdot\boldsymbol{\theta})$
$=(E_o/k_BT_o)$
$=$Invariant$_{(\text{dimensionless})}$
Just a number

4-Acceleration
$\mathbf{A}=A^\mu=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$
$=d\mathbf{U}/d\tau=d^2\mathbf{R}/d\tau^2$

4-Acceleration$_{MCRF}$
$\mathbf{A}_{MCRF}=A_{MCRF}{}^\mu=(0,\mathbf{a})_{MCRF}$

$\mathbf{A}\cdot\mathbf{A}= -(a)^2= -(a_o)^2$

Invariant Distribution Function
$N_i = 1/[e^{(E_i/k_BT)} \pm 1]$
$= 1/[e^{(\mathbf{P}_i\cdot\mathbf{\Theta})} \pm 1]$
(-) → Bose-Einstein
(+) → Fermi-Dirac

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})=m_o\mathbf{U}$
$\mathbf{P}\cdot\mathbf{P}=(m_oc)^2=(E_o/c)^2$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

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(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.

$\text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

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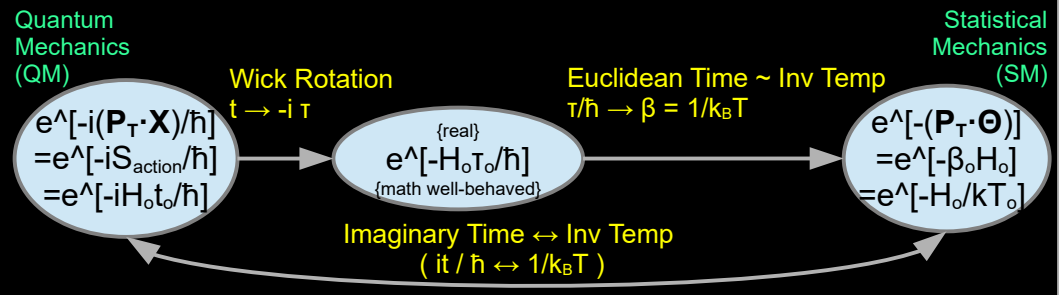
The QM/QFT↔SM Correspondence, via the Wick Rotation

The operator which governs how a quantum system evolves in time, the time evolution operator, and the density operator, a time-independent object which describes the statistical state of a many-particle system in an equilibrium state (with temperature T) can be related via arithmetic substitutions:



where τ, called Euclidean Time (Imaginary Time) is cyclic with period β, ($0 \le \tau \le +\beta$).

In Quantum Mechanics (or Quantum Field Theory), the Hamiltonian H acts as the generator of the Lie group of time translations while in Statistical Mechanics the role of the same Hamiltonian H is as the Boltzmann weight in an ensemble.

Time Evolution Operator

$U(t) = \sum_{n=0..\infty} [\, e^{-(i E_n t / \hbar)}] \, | n \rangle \langle n | = e^{-(i H t / \hbar)}$

Partition Function (time-independent function of state)

$Z = \sum_{n=0..\infty} [\, e^{-(E_n / k_B T)}] = \mathrm{Trace}[\, e^{-(i H t / \hbar)}]$

In the Matsubara Formalism, the basic idea (due to Felix Bloch) is that the expectation values of operators in a canonical ensemble:

$\langle A \rangle = \mathrm{Tr}[\exp(-\beta H) A] \,/\, \mathrm{Tr}[\exp(-\beta H)]$

may be written as expectation values in ordinary quantum field theory (QFT), where the configuration is evolved by an imaginary time $\tau = -i t$ ($0 \le \tau \le \beta$).

One can therefore switch to a spacetime with Euclidean signature, where the above trace (Tr [..]) leads to the requirement that all bosonic and fermionic fields be periodic and antiperiodic, respectively, with respect to the Euclidean time direction with periodicity $\beta = \hbar / (k_B T)$.

This allows one to perform calculations with the same tools as in ordinary quantum field theory, such as functional integrals and Feynman diagrams, but with compact Euclidean time.

Note that the definition of normal ordering has to be altered.
In momentum space, this leads to the replacement of continuous frequencies by discrete imaginary (Matsubara) frequencies:

Bosonic $\omega_n = (n)(2\pi/\beta)$

Fermionic $\omega_n = (n+1/2)(2\pi/\beta)$

and, through the de Broglie relation $E = \hbar\omega$,
to a discretized EM thermal energy spectrum $E_n = \hbar\omega_n = n(2\pi k_B T)$.

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Quantum Mechanics (QM)

$e^{[-i(\mathbf{P}_T\cdot\mathbf{R})/\hbar]}$
$=e^{[iS_{action}/\hbar]}$
$=e^{[-iH_o t_o/\hbar]}$

Wick Rotation
$t \rightarrow -i\,\tau$

{real}
$e^{[-H_o\tau_o/\hbar]}$
{math well-behaved}

Euclidean Time ~ Inv Temp
$\tau/\hbar \rightarrow \beta = 1/k_B T$

Statistical Mechanics (SM)

$e^{[-(\mathbf{P}_T\cdot\mathbf{\Theta})]}$
$=e^{[-\beta_o H_o]}$
$=e^{[-H_o/kT_o]}$

Imaginary Time ↔ Inv Temp
($it/\hbar \leftrightarrow 1/k_B T$)

$$S_{action} = -(\mathbf{P}_T\cdot\mathbf{R})$$
$$-\int[\mathbf{P}_T\cdot d\mathbf{R}]$$
$$= -\int[\mathbf{P}_T\cdot\mathbf{U}]d\tau = \int L dt$$
$$= -\int[(H/c,\mathbf{p}_T)\cdot\gamma(c,\mathbf{u})]d\tau$$
$$= -\int[\gamma(H-\mathbf{p}_T\cdot\mathbf{u}]d\tau$$

$$\mathbf{P}\cdot\mathbf{\Theta}$$
$$=(E/c,\mathbf{p})\cdot(c/k_B T,\boldsymbol{\theta})$$
$$=(E/k_B T-\mathbf{p}\cdot\boldsymbol{\theta})$$
$$=(E_o/k_B T_o)$$

where τ, called Euclidean Time (Imaginary Time) is cyclic with period β, ($0 \le \tau \le +\beta$).

In Quantum Mechanics (or Quantum Field Theory), the Hamiltonian H acts as the generator of the Lie group of time translations while in Statistical Mechanics the role of the same Hamiltonian H is as the Boltzmann weight in an ensemble.

4-Position
$\mathbf{R}=R^\mu=(ct,\mathbf{r})\in$<Event>
$\rightarrow(ct,x,y,z)$
alt. notation $\mathbf{X}=X^\mu$

i

Covariant Wick Rotation

4-ImaginaryPosition
$\mathbf{R}_{im}=R_{im}{}^\mu=i(ct,\mathbf{r})$
$=(ict,i\mathbf{r})=(c\tau,i\mathbf{r})$

1/ħ

Covariant Euclidean Time ~ Inv Temp

4-ThermalVector
4-InverseTemperatureMomentum
$\mathbf{\Theta}=\Theta^\mu=(\theta^0,\boldsymbol{\theta})=(c/k_B T,\mathbf{u}/k_B T)=(\theta_o/c)\mathbf{U}$
$=(1/k_B T)(c,\mathbf{u})=(1/k_B\gamma T)\mathbf{U}=(1/k_B T_o)\mathbf{U}=(\beta_o)\mathbf{U}$

-i = 1/i

ħ

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4-Momentum
$\mathbf{P}=P^\mu=(mc,\mathbf{p})=(mc,m\mathbf{u})=m_o\mathbf{U}$
$=(E/c,\mathbf{p})=(E_o/c^2)\mathbf{U}$
[kg·m/s] = [N·s]

1/ħ — Einstein de Broglie $\mathbf{P}=\hbar\mathbf{K}$

4-WaveVector
$\mathbf{K}=K^\mu=(\omega/c,\mathbf{k})=(\omega_o/c^2)\mathbf{U}$
$=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})=(1/c\mathcal{T},\hat{\mathbf{n}}/\lambda)$
[1/m]

-i = 1/i — Complex Plane-Waves $\mathbf{K}=i\partial$

4-Gradient
$\partial=\partial_R=\partial/\partial R_\mu=\partial^\mu=(\partial_t/c,-\nabla)$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$
[1/m]

ħ — i

$S_{action}=-\int[\mathbf{P_T}\cdot d\mathbf{R}]$
$=-\int[\mathbf{P_T}\cdot\mathbf{U}]d\tau$
$=-\int[(H/c,\mathbf{p_T})\cdot\gamma(c,\mathbf{u})]d\tau$
$=-\int[\gamma(H-\mathbf{p_T}\cdot\mathbf{u}]d\tau$
$=-\int[H_o]d\tau$

Einstein-de Broglie: $\mathbf{P}=\hbar\mathbf{K}\rightarrow\{E=\hbar\omega : \mathbf{p}=\hbar\mathbf{k}\}$
Complex Plane-Wave: $\mathbf{K}=i\partial\rightarrow\{\omega=i\partial_t : \mathbf{k}=-i\nabla\}$
==========
Schrödinger Relations: $\mathbf{P}=i\hbar\partial\rightarrow\{E=i\hbar\partial_t : \mathbf{p}=-\hbar\nabla\}$

Wick Rotation: $\mathbf{R}=-i\mathbf{R_{im}}\rightarrow\{t=-i\tau : \mathbf{r}=-i(i\mathbf{r})\}$
CyclicTemp: $\mathbf{R_{im}}=\hbar\Theta\rightarrow\{\tau=\hbar/k_BT : i\mathbf{r}=\hbar\mathbf{u}/k_BT\}$
==========
TimeTemp: $\mathbf{R}=-i\hbar\Theta\rightarrow\{t=-i\hbar/k_BT : \mathbf{r}=-i\hbar\mathbf{u}/k_BT\}$

Inverses

i — Covariant Wick Rotation $\mathbf{R}=-i\mathbf{R_{im}}$

4-Position
$\mathbf{R}=R^\mu=(ct,\mathbf{r})\in$ <Event>
$\rightarrow(ct,x,y,z)$
alt. notation $\mathbf{X}=X^\mu$
[m]

4-ImaginaryPosition
$\mathbf{R_{im}}=R_{im}{}^\mu=i(ct,\mathbf{r})$
$=(ict,i\mathbf{r})=(c\tau,i\mathbf{r})$
[m]

1/ħ — Covariant Euclidean Time ~ Inv Temp $\mathbf{R_{im}}=\hbar\Theta$

4-ThermalVector
4-InverseTemperatureMomentum
$\Theta=\Theta^\mu=(\theta^0,\boldsymbol{\theta})=(c/k_BT,\mathbf{u}/k_BT)=(\theta_o/c)\mathbf{U}$
$=(1/k_BT)(c,\mathbf{u})=(1/k_B\gamma T)\mathbf{U}=(1/k_BT_o)\mathbf{U}$
[s/kg·m] = [1/N·s]

-i = 1/i — ħ

SpaceTime Dimension
$\partial\cdot\mathbf{R}=\partial_\mu R^\mu=4$

Boltzmann Distribution
$\mathbf{P}\cdot\Theta=(E/c,\mathbf{p})\cdot(c/k_BT,\boldsymbol{\theta})$
$=(E/k_BT-\mathbf{p}\cdot\boldsymbol{\theta})=(E_o/k_BT_o)$

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$\mathbf{R}=R^\mu=(ct,\mathbf{r})\in$ <Event>
$\rightarrow(ct,x,y,z)$
alt. notation $\mathbf{X}=X^\mu$

Inverses
$\{R^\mu,\partial/\partial R_\mu\}$
[m] · [1/m] = Dimensionless

SpaceTime Dimension
$\partial\cdot\mathbf{R} = \partial_\mu R^\mu = 4$

$\partial^\mu[R^\nu]=\eta^{\mu\nu}$
Minkowski Metric

4-Gradient
$\partial=\partial_R=\partial/\partial R_\mu=\partial^\mu=(\partial_t/c,-\nabla)$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

Schrödinger QM Relation
$\mathbf{P} = i\hbar\partial$
$(E/c,\mathbf{p}) = i\hbar(\partial_t/c,-\nabla)$

$-i = 1/i$

Complex Plane-Waves
$\mathbf{K} = i\partial$

Covariant Wick Rotation
$\mathbf{R} = -i\mathbf{R}_{im}$

i

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

ProperTime Derivative

$\mathbf{U}\cdot\partial$
$= \gamma d/dt$
$= d/d\tau$

4-WaveVector
$\mathbf{K}=K^\mu=(\omega/c,\mathbf{k})=(\omega_o/c^2)\mathbf{U}$
$=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})=(1/c\mathrm{T},\hat{\mathbf{n}}/\lambda)$

Inverses
$\{K^\mu,R_{im}{}^\mu\}$
[1/m] · [m] = Dimensionless

4-ImaginaryPosition
$\mathbf{R}_{im}=R_{im}{}^\mu=i(ct,\mathbf{r})$
$=(ict,i\mathbf{r})=(c\tau,i\mathbf{r})$

Covariant ImaginaryTime ~ Inv Temp
$\mathbf{R}_{im} = \hbar\Theta$

$1/\hbar$

Einstein
de Broglie
$\mathbf{P} = \hbar\mathbf{K}$

$1/\hbar$

Covariant Time ~ Inv Temp
$\mathbf{R} = i\hbar\Theta$
$(ct,\mathbf{r}) = i\hbar(c/k_BT,\mathbf{u}/k_BT)$

4-Velocity
$\mathbf{U}=U^\mu=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau=c\mathbf{T}$

Energy Factors
E_o
$\sim \hbar\omega_o$
$\sim m_oc^2$
$\sim k_BT_o$

ω_o/c^2 m_o $1/k_BT_o$

4-Momentum
$\mathbf{P}=P^\mu=(mc,\mathbf{p})=(mc,m\mathbf{u})=m_o\mathbf{U}$
$=(E/c,\mathbf{p})=(E_o/c^2)\mathbf{U}$

Inverses
$\{P^\mu,\Theta^\mu\}$
[kg·m/s]·[s/kg·m] = Dimensionless

Boltzmann Distribution
$\mathbf{P}\cdot\Theta = (E/c,\mathbf{p})\cdot(c/k_BT,\boldsymbol{\theta})$
$= (E/k_BT-\mathbf{p}\cdot\boldsymbol{\theta}) = (E_o/k_BT_o)$

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The 4-EntropyVector is used in Relativistic Thermodynamics.

Pure Entropy is a Lorentz Scalar in all frames

4-Gradient $\partial=(\partial_t/c,-\nabla)$

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\partial^\mu[R^\nu]=\eta^{\mu\nu}$ Minkowski Metric

4-Position $\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

$\partial\cdot\mathbf{A}=0$ Lorenz Gauge

$\partial\cdot\mathbf{N}=0$ Conservation of Particle #

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$

$\partial^\mu A^\nu-\partial^\nu A^\mu=F^{\mu\nu}$ EM Faraday 4-Tensor

n_o Rest Number Density

EM φ_o/c^2

4-PureEntropyFlux $\mathbf{S}_{ent\_pure}=S_{ent}\mathbf{N}=n_oS_{ent}\mathbf{U}$

Rest Entropy = Entropy

$S_{ent}=k_B\ln[\Omega]$

4-HeatEntropyFlux
$\mathbf{S}_{ent\_heat}=(s,\mathbf{s})=S_{ent}\mathbf{N}+\mathbf{Q}/T_o$
$\mathbf{S}_{ent\_heat}=(s,\mathbf{s})=S_{ent}\mathbf{N}+E_o\mathbf{N}/T_o$
$\mathbf{S}_{ent\_heat}=(s,\mathbf{s})=n_o(S_{ent}+E_o/T_o)\mathbf{U}$

4-NumberFlux $\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$

4-EMVectorPotential $\mathbf{A}=(\varphi/c,\mathbf{a})$

Rest Inverse Temperature $1/T_o$

E_o Rest Energy

4-HeatEnergyFlux
$\mathbf{Q}=(\rho_Ec,\mathbf{q})=\rho_E(c,\mathbf{u})=E_o\mathbf{N}$
$n_oE_o\mathbf{U}=c^2\mathbf{G}$

EM Charge q

Rest Charge Density ρ_o

$(\partial\cdot\partial)\mathbf{A}-\partial(\partial\cdot\mathbf{A})=\mu_o\mathbf{J}$ Maxwell EM Wave Eqn

ε_oc^2 $1/\mu_o$

4-ChargeFlux 4-CurrentDensity
$\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})=q\mathbf{N}$

Electric:Magnetic $1/(\varepsilon_o\mu_o)=c^2$

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SRQM Interpretation:
** Transition to QM **

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Up to this point, we have mostly been exploring the SR aspects of 4-Vectors.

It is now time to show how RQM and QM fit into the works...

This is SRQM, [SR → QM]

RQM & QM are derivable from principles of SR
Let that sink in...
Quantum Mechanics is derivable from Special Relativity

GR → SR → RQM → QM → {CM & EM}

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Special Relativity → Quantum Mechanics
RoadMap of SR→QM

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

4-Gradient=***Alteration*** of SR <Events>
SR SpaceTime "Flat" Minkowski 4D Metric
SR SpaceTime Dimension=4
SR Lorentz Transforms
SR Action → 4-Momentum
SR Phase → 4-WaveVector
SR ProperTime Derivative
SR & QM Invariant Waves

START HERE: 4-Position=***Location*** of SR <Events> in SpaceTime
follow the arrows

$\partial^\mu[R^\nu]=\eta^{\mu\nu}$ Minkowski Metric

$\partial_\nu[R^{\mu'}]=\Lambda^{\mu'}{}_\nu$ Lorentz Transform

$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4$

$Tr[\eta^{\mu\nu}]=4$

SpaceTime $\partial\cdot R=\partial_\mu R^\mu=4$ Dimension

4-Position R^μ
$R=(ct,r)\in$<Event>

$U\cdot\partial$ [..]
$\gamma d/dt$ [..]
$d/d\tau$ [..]
ProperTime Derivative

4-Velocity=***Motion*** of SR <Events> in SpaceTime as both particles & waves

$R\cdot R=(ct)^2-r\cdot r$
$=(c\tau)^2=(i|r_o|)^2$

$R\cdot U=c^2\tau$

4-Gradient ∂^μ
$\partial=\partial/\partial R_\mu$
$=(\partial_t/c,-\nabla)=-iK$

$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$
$=-(m_oc/\hbar)^2=-(\omega_o/c)^2$
$=(\partial_\tau/c)^2$

ProperTime
$U\cdot\partial=d/d\tau=\gamma d/dt$
Derivative

4-Velocity U^μ
$U=\gamma(c,u)=dR/d\tau$

Matter Wave
$v_{group}*v_{phase}=c^2$
Rest Angular Frequency ω_o

$U\cdot U=\gamma^2(c^2-u\cdot u)$
$=(c)^2$

$-K\cdot R=\Phi_{phase,free}$
SR Phase

$-P\cdot R=S_{action,free}$
SR Action

Phase & Action Lorentz Scalars

$K\cdot U=\omega_o$

ω_o/c^2

$-\partial[\Phi_{phase,free}]=K$

$-\partial[S_{action,free}]=P$

4-WaveVector Complex Plane-Waves
$K_T=-\partial[\Phi]$
$K=i\partial$

Hamilton-Jacobi
$P_T=-\partial[S]$

$(-i)$

Einstein
$E=mc^2=\gamma m_oc^2=\gamma E_o$
Rest Mass m_o : Rest Energy E_o

$P\cdot U=E_o$

m_o
E_o/c^2

SR d'Alembertian & Klein-Gordon Relativistic Quantum Wave Relation
Schrödinger QWE is {|v|<<c} limit of KG QWE
[SR → QM]

4-WaveVector=***Substantiation*** of SR Wave <Events>
oscillations proportional to mass:energy & 3-momentum

4-WaveVector K^μ
$K=(\omega/c,k)=(\omega/c,\omega\hat{n}/v_{phase})$
$=(1/c\mathcal{T},\hat{n}/\lambda)=(\omega_o/c^2)U=P/\hbar$

4-Momentum P^μ
$P=(mc,p)=(E/c,p)=m_oU$

4-Momentum=***Substantiation*** of SR Particle <Events>
mass:energy & 3-momentum

Einstein, de Broglie Relation
Dirac:Planck Constant $\hbar=h/2\pi$
$P=\hbar K$

$P\cdot K=m_o\omega_o$

$K\cdot K=(\omega/c)^2-k\cdot k$
$=(m_oc/\hbar)^2=(\omega_o/c)^2=(1/c\mathcal{T}_o)^2$

$P\cdot P=(E/c)^2-p\cdot p$
$=(m_oc)^2=(E_o/c)^2$

ω_o/E_o
$(1/\hbar)$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
(QM Principles)

$Trace[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Special Relativity → Quantum Mechanics
RoadMap of SR→QM (w/ EM Potential)

4-Vector SRQM Interpretation of QM

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SR → RQM Klein-Gordon
Relativistic Quantum
Particle in EM Potential
d'Alembertian Wave Equation

$\partial\cdot\partial = (\partial_t/c)^2-\nabla\cdot\nabla$
$= (\partial_T+(iq/\hbar)\mathbf{A})\cdot(\partial_T+(iq/\hbar)\mathbf{A})$
$= -(\omega_o/c)^2 = -(m_oc/\hbar)^2$
$= (\partial_\tau/c)^2$

Limit: { $|\mathbf{v}|<<c$ }
$(i\hbar\partial_{tT}) \sim [\, q\varphi + (m_oc^2) + (i\hbar\nabla_T+q\mathbf{a})^2/(2m_o)\,]$
$(i\hbar\partial_{tT}) \sim [\, V + (i\hbar\nabla_T+q\mathbf{a})^2/(2m_o)\,]$
with potential $V = q\varphi + (m_oc^2)$
=Schrödinger QM Equation (EM potential)
[SR → QM]

4-WaveVector=***Substantiation*** of SR Wave <Events>
oscillations proportional to mass:energy & 3-momentum

$\mathbf{K}\cdot\mathbf{K} = (\omega/c)^2-\mathbf{k}\cdot\mathbf{k}$
$= (\mathbf{K}_T-(q/\hbar)\mathbf{A})\cdot(\mathbf{K}_T-(q/\hbar)\mathbf{A})$
$= (m_oc/\hbar)^2 = (\omega_o/c)^2$

START HERE: 4-Position=***Location*** of SR <Events> in SpaceTime
follow the arrows

$\partial^\mu[R^\nu]=\eta^{\mu\nu}$ Minkowski Metric

$\partial_\nu[R^{\mu'}]=\Lambda^{\mu'}{}_\nu$ Lorentz Transform

$\partial\cdot\mathbf{R}=4$ SpaceTime Dim

4-Position R^μ
$\mathbf{R}=(ct,\mathbf{r})$
$\in$<Event>

$\mathbf{R}\cdot\mathbf{R} = (ct)^2-\mathbf{r}\cdot\mathbf{r}$
$= (c\tau)^2 = (i|\mathbf{r}_o|)^2$

$\mathbf{U}\cdot\mathbf{U} = \gamma^2(c^2-\mathbf{u}\cdot\mathbf{u})$
$= (c)^2$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$
ProperTime Derivative

EM Faraday
$\partial^\mu A^\nu-\partial^\nu A^\mu=F^{\mu\nu}$
4-Tensor

4-Velocity U^μ
$\mathbf{U}=\gamma(c,\mathbf{u})=d\mathbf{R}/d\tau$

4-Velocity=***Motion*** of SR <Events> in SpaceTime as both particles & waves

4-Gradient ∂^μ
$\partial=(\partial_t/c,-\nabla)= -i\mathbf{K}$

ProperTime
$\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$
Derivative

$-\mathbf{K}\cdot\mathbf{R}=\Phi_{phase,free}$
$-\mathbf{K}_T\cdot\mathbf{R}=\Phi_{phase}$
SR Phase

$-\mathbf{P}\cdot\mathbf{R}=S_{action,free}$
$-\mathbf{P}_T\cdot\mathbf{R}=S_{action}$
SR Action

Phase & Action Lorentz Scalars

Matter Wave
$v_{group}*v_{phase}= c^2$
Rest Angular Frequency ω_o

φ_o/c^2
EM

4-EMVectorPotential A^μ
$\mathbf{A}=(\varphi/c,\mathbf{a})=(\varphi_o/c^2)\mathbf{U}$

$-\partial[\Phi_{phase,free}]=\mathbf{K}$
$-\partial[\Phi_{phase}]=\mathbf{K}_T$

(-i)

4-WaveVector
Complex
Plane-Waves
$\mathbf{K}_T = -\partial[\Phi]$, $\mathbf{K} = i\partial$

$-\partial[S_{action,free}]=\mathbf{P}$
$-\partial[S_{action}]=\mathbf{P}_T$
Hamilton-Jacobi
$\mathbf{P}_T = -\partial[S]$

ω_o/c^2

m_o
E_o/c^2

Einstein
$E = mc^2 = \gamma m_oc^2= \gamma E_o$

EM Charge
q

4-WaveVector K^μ
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega_o/c^2)\mathbf{U}=\mathbf{P}/\hbar$

4-Momentum P^μ
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})=m_o\mathbf{U}$

4-PotentialMomentum Q^μ
$\mathbf{Q}=(V/c,\mathbf{q})=q(\varphi/c,\mathbf{a})=q\mathbf{A}$

Einstein, de Broglie
$\mathbf{P} = \hbar\mathbf{K}$

ω_o/E_o
($1/\hbar$)

4-TotMomentum Conservation
$\mathbf{P}_T = (\mathbf{P}+\mathbf{Q}) = (\mathbf{P}+q\mathbf{A})$

Minimal Coupling
$\mathbf{P} = (\mathbf{P}_T-q\mathbf{A}) = (\mathbf{P}_T-\mathbf{Q})$

4-Momentum=***Substantiation*** of SR Particle <Events>
mass:energy & 3-momentum

$\mathbf{P}\cdot\mathbf{P} = (E/c)^2-\mathbf{p}\cdot\mathbf{p}$
$= (\mathbf{P}_T-q\mathbf{A})\cdot(\mathbf{P}_T-q\mathbf{A})$
$= (m_oc)^2 = (E_o/c)^2$

4-TotalMomentum P_T^μ
$\mathbf{P}_T=(E_T/c,\mathbf{p}_T)=((E+q\varphi)/c,\mathbf{p}+q\mathbf{a})=\mathbf{P}+\mathbf{Q}$

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(0,0)-Tensor S or S_o
Lorentz Scalar

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Quantum Principles

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM Basic Idea (part 1)

SR → Relativistic Wave Eqn

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

The basic idea is to show that Special Relativity plus a few empirical facts lead to Relativistic Wave Equations, and thus RQM, without using any assumptions or axioms from Quantum Mechanics.

Start only with the concepts of SR, no concepts from QM:

(1) SR provides the ideas of Invariant Intervals and (c) as a Physical Constant, as well as: Poincaré Invariance, Minkowski 4D SpaceTime, ProperTime, ProperLength, Physical SR 4-Vectors.

Note empirical facts which can relate the SR 4-Vectors from the following:

(2a) Elementary matter particles each have RestMass, (m_o), a physical constant which can be measured by experiment: eg. in collisions, cyclotrons, Compton Scattering, etc.

(2b) There is a physical constant, (ħ), which can be measured by classical experiment – eg. the Photoelectric Effect, the inverse Photoelectric Effect, LED's=Injection Electroluminescence, Duane-Hunt Law in Bremsstralung, the Watt/Kibble-Balance, Electron Diffraction, Incandescence, Stern-Gerlach, Compton Scattering, Atomic Line Spectra, etc. All known particles types obey this constant.

(2c) The use of imaginary:complex numbers (i) and differential operators { ∂_t & $\nabla \rightarrow (\partial_x,\partial_y,\partial_z)$ } in wave-type equations comes from pure mathematics: not necessary to assume any QM Axioms

These few things are enough to derive the RQM Klein-Gordon equation, the most basic of the relativistic wave equations. Taking the low-velocity limit { $|\mathbf{v}| \ll c$ }(a standard SR technique) leads to the Schrödinger Equation, the basic QM equation.

SRQM Basic Idea (part 2)

Klein-Gordon RWE implies QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

If one has a Relativistic Wave Equation, such as the Klein-Gordon equation, then one has RQM, and thence QM via the low-velocity limit { |**v**|<<c }.

The physical and mathematical properties of QM, usually regarded as axiomatic, are inherent in the Klein-Gordon RWE itself (i.e. derivable from it).

QM Principles emerge not from { QM Axioms + SR → RQM },
but from { SR + Empirical Facts → RQM }.

The result is a paradigm shift from the idea of { SR and QM as separate theories }
to { QM derived from SR } – leading to a new interpretation of QM:
The SRQM or [SR→QM] Interpretation.

GR → (low-mass limit = {curvature ~ 0} limit) → SR
SR → (+ a few empirical facts giving Lorentz Invariant Scalars) → RQM
RQM → (low-velocity limit { |**v**|<<c }) → QM

The results of this analysis will be facilitated by the use of SR 4-Vectors

SRQM 4-Vector Study:
Basic 4-Vectors on the path to QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

SR 4-Vector	Dimens. Units (SI)	Definition Component Notation	Properties united in SR:4D SpaceTime Time·Space
4-Position **R**	[m]	$\mathbf{R} = R^\mu = (r^\mu) = (r^0, r^i) \in$ <Event> $= (ct, r) \rightarrow (ct, x, y, z)$	**Time**, **Space** *(c*when, where) = SR location of <Event>*
4-Velocity **U**	[m/s]	$\mathbf{U} = U^\mu = (u^\mu) = (u^0, u^i) = \gamma(c, \mathbf{u})$	**Temporal velocity**, **Spatial velocity** *SR motion of <Event>, max speed = c*
4-Momentum **P**	[kg·m/s] [N·s]	$\mathbf{P} = P^\mu = (p^\mu) = (p^0, p^i) = (E/c, \mathbf{p}) = (mc, \mathbf{p})$	**Mass:Energy**, **Momentum** *used in 4-Momentum Conservation* $\Sigma\, \mathbf{P}_{final} = \Sigma\, \mathbf{P}_{initial}$
4-WaveVector **K**	[{rad}/m]	$\mathbf{K} = K^\mu = (k^\mu) = (k^0, k^i) = (\omega/c, \mathbf{k}) = (\omega/c, \omega\hat{\mathbf{n}}/v_{phase}) = (1/c\mathrm{T}, \hat{\mathbf{n}}/\lambda) = 2\pi(1/cT, \hat{\mathbf{n}}/\lambda)$	**Angular Frequency**, **WaveNumber** *used in Relativistic Doppler Shift* $\omega_{obs} = \omega_{emit} / [\gamma(1 - \beta \cos[\theta])]$, $k = \omega/c$ for photons
4-Gradient ∂	[1/m]	$\partial = \partial^\mu = (\partial^\mu) = (\partial^0, \partial^i) = (\partial_t/c, -\nabla) \rightarrow (\partial_t/c, -\partial_x, -\partial_y, -\partial_z) \rightarrow (\partial/\partial ct, -\partial/\partial x, -\partial/\partial y, -\partial/\partial z)$	**Temporal Partial**, **Spatial Partial** *used in SR Continuity Eqns., ProperTime eg.* $\partial \cdot \mathbf{A} = 0$ *means* $\mathbf{A}$ *is locally conserved*

All of these are standard SR 4-Vectors, which can be found and used in a totally relativistic context, with no mention or need of QM.
I want to emphasize that these objects are ALL *relativistic* in origin.

SRQM 4-Vector Study:
SR Lorentz Scalar Invariants

SciRealm.org
John B. Wilson
SciRealm@aol.com
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SR 4-Vector	Lorentz Scalar Invariant	What it means in SR...
4-Position **R** = (ct,r)	**R·R** = $(ct)^2$ - **r·r** = $(ct_o)^2$ = $(c\tau)^2$	SR Invariant Interval, ProperTime
4-Velocity **U** = γ(c,**u**)	**U·U** = $\gamma^2(c^2$ - **u·u**) = c^2	<Event> Motion Invariant Magnitude (c)
4-Momentum **P** = (E/c,**p**)	**P·P** = $(E/c)^2$ - **p·p** = $(E_o/c)^2$ = $(m_oc)^2$	Einstein Invariant Mass:Energy Relation
4-WaveVector **K** = (ω/c,**k**)	**K·K** = $(\omega/c)^2$ - **k·k** = $(\omega_o/c)^2$	Wave/Dispersion Invariance Relation
4-Gradient ∂ = (∂_t/c,-∇)	$\partial\cdot\partial$ = $(\partial_t/c)^2$ - $\nabla\cdot\nabla$ = $(\partial_\tau/c)^2$	The d'Alembert Invariant Operator

All 4-Vectors have invariant magnitudes, found by taking the scalar product of the 4-Vector with itself. Quite often a simple expression can be found by examining the case when the spatial part is zero. This is usually found when the 3-velocity is zero. The temporal part is then specified by its "rest" value.

For example: **P·P** = $(E/c)^2$-**p·p** = $(E_o/c)^2$ = $(m_oc)^2$
E = $\sqrt{[(E_o)^2 + \mathbf{p}\cdot\mathbf{p}\, c^2]}$, from above relation
E = γE_o , using {$\gamma = 1/\sqrt{[1-\beta^2]} = \sqrt{[1+\gamma^2\beta^2]}$} and {$\beta$=v/c}
meaning the relativistic energy E is equal to the relative gamma factor γ * the rest energy E_o

SR + A few empirical facts: SRQM Overview

SciRealm.org
John B. Wilson
SciRealm@aol.com
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SR 4-Vector	Empirical Fact	What it means in SR...
4-Position **R** = (ct,**r**); alt. **X** = (ct,**x**)	**R** ∈ <Event>; alt. **X**	Location of 4D SpaceTime <Event>
4-Velocity **U** = γ(c,**u**)	**U** = d**R**/dτ	Motion of 4D SpaceTime <Event>
4-Momentum **P** = (E/c,**p**) = (mc,**p**)	**P** = m_o**U**	<Events> described as Particles
4-WaveVector **K** = (ω/c,**k**)	**K** = **P**/ħ	<Events> described as Waves
4-Gradient ∂ = (∂_t/c,-∇)	∂ = -i**K**	Alteration of 4D SpaceTime <Event>

The Axioms of SR, which is actually a GR limiting-case, lead us to the use of Minkowski SpaceTime and Physical 4-Vectors, which are elements of Minkowski Space = (4D Time·Space).

Empirical Observation leads us to the transformation relations between the components of these SR 4-Vectors, and to the chain of relations between the 4-Vectors themselves. These relations all use Lorentz Scalar Invariant Constants, whose values are measured empirically.
They are manifestly invariant relations, true in all reference frames...

The combination of these SR objects and their relations is enough to derive RQM.

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Chart:
Special Relativity → Quantum Mechanics
SR→QM Interpretation Simplified

4-Vector SRQM Interpretation of QM

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John B. Wilson
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SRQM: The [SR→QM] Interpretation of Quantum Mechanics

Special Relativity (SR) Axioms: Invariant Interval + LightSpeed (c) as Universal Physical Constant lead to SR, although technically SR is itself the Minkowski-SpaceTime low-curvature:"flat" limiting-case of GR.

{ $c,\tau,m_o,\hbar,i$ } = {c: SpeedOfLight, τ: ProperTime, m_o: RestMass, $\hbar$: Dirac/PlanckReducedConstant($\hbar=h/2\pi$), i: ImaginaryNumber}: are all Empirically-Measured SR Lorentz-Invariant Physical and/or Mathematical Constants $i = +\sqrt{[-1]} = (0,1)_{complex\#}$

Standard SR 4-Vectors:		*Related by these SR Lorentz Invariants:*	
4-Position	$\mathbf{R} = (ct,\mathbf{r})$	$\in$ <Event> $\in$ <Time·Space>	$(\mathbf{R}\cdot\mathbf{R}) = (c\tau)^2 = (i\lvert\mathbf{r_o}\rvert)^2$
4-Velocity	$\mathbf{U} = \gamma(c,\mathbf{u})$	$= (\mathbf{U}\cdot\partial)\mathbf{R} = (d/d\tau)\mathbf{R} = d\mathbf{R}/d\tau$	$(\mathbf{U}\cdot\mathbf{U}) = (c)^2$
4-Momentum	$\mathbf{P} = (E/c,\mathbf{p})$	$= m_o\mathbf{U}$	$(\mathbf{P}\cdot\mathbf{P}) = (m_oc)^2$
4-WaveVector	$\mathbf{K} = (\omega/c,\mathbf{k})$	$= \mathbf{P}/\hbar$	$(\mathbf{K}\cdot\mathbf{K}) = (m_oc/\hbar)^2$
4-Gradient	$\partial = (\partial_t/c,-\nabla)$	$= -i\mathbf{K}$	$(\partial\cdot\partial) = (-im_oc/\hbar)^2 = -(m_oc/\hbar)^2$ = QM Relation → RQM → QM

KG Equation: $|v|<<c$

SR + Empirically Measured Physical Constants lead to RQM via the (KG) Klein-Gordon Quantum Eqn, and thence to QM via the low-velocity limit { $|\mathbf{v}| << c$ }, giving the Schrödinger Eqn. This fundamental KG Relation also leads to the other

Quantum Wave Equations:	RQM (massless, no rest-frame, Lorentzian) { $\lvert\mathbf{v}\rvert = c : m_o = 0$ }	RQM (with non-zero mass, Lorentzian) { $0 <= \lvert\mathbf{v}\rvert < c : m_o > 0$ }	QM (limit-case from RQM, Galilean) { $0 <= \lvert\mathbf{v}\rvert << c : m_o > 0$ }
spin=0 boson field = 4-Scalar:	Free Scalar Wave (Higgs)	Klein-Gordon (KG)	Schrödinger (regular QM)
spin=1/2 fermion field = 4-Spinor:	Weyl	Dirac (RQM w/ EM charge)	Pauli (QM w/ EM charge)
spin=1 boson field = 4-Vector:	Maxwell (EM photonic)	Proca	

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(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

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(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)

SRQM Diagram:
RoadMap of SR (4-Vectors)

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John B. Wilson
SciRealm@aol.com
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4-Position
R=(ct,**r**)
∈<Event>

4-Velocity
U=γ(c,**u**)

4-Gradient
∂=(∂_t/c,-∇)

4-WaveVector
K=(ω/c,**k**)

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P=(mc,**p**)=(E/c,**p**)

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(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor V^{μ} = **V** = (v^0,**v**)
SR 4-CoVector:OneForm
(0,1)-Tensor V_{μ} = (v_0,-**v**)

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Trace[$T^{\mu\nu}$] = $\eta_{\mu\nu}T^{\mu\nu}$ = $T^{\mu}{}_{\mu}$ = T
V·V = $V^{\mu}\eta_{\mu\nu}V^{\nu}$ = [$(v^0)^2$ - **v·v**] = $(v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM Diagram:
RoadMap of SR (Connections)

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John B. Wilson
SciRealm@aol.com
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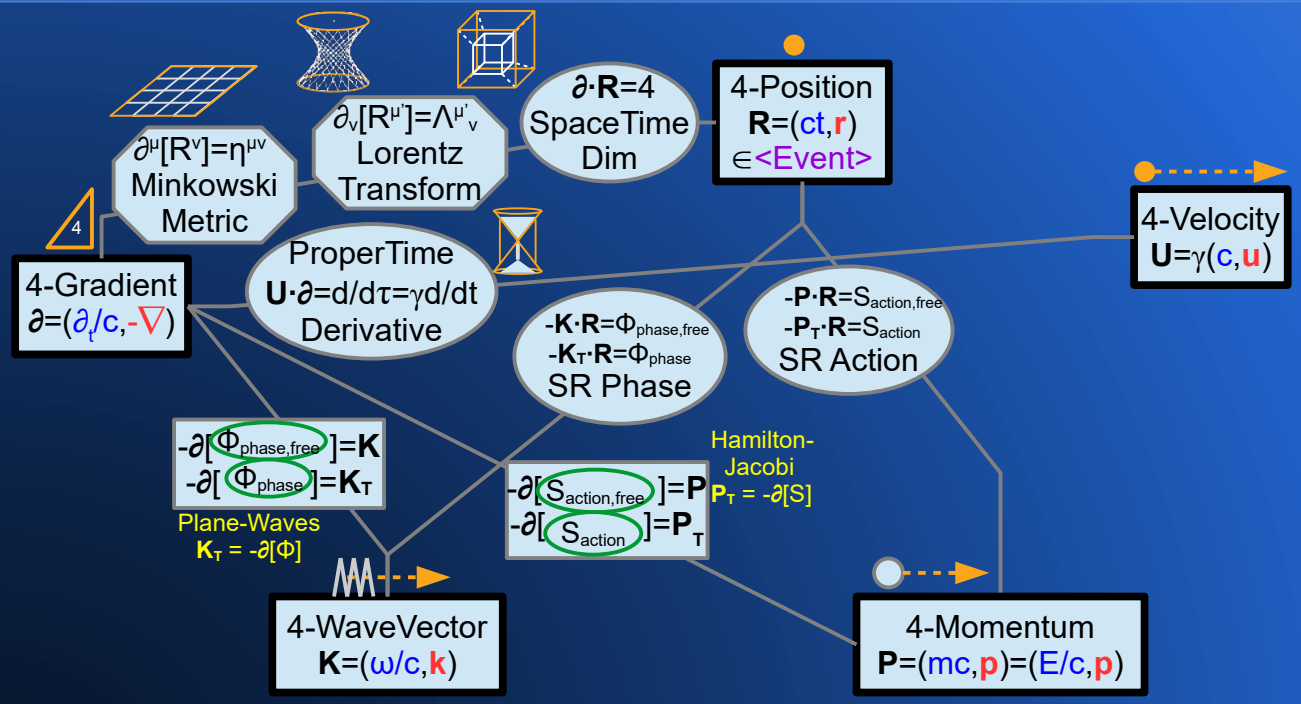

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Lorentz Scalar

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= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

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SRQM Diagram:
RoadMap of SR (Free Particle)

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SR Proper Time Derivative
SR Invariant Waves

START HERE: <Events> have 4-Position=***Location*** in SR SpaceTime

<Events> have 4-Velocity=***Motion*** in SR SpaceTime as both particles & waves

SR Wave <Events> have 4-WaveVector=***Substantiation*** oscillations proportional to mass:energy & 3-momentum

SR Particle <Events> have 4-Momentum=***Substantiation*** mass:energy & 3-momentum

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram: RoadMap of SR (Free Particle) with Magnitudes

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

4-Gradient=***Alteration*** of SR <Events>
SR SpaceTime "Flat" Minkowski 4D Metric
SR SpaceTime Dimension=4
SR Lorentz Transforms
SR Action → 4-Momentum
SR Phase → 4-WaveVector
SR Proper Time Derivative
SR Invariant Waves

START HERE: <Events> have 4-Position=***Location*** in SR SpaceTime

$\mathbf{R}\cdot\mathbf{R} = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (c\tau)^2 = (i|\mathbf{r}_o|)^2$

$\mathbf{U}\cdot\mathbf{U} = \gamma^2(c^2 - \mathbf{u}\cdot\mathbf{u}) = (c)^2$

$\partial^\mu[R^\nu] = \eta^{\mu\nu}$ Minkowski Metric

$\partial_\nu[R^{\mu'}] = \Lambda^{\mu'}{}_\nu$ Lorentz Transform

$\partial\cdot\mathbf{R} = 4$ SpaceTime Dim

4-Position $\mathbf{R} = (ct, \mathbf{r}) \in$ <Event>

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

ProperTime Derivative

4-Velocity $\mathbf{U} = \gamma(c, \mathbf{u})$

<Events> have 4-Velocity=***Motion*** in SR SpaceTime as both particles & waves

4-Gradient $\partial = (\partial_t/c, -\nabla)$

ProperTime $\mathbf{U}\cdot\partial = d/d\tau = \gamma d/dt$ Derivative

$-\mathbf{K}\cdot\mathbf{R} = \Phi_{phase,free}$
$-\mathbf{K}_T\cdot\mathbf{R} = \Phi_{phase}$
SR Phase

$-\mathbf{P}\cdot\mathbf{R} = S_{action,free}$
$-\mathbf{P}_T\cdot\mathbf{R} = S_{action}$
SR Action

$\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = (\partial_\tau/c)^2$

d'Alembertian Free Particle Wave Equation

$-\partial[\Phi_{phase,free}] = \mathbf{K}$
$-\partial[\Phi_{phase}] = \mathbf{K}_T$

Plane-Waves $\mathbf{K}_T = -\partial[\Phi]$

$-\partial[S_{action,free}] = \mathbf{P}$
$-\partial[S_{action}] = \mathbf{P}_T$

Hamilton-Jacobi $\mathbf{P}_T = -\partial[S]$

ω_o/c^2 Wave Velocity $v_{group} * v_{phase} = c^2$

m_o E_o/c^2 Einstein $E = mc^2 = \gamma m_o c^2 = \gamma E_o$

4-WaveVector $\mathbf{K} = (\omega/c, \mathbf{k})$

4-Momentum $\mathbf{P} = (mc, \mathbf{p}) = (E/c, \mathbf{p})$

ω_o/E_o

SR Wave <Events> have 4-WaveVector=***Substantiation*** oscillations proportional to mass:energy & 3-momentum

$\mathbf{K}\cdot\mathbf{K} = (\omega/c)^2 - \mathbf{k}\cdot\mathbf{k} = (\omega_o/c)^2$

SR Particle <Events> have 4-Momentum=***Substantiation*** mass:energy & 3-momentum

$\mathbf{P}\cdot\mathbf{P} = (E/c)^2 - \mathbf{p}\cdot\mathbf{p} = (m_o c)^2 = (E_o/c)^2$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
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Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

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SRQM Diagram:
RoadMap of SR (EM Potential)

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START HERE: <Events> have 4-Position=***Location*** in SR SpaceTime

$\partial^\mu[R^\nu]=\eta^{\mu\nu}$ Minkowski Metric

$\partial_\nu[R^{\mu'}]=\Lambda^{\mu'}{}_\nu$ Lorentz Transform

$\partial\cdot\mathbf{R}=4$ SpaceTime Dim

4-Position $\mathbf{R}=(ct,\mathbf{r})\in$<Event>

$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2=(i|\mathbf{r}_o|)^2$

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

$\mathbf{U}\cdot\mathbf{U}=\gamma^2(c^2-\mathbf{u}\cdot\mathbf{u})=(c)^2$

EM Faraday $\partial^\mu A^\nu-\partial^\nu A^\mu=F^{\mu\nu}$ 4-Tensor

ProperTime Derivative

4-Velocity $\mathbf{U}=\gamma(c,\mathbf{u})$

<Events> have 4-Velocity=***Motion*** in SR SpaceTime as both particles & waves

4-Gradient $\partial=(\partial_t/c,-\nabla)$

ProperTime $\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$ Derivative

$-\mathbf{K}\cdot\mathbf{R}=\Phi_{phase,free}$
$-\mathbf{K}_T\cdot\mathbf{R}=\Phi_{phase}$
SR Phase

$-\mathbf{P}\cdot\mathbf{R}=S_{action,free}$
$-\mathbf{P}_T\cdot\mathbf{R}=S_{action}$
SR Action

φ_o/c^2 EM

4-EMVectorPotential $\mathbf{A}=(\varphi/c,\mathbf{a})$

$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla=(\partial_\tau/c)^2$

d'Alembertian Particle Wave Equation in EM Potential

$-\partial[\Phi_{phase,free}]=\mathbf{K}$
$-\partial[\Phi_{phase}]=\mathbf{K}_T$
Plane-Waves $\mathbf{K}_T=-\partial[\Phi]$

$-\partial[S_{action,free}]=\mathbf{P}$
$-\partial[S_{action}]=\mathbf{P}_T$

Hamilton-Jacobi $\mathbf{P}_T=-\partial[S]$

ω_o/c^2 Wave Velocity $v_{group}*v_{phase}=c^2$

m_o E_o/c^2 Einstein $E=mc^2=\gamma m_o c^2=\gamma E_o$

EM Charge q

4-WaveVector $\mathbf{K}=(\omega/c,\mathbf{k})$

4-Momentum $\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

4-PotentialMomentum $\mathbf{Q}=(V/c,\mathbf{q})=q(\varphi/c,\mathbf{a})$

ω_o/E_o

4-TotMom Conservation $\mathbf{P}_T=(\mathbf{P}+\mathbf{Q})=(\mathbf{P}+q\mathbf{A})$

Minimal Coupling $\mathbf{P}=(\mathbf{P}_T-q\mathbf{A})=(\mathbf{P}_T-\mathbf{Q})$

4-TotalMomentum $\mathbf{P}_T=(E_T/c,\mathbf{p}_T)=((E+q\varphi)/c,\mathbf{p}+q\mathbf{a})$

4-WaveVector=***Substantiation*** of SR Wave <Events> oscillations proportional to mass:energy & 3-momentum

$\mathbf{K}\cdot\mathbf{K}=(\omega/c)^2-\mathbf{k}\cdot\mathbf{k}=(\mathbf{K}_T-(q\omega_o/E_o)\mathbf{A})\cdot(\mathbf{K}_T-(q\omega_o/E_o)\mathbf{A})=(\omega_o/c)^2$

SR Particle <Events> have 4-Momentum=***Substantiation*** mass:energy & 3-momentum

$\mathbf{P}\cdot\mathbf{P}=(E/c)^2-\mathbf{p}\cdot\mathbf{p}=(\mathbf{P}_T-q\mathbf{A})\cdot(\mathbf{P}_T-q\mathbf{A})=(m_oc)^2=(E_o/c)^2$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
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$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Special Relativity → Quantum Mechanics
RoadMap of SR→QM (w/ EM Potential)

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SR & QM Invariant Waves

SR → RQM Klein-Gordon
Relativistic Quantum
Particle in EM Potential
d'Alembertian Wave Equation

$\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla$
$= (\partial_T + (iq/\hbar)\mathbf{A})\cdot(\partial_T + (iq/\hbar)\mathbf{A})$
$= -(\omega_o/c)^2 = -(m_o c/\hbar)^2$
$= (\partial_\tau/c)^2$

Limit: { |**v**|<<c }
$(i\hbar\partial_{tT}) \sim [\, q\varphi + (m_o c^2) + (i\hbar\nabla_T + q\mathbf{a})^2/(2m_o)\,]$
$(i\hbar\partial_{tT}) \sim [\, V + (i\hbar\nabla_T + q\mathbf{a})^2/(2m_o)\,]$
with potential $V = q\varphi + (m_o c^2)$
=Schrödinger QM Equation (EM potential)
[SR → QM]

4-WaveVector=***Substantiation*** of SR Wave <Events>
oscillations proportional to mass:energy & 3-momentum

$\mathbf{K}\cdot\mathbf{K} = (\omega/c)^2 - \mathbf{k}\cdot\mathbf{k}$
$= (\mathbf{K}_T - (q/\hbar)\mathbf{A})\cdot(\mathbf{K}_T - (q/\hbar)\mathbf{A})$
$= (m_o c/\hbar)^2 = (\omega_o/c)^2$

START HERE: 4-Position=***Location*** of SR <Events> in SpaceTime
follow the arrows

$\partial^\mu[R^\nu] = \eta^{\mu\nu}$ Minkowski Metric

$\partial_\nu[R^{\mu'}] = \Lambda^{\mu'}{}_\nu$ Lorentz Transform

$\partial\cdot\mathbf{R} = 4$ SpaceTime Dim

4-Position R^μ
$\mathbf{R} = (ct, \mathbf{r}) \in$ <Event>

$\mathbf{R}\cdot\mathbf{R} = (ct)^2 - \mathbf{r}\cdot\mathbf{r}$
$= (c\tau)^2 = (i|\mathbf{r}_o|)^2$

$\mathbf{U}\cdot\mathbf{U} = \gamma^2(c^2 - \mathbf{u}\cdot\mathbf{u})$
$= (c)^2$

EM Faraday
$\partial^\mu A^\nu - \partial^\nu A^\mu = F^{\mu\nu}$
4-Tensor

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$
ProperTime Derivative

4-Velocity U^μ
$\mathbf{U} = \gamma(c, \mathbf{u}) = d\mathbf{R}/d\tau$

4-Velocity=***Motion*** of SR <Events> in SpaceTime as both particles & waves

4-Gradient ∂^μ
$\partial = (\partial_t/c, -\nabla) = -i\mathbf{K}$

ProperTime
$\mathbf{U}\cdot\partial = d/d\tau = \gamma d/dt$
Derivative

$-\mathbf{K}\cdot\mathbf{R} = \Phi_{phase,free}$
$-\mathbf{K}_T\cdot\mathbf{R} = \Phi_{phase}$
SR Phase

$-\mathbf{P}\cdot\mathbf{R} = S_{action,free}$
$-\mathbf{P}_T\cdot\mathbf{R} = S_{action}$
SR Action

Phase & Action Lorentz Scalars

Matter Wave
$v_{group} * v_{phase} = c^2$
Rest Angular Frequency ω_o

φ_o/c^2
EM

ω_o/c^2

m_o
E_o/c^2
Einstein
$E = mc^2 = \gamma m_o c^2 = \gamma E_o$

4-EMVectorPotential A^μ
$\mathbf{A} = (\varphi/c, \mathbf{a}) = (\varphi_o/c^2)\mathbf{U}$

EM Charge q

$-\partial[\Phi_{phase,free}] = \mathbf{K}$
$-\partial[\Phi_{phase}] = \mathbf{K}_T$

(-i)

4-WaveVector
Complex
Plane-Waves
$\mathbf{K}_T = -\partial[\Phi]$, $\mathbf{K} = i\partial$

$-\partial[S_{action,free}] = \mathbf{P}$
$-\partial[S_{action}] = \mathbf{P}_T$
Hamilton-Jacobi
$\mathbf{P}_T = -\partial[S]$

4-WaveVector K^μ
$\mathbf{K} = (\omega/c, \mathbf{k}) = (\omega_o/c^2)\mathbf{U} = \mathbf{P}/\hbar$

4-Momentum P^μ
$\mathbf{P} = (mc, \mathbf{p}) = (E/c, \mathbf{p}) = m_o\mathbf{U}$

4-PotentialMomentum Q^μ
$\mathbf{Q} = (V/c, \mathbf{q}) = q(\varphi/c, \mathbf{a}) = q\mathbf{A}$

Einstein, de Broglie
$\mathbf{P} = \hbar\mathbf{K}$

ω_o/E_o
(1/ħ)

4-TotMomentum Conservation
$\mathbf{P}_T = (\mathbf{P} + \mathbf{Q}) = (\mathbf{P} + q\mathbf{A})$

Minimal Coupling
$\mathbf{P} = (\mathbf{P}_T - q\mathbf{A}) = (\mathbf{P}_T - \mathbf{Q})$

4-Momentum=***Substantiation*** of SR Particle <Events>
mass:energy & 3-momentum

$\mathbf{P}\cdot\mathbf{P} = (E/c)^2 - \mathbf{p}\cdot\mathbf{p}$
$= (\mathbf{P}_T - q\mathbf{A})\cdot(\mathbf{P}_T - q\mathbf{A})$
$= (m_o c)^2 = (E_o/c)^2$

4-TotalMomentum P_T^μ
$\mathbf{P}_T = (E_T/c, \mathbf{p}_T) = ((E + q\varphi)/c, \mathbf{p} + q\mathbf{a}) = \mathbf{P} + \mathbf{Q}$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
Quantum Principles

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM Study:
The Empirical 4-Vector Facts

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SR 4-Vector	Empirical Fact	Discoverer	Physics
4-Position	$\mathbf{R} \in$ <Event>	Newton+ Einstein	[t & **r**] Time & Space <time> & <location> [**R**=(ct,**r**)] SpaceTime as 4D=(1+3)D
4-Velocity	$\mathbf{U} = d\mathbf{R}/d\tau$	Newton Einstein	[$\mathbf{v}=\dot{\mathbf{r}}=d\mathbf{r}/dt$] Calculus of motion [$\mathbf{U}=\gamma(c,\mathbf{u})=d\mathbf{R}/d\tau$] Gamma & Proper Time
4-Momentum	$\mathbf{P} = m_o\mathbf{U}$	Newton Einstein	[$\mathbf{p}=m\mathbf{v}$] Classical Mechanics [$\mathbf{P}=(E/c,\mathbf{p})=m_o\mathbf{U}$] SR Mechanics
4-WaveVector	$\mathbf{K} = \mathbf{P}/\hbar$	Planck Einstein de Broglie ?	[h] Photon Thermal Distribution [$E=h\nu=\hbar\omega$] Photoelectric Effect ($\hbar=h/2\pi$) [$\mathbf{p}=\hbar\mathbf{k}$] Matter Waves [$\mathbf{P}=(E/c,\mathbf{p})=\hbar\mathbf{K}=\hbar(\omega/c,\mathbf{k})$] as 4-Vector Math
4-Gradient	$\partial = -i\mathbf{K}$	Schrödinger	[$\omega=i\partial_t$, $\mathbf{k}=-i\nabla$] (SR) Wave Mechanics [$\mathbf{P}=(E/c,\mathbf{p})=i\hbar\partial=i\hbar(\partial_t/c,-\nabla)$] (QM) 4-Vecctor

(1) The SR 4-Vectors and their components are related to each other via constants
(2) We have not taken any 4-vector relation as axiomatic, the constants come from experiment.
(3) c, τ, m_o, $\hbar$ come from physical experiments, (-i) comes from the general mathematics of waves

SRQM Study:
4-Vector Relations Explained

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SR 4-Vector	Empirical Fact	What it means in SRQM...	Lorentz Invariant
4-Position **R** = (ct,**r**)	**R** ∈ <Event>	SpaceTime:LightCone = Unified Concepts	c = LightSpeed
4-Velocity **U** = γ(c,**u**)	**U** = d**R**/dτ	Velocity is ProperTime Derivative of **R**	$\tau = t_o$ = ProperTime
4-Momentum **P** = (E/c,**p**)	**P** = m_o**U**	Mass:Energy-Momentum Equivalence	m_o = RestMass
4-WaveVector **K** = (ω/c,**k**)	**K** = **P**/ħ	Wave-Particle Duality	ħ = UniversalAction
4-Gradient ∂ = (∂_t/c,-∇)	∂ = -i**K**	Unitary Evolution, Operator Formalism	i = ComplexSpace

Three old-paradigm QM Axioms:
Particle-Wave Duality [(**P**)=ħ(**K**)], Unitary Evolution [∂=(-i)**K**], Operator Formalism [(∂)=-i**K**] are actually just empirically-found constant relations between known SR 4-Vectors.
Note that these constants are in fact all Lorentz Scalar Invariants.

Minkowski Space and 4-Vectors also lead to idea of Lorentz Invariance. A Lorentz Invariant is a quantity that always has the same value, independent of the motion or orientation of inertial observers.
Lorentz Invariants can typically be derived using the scalar product relation.
U·**U** = c^2, **U**·∂ = d/dτ , **P**·**U** = $E_o = m_o c^2$, etc.

A very important Lorentz invariant is the Proper Time τ, which is defined as the time displacement between two points on a worldline that is at-rest wrt. an observer. It is used in the relations between 4-Position **R**, 4-Velocity **U** = d**R**/dτ, and 4-Acceleration **A** = d**U**/dτ.

SRQM: The SR Path to RQM
Follow the Invariants...

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SR 4-Vector	Lorentz Invariant	What it means in SRQM...
4-Position $\mathbf{R} = (ct,\mathbf{r}) \in$ <Event>	$\mathbf{R}\cdot\mathbf{R} = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (c\tau)^2$	SR Invariant Interval
4-Velocity $\mathbf{U} = \gamma(c,\mathbf{u}) = d\mathbf{R}/d\tau$	$\mathbf{U}\cdot\mathbf{U} = \gamma^2(c^2 - \mathbf{u}\cdot\mathbf{u}) = c^2$	Events move into future at magnitude c
4-Momentum $\mathbf{P} = (E/c,\mathbf{p}) = m_o\mathbf{U}$	$\mathbf{P}\cdot\mathbf{P} = (m_oc)^2 = (E_o/c)^2$	Einstein Mass:Energy Relation
4-WaveVector $\mathbf{K} = (\omega/c,\mathbf{k}) = \mathbf{P}/\hbar$	$\mathbf{K}\cdot\mathbf{K} = (m_oc/\hbar)^2 = (\omega_o/c)^2$	Matter-Wave Dispersion Relation
4-Gradient $\partial = (\partial_t/c,-\nabla) = -i\mathbf{K}$	$\partial\cdot\partial = (-im_oc/\hbar)^2 = -(m_oc/\hbar)^2$	The Klein-Gordon Equation → RQM!

$\mathbf{U} = d\mathbf{R}/d\tau$
Remember, everything after 4-Velocity was just a constant times the last 4-vector, and the Invariant Magnitude of the 4-Velocity is itself a constant
$\mathbf{P} = m_o\mathbf{U}$, $\mathbf{K} = \mathbf{P}/\hbar$, $\partial = -i\mathbf{K}$, so e.g. $\mathbf{P}\cdot\mathbf{P} = m_o\mathbf{U}\cdot m_o\mathbf{U} = m_o^2\mathbf{U}\cdot\mathbf{U} = (m_oc)^2$

The last equation is the Klein-Gordon RQM Equation, which we have just derived without invoking any QM axioms, only SR plus a few empirical facts

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM: Some Basic 4-Vectors
4-Momentum, 4-WaveVector, 4-Position, 4-Velocity, 4-Gradient, Wave-Particle

4-Vector SRQM Interpretation of QM

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$\mathbf{P}\cdot\mathbf{P} = (m_o c)^2 = (E_o/c)^2$

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$
$\mathbf{P}=-\partial[S_{action,free}]$

$\int \mathbf{P}\cdot d\mathbf{R} = -S_{action,free}$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

Rest Mass:Energy Einstein
$E=\gamma E_o=\gamma m_o c^2=mc^2$

m_o
E_o/c^2

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{U}\cdot\mathbf{U} = (c)^2$

$\mathbf{P}\cdot\mathbf{U} = E_o$

Einstein
de Broglie
$\mathbf{P} = \hbar\mathbf{K}$

$\omega_o/E_o = (1/\hbar)$
or
$E_o/\omega_o = (\hbar)$

$\mathbf{K}\cdot\mathbf{U} = \omega_o$

ω_o/c^2

Wave Velocity
$v_{group} * v_{phase} = c^2$
RestAngFrequency

$\int \mathbf{K}\cdot d\mathbf{R} = -\Phi_{phase,plane}$

$\mathbf{P}\cdot\mathbf{K} = m_o\omega_o$

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$
$\mathbf{K}=-\partial[\Phi_{phase,plane}]$

$\mathbf{K}\cdot\mathbf{K} = (\omega_o/c)^2$

$..[-S_{action,free}]$

Hamilton-Jacobi
$\mathbf{P} = -\partial[S_{action,free}]$

$..[-\Phi_{phase,plane}]$

$\mathbf{K} = -\partial[\Phi_{phase,plane}]$
WaveVector Gradient

4-Gradient
$\partial=(\partial_t/c,-\nabla)\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

d'Alembertian
$\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = (\partial_\tau/c)^2$

SpaceTime
$\partial\cdot\mathbf{R}=4$
Dimension

$\partial[\mathbf{R}]=\eta^{\mu\nu}\rightarrow \mathrm{Diag}[1,-1,-1,-1]$
Minkowski Metric

ProperTime
$\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$
Derivative

Treating motion like a particle
Moving particles have a 4-Velocity
4-Momentum is the negative 4-Gradient of the SR Action (S)

Treating motion like a wave
Moving waves have a 4-Velocity
4-WaveVector is the negative 4-Gradient of the SR Phase (Φ)

See Hamilton-Jacobi Formulation of Mechanics
for info on the Lorentz Scalar Invariant SR Action.
{ $\mathbf{P} = (E/c,\mathbf{p}) = -\partial[S] = (-\partial/c\partial t[S],\nabla[S])$ }
{temporal component} $E = -\partial/\partial t[S] = -\partial_t[S]$
{spatial component} $\mathbf{p} = \nabla[S]$
Note This is the Action (S_{action}) for a free particle.
Generally Action is for the 4-TotalMomentum $\mathbf{P}_T$ of a system.

See SR Wave Definition
for info on the Lorentz Scalar Invariant SR WavePhase.
{ $\mathbf{K} = (\omega/c,\mathbf{k}) = -\partial[\Phi] = (-\partial/c\partial t[\Phi],\nabla[\Phi])$ }
{temporal component} $\omega = -\partial/\partial t[\Phi] = -\partial_t[\Phi]$
{spatial component} $\mathbf{k} = \nabla[\Phi]$
Note This is the Phase (Φ) for a single free plane-wave.
Generally WavePhase is for the 4-TotalWaveVector $\mathbf{K}_T$ of a system.

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
Quantum Principles

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM: Wave-Particle Diffraction/Interference Types

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

The 4-Vector Wave-Particle relation is inherent in all particle types: Einstein-de Broglie $\mathbf{P} = (E/c,\mathbf{p}) = \hbar\mathbf{K} = \hbar(\omega/c,\mathbf{k})$.

All waves can superpose, interfere, diffract: Water waves, gravitational waves, photonic waves of all frequencies, etc.
In all cases: experiments using single particles build the diffraction/interference pattern over the course many iterations.

Photon/light Diffraction: Photonic particles diffracted by matter particles.
Photons of any frequency encounter a translucent "solid=matter" object, grating, or edge.
Most often encountered are diffraction gratings and the famous double-slit experiment

Matter Diffraction: Matter particles diffracted by matter particles.
Electrons, neutrons, atoms, small molecules, buckyballs (fullerenes), macromolecules, etc.
have been shown to diffract through crystals.
Crystals may be solid single pieces or in powder form.

Kapitsa-Dirac Diffraction: Matter particles diffracted by photonic standing waves.
Electrons, atoms, super-sonic atom beams have been diffracted from resonant standing waves of light.

Photonic-Photonic Diffraction?: Delbruck scattering & Light-by-light scattering
Light-by-light scattering/two-photon physics/gamma-gamma physics.
Normally, photons do not interact, but at high enough relative energy, virtual particles can form which allow interaction.

$\mathbf{P}\cdot\mathbf{P} = (m_o c)^2 = (E_o/c)^2$

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$
$\mathbf{P}=-\partial[S_{action,free}]$

$\downarrow \omega_o/E_o = (1/\hbar)$
or
$\uparrow E_o/\omega_o = (\hbar)$

Einstein de Broglie
$\mathbf{P} = \hbar\mathbf{K}$

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$
$\mathbf{K}=-\partial[\Phi_{phase,plane}]$

$\mathbf{K}\cdot\mathbf{K} = (\omega_o/c)^2$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

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= Lorentz Scalar Invariant

SRQM:
Hold on, aren't you getting the "ħ" from a QM Axiom?

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John B. Wilson
SciRealm@aol.com
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SR 4-Vector	SR Empirical Fact	What it means...
4-WaveVector	$\mathbf{K} = (\omega/c, \mathbf{k}) = (\omega/c, \omega\hat{\mathbf{n}}/v_{phase}) = (\omega_o/c^2)\mathbf{U}$	Wave-Particle Duality

(ħ) is actually an empirically measurable quantity, just like (e) or (c). It can be measured classically from the photoelectric effect, from the inverse photoelectric effect, from LED's (injection electroluminescence), from Atomic Line Spectra (Bohr Atom), from the Duane-Hunt Law in Bremsstrahlung, from Electron Diffraction in crystals, from the Watt/Kibble-Balance, from Incandescent Blackbody Intensity-Temperature Relations, from Compton Scattering, from the Stern-Gerlach experiment, etc.
See (http://scirealm.org/Physics-PlanckConstantViaLEDs.html)

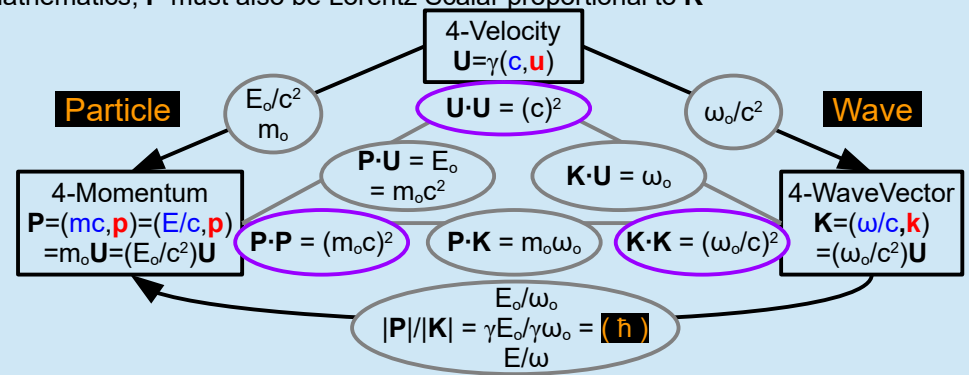
For the LED experiment, one uses several different LED's, each with its own characteristic wavelength.
One then makes a chart of each LED's wavelength (λ) vs the threshold or activation voltage (V) needed to make that individual LED emit.
*One finds that: { λ = h*c / (e*V) } or { e*V = h*c / λ }, where (e)=ElectronCharge, (c)=LightSpeed, and (h) is simply a slope that can be measured.*
Consider this as a [black-box] *where no assumption about QM is made. However, we know the* <u>*classical*</u> *SR relations { E = e*V }, and { λ*f = c }.*
*The data force one to conclude that { E = h*f = ħ*ω = ħω }.*
Applying our 4-Vector knowledge, we recognize this as the temporal components of an SR 4-Vector relation. (E/c,...) = ħ(ω/c,...)
Due to manifest Tensor Invariance, this means that 4-Momentum $\mathbf{P} = (E/c, \mathbf{p}) = \hbar\mathbf{K} = \hbar(\omega/c, \mathbf{k}) = \hbar$**4-WaveVector* $\mathbf{K}$.

The spatial component (due to De Broglie) follows naturally from the temporal component (due to Einstein) via to the nature of 4-Vector (Tensor) mathematics.

This is also derivable from pure SR 4-Vector (Tensor) arguments: $\mathbf{P} = m_o\mathbf{U} = (E_o/c^2)\mathbf{U}$ and $\mathbf{K} = (\omega_o/c^2)\mathbf{U}$
Since **P** and **K** are both Lorentz Scalar proportional to **U**, then by the rules of tensor mathematics, **P** must also be Lorentz Scalar proportional to **K**
i.e. Tensors obey certain mathematical structures:
Transitivity{if a~b and b~c, then a~c} & Euclideaness: {if a~c and b~c, then a~b}

This invariant proportional constant is empirically measured to be (ħ),
for each known particle type, whether massive (m_o>0) or massless (m_o=0):
$\mathbf{P} = m_o\mathbf{U} = (E_o/c^2)\mathbf{U} = (E_o/c^2)/(\omega_o/c^2)\mathbf{K} = (E_o/\omega_o)\mathbf{K} = (\gamma E_o/\gamma\omega_o)\mathbf{K} = (E/\omega)\mathbf{K} = (\hbar)\mathbf{K}$

also from standard SR Lorentz 4-Vector Scalar Products:
$(\mathbf{P}\cdot\mathbf{U})/(\mathbf{K}\cdot\mathbf{U}) = E_o/\omega_o \rightarrow |\mathbf{P}|/|\mathbf{K}| = E_o/\omega_o = (\hbar)$
$(\mathbf{P}\cdot\mathbf{K})/(\mathbf{K}\cdot\mathbf{K}) = m_o\omega_o/(\omega_o/c)^2 \rightarrow |\mathbf{P}|/|\mathbf{K}| = E_o/\omega_o = (\hbar)$
$(\mathbf{P}\cdot\mathbf{P})/(\mathbf{K}\cdot\mathbf{P}) = (m_oc)^2/(m_o\omega_o) \rightarrow |\mathbf{P}|/|\mathbf{K}| = E_o/\omega_o = (\hbar)$
$(\mathbf{P}\cdot\mathbf{R})/(\mathbf{K}\cdot\mathbf{R}) = (-S_{action,free})/(-\Phi_{phase,plane}) \rightarrow |\mathbf{P}|/|\mathbf{K}| = E_o/\omega_o = (\hbar)$



SRQM:
Hold on, aren't you getting the "K" from a QM Axiom?

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John B. Wilson
SciRealm@aol.com
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SR 4-Vector	SR Empirical Fact	What it means...
4-WaveVector	$\mathbf{K} = (\omega/c, \mathbf{k}) = (\omega/c, \omega\hat{\mathbf{n}}/v_{phase}) = (\omega_o/c^2)\mathbf{U}$	Wave-Particle Duality

K is a standard SR 4-Vector, used in generating the SR formulae:

Relativistic Doppler Effect:

$\omega_{obs} = \omega_{emit} / [\gamma(1 - \beta \cos[\theta])]$, $\quad |\mathbf{k}| = k = \omega/c$ for photons

Relativistic Aberration Effect:

$\cos[\theta_{obs}] = (\cos[\theta_{emit}] + |\boldsymbol{\beta}|) / (1 + |\boldsymbol{\beta}|\cos[\theta_{emit}])$

The 4-WaveVector **K** can be derived in terms of periodic motion, where families of surfaces move through space as time increases, or alternately, as families of hypersurfaces in SpaceTime, formed by all events passed by the wave surface. The 4-WaveVector is everywhere in the direction of propagation of the wave surfaces.

$\mathbf{K} = -\partial[\Phi_{phase,planewave}]$

From this structure, one obtains relativistic/wave optics without ever mentioning QM.

SRQM:
Hold on, aren't you getting the "-i" from a QM Axiom?

SciRealm.org
John B. Wilson
SciRealm@aol.com
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SR 4-Vector	SR Empirical Fact	What it means...
4-Gradient	$\partial = (\partial_t/c, -\nabla) = -i\mathbf{K}$	Unitary Evolution of States Operator Formalism

[$\partial = -i\mathbf{K}$] gives the sub-equations [$\partial_t = -i\omega$] and [$\nabla = i\mathbf{k}$], and is certainly the main equation that relates QM and SR by allowing Operator Formalism. But, this is a basic equation regarding the general mathematics of plane-waves; not just quantum-waves, but anything that can be mathematically described by plane-waves and superpositions of plane-waves…
This includes purely SR waves, an example of which would be EM plane-waves (i.e. photons)...

$\psi(t,\mathbf{r}) = ae^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$: Standard mathematical plane-wave equation

$\partial_t[\psi(t,\mathbf{r})] = \partial_t[ae^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}] = (-i\omega)[ae^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}] = (-i\omega)\psi(t,\mathbf{r})$, or [$\partial_t = -i\omega$]
$\nabla[\psi(t,\mathbf{r})] = \nabla[ae^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}] = (i\mathbf{k})[ae^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}] = (i\mathbf{k})\psi(t,\mathbf{r})$, or [$\nabla = i\mathbf{k}$]

In the more economical SR notation:
$\partial[\psi(\mathbf{R})] = \partial[ae^{(-i\mathbf{K}\cdot\mathbf{R})}] = (-i\mathbf{K})[ae^{(-i\mathbf{K}\cdot\mathbf{R})}] = (-i\mathbf{K})\psi(\mathbf{R})$, or in 4-Vector shorthand [$\partial = -i\mathbf{K}$]

This one is more of a mathematical empirical fact, but regardless, it is not axiomatic.
It can describe purely SR waves, again without any mention of QM.

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla$

4-WaveVector
$\mathbf{K} = (\omega/c = 2\pi\nu/c, \mathbf{k}) = (\omega/c, \omega\hat{\mathbf{n}}/v_{phase}) = (1/c\mathrm{T}, \hat{\mathbf{n}}/\lambda) = 2\pi(1/c\mathrm{T}, \hat{\mathbf{n}}/\lambda) = (\omega_o/c^2)\mathbf{U}$

$\mathbf{K}\cdot\mathbf{K} = (\omega_o/c)^2$

SRQM:
Hold on, aren't you getting the "∂" from a QM Axiom?

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John B. Wilson
SciRealm@aol.com
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SR 4-Vector	SR Empirical Fact	What it means...
4-Gradient	$\partial = (\partial_t/c, -\nabla) = -i\mathbf{K}$	4D Gradient Operator

[$\partial = (\partial_t/c, -\nabla)$] is the SR 4-Vector Gradient Operator. It occurs in a purely relativistic context without ever mentioning QM.

$\partial\cdot\mathbf{X} = \partial^\mu \eta_{\mu\nu} X^\nu = (\partial_t/c, -\nabla)\cdot(ct, \mathbf{x}) = (\partial_t/c[ct] - (-\nabla\cdot\mathbf{x})) = (\partial_t[t] + \nabla\cdot\mathbf{x})\ (1)+(3) = 4$

The 4-Divergence of the 4-Position gives the dimensionality of SpaceTime.

$\partial[\mathbf{X}] = \partial^\mu[X^\nu] = (\partial_t/c, -\nabla)[(ct, \mathbf{x})] = (\partial_t/c[ct], -\nabla[\mathbf{x}]) = \mathrm{Diag}[1, -\mathbf{I}_{(3)}] = \eta^{\mu\nu}$

The 4-Gradient acting on the 4-Position gives the Minkowski Metric Tensor

$\partial\cdot\mathbf{J} = \partial^\mu \eta_{\mu\nu} J^\nu = (\partial_t/c, -\nabla)\cdot(\rho c, \mathbf{j}) = (\partial_t/c[\rho c] - (-\nabla\cdot\mathbf{j})) = (\partial_t[\rho] + \nabla\cdot\mathbf{j}) = 0$

The 4-Divergence of the 4-CurrentDensity is equal to 0 for a conserved current. It can be rewritten as ($\partial_t[\rho] = -\nabla\cdot\mathbf{j}$), which means that the time change of ChargeDensity is balanced by the space change or divergence of CurrentDensity. It is a Continuity Equation, giving local conservation of ChargeDensity. It is related to Noether's Theorem.

SRQM:
Hold on, doesn't using "∂" in an Equation of Motion presume a QM Axiom?

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John B. Wilson
SciRealm@aol.com
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SR 4-Vector	SR Empirical Fact	What it means...
4-(Position)Gradient	$\partial_R = \partial = (\partial_t/c, -\nabla) = -i\mathbf{K}$	4D Gradient Operator

Klein-Gordon Relativistic Quantum Wave Equation
$\partial\cdot\partial[\Psi] = -(m_o c/\hbar)^2[\Psi] = -(\omega_o/c)^2[\Psi]$

Relativistic Euler-Lagrange Equations
$\partial_R[L] = (d/d\tau)\partial_U[L]$: {particle format}
$\partial_{[\Phi]}[\mathcal{L}] = (\partial_R)\, \partial_{[\partial_R(\Phi)]}[\mathcal{L}]$: {density format}

$[\partial = (\partial_t/c, -\nabla)]$ is the SR 4-Vector (Position)Gradient Operator.
It occurs in a purely relativistic context without ever mentioning QM.
There is a long history of using the gradient operator on classical physics functions, in this case the Lagrangian. And, in fact, it is another area where the same mathematics is used in both classical and quantum contexts.

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram: RoadMap of SR→QM

QM Schrödinger Relation

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
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The QM Schrödinger Relation
$\mathbf{P} = i\hbar\partial$

This is derived from the combination of:

The Einstein-de Broglie Relation
$\mathbf{P} = \hbar\mathbf{K}$

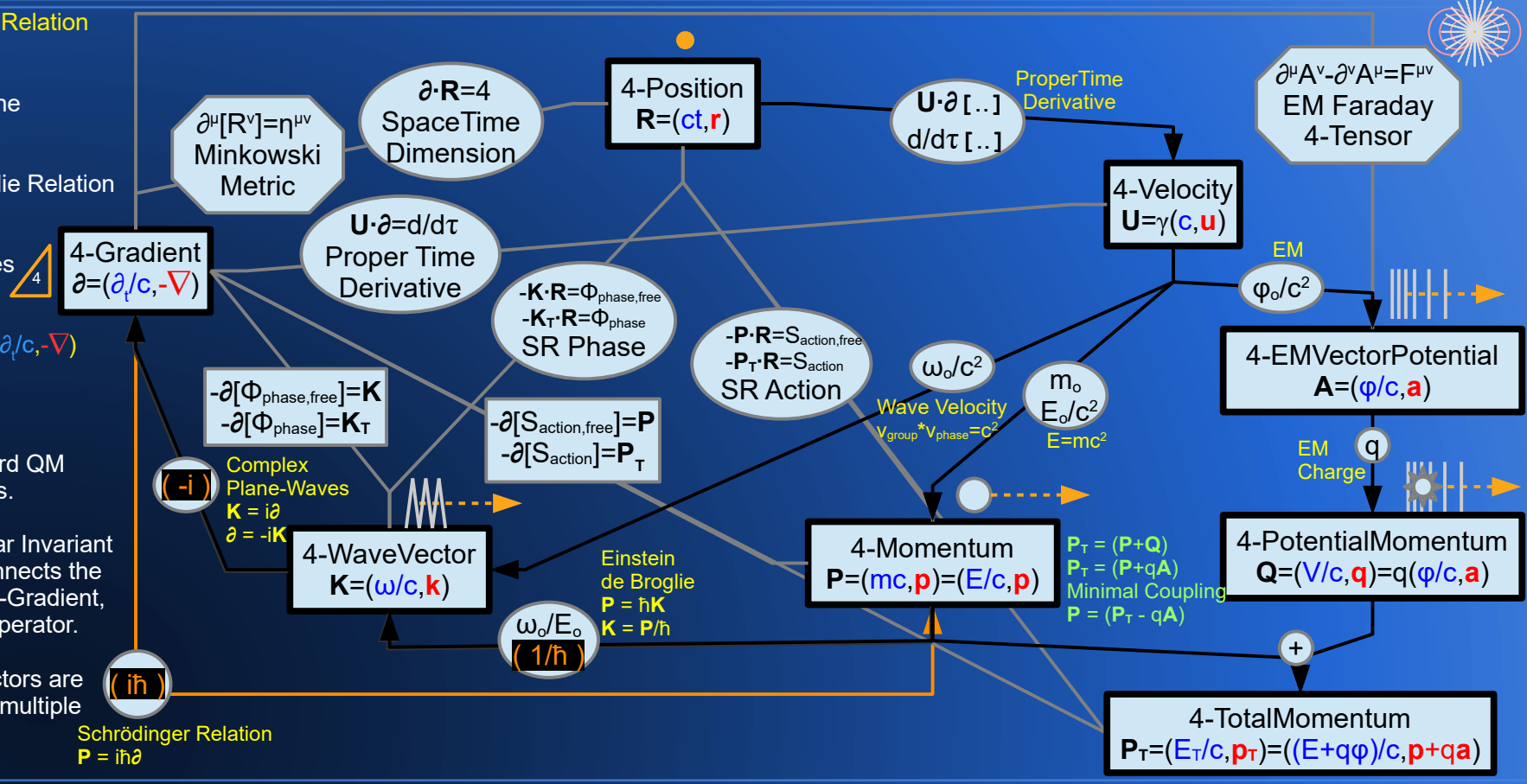
Complex Plane-Waves
$\mathbf{K} = i\partial$

$\mathbf{P} = (E/c,\mathbf{p}) = i\hbar\partial = i\hbar(\partial_t/c,-\nabla)$
{temporal} $E = i\hbar\partial_t$
{spatial} $\mathbf{p} = -i\hbar\nabla$

These are the standard QM Schrödinger Relations.

It is this Lorentz Scalar Invariant relation ($i\hbar$) which connects the 4-Momentum to the 4-Gradient, making it into a QM operator.

Note that these 4-Vectors are already connected in multiple ways in standard SR.



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Quantum Principles

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= Lorentz Scalar Invariant

SRQM:
Review of SR 4-Vector Mathematics

SciRealm.org
John B. Wilson
SciRealm@aol.com
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4-Gradient $\partial = (\partial_t/c, -\nabla)$ — $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = -(\omega_o/c)^2$

4-Position $\mathbf{X} = (ct, \mathbf{x})$ — $\mathbf{X}\cdot\mathbf{X} = ((ct)^2 - \mathbf{x}\cdot\mathbf{x}) = (ct_o)^2 = (c\tau)^2$: Invariant Interval Measure

4-Velocity $\mathbf{U} = \gamma(c, \mathbf{u})$ — $\mathbf{U}\cdot\mathbf{U} = \gamma^2(c^2 - \mathbf{u}\cdot\mathbf{u}) = (c)^2$

4-Momentum $\mathbf{P} = (E/c, \mathbf{p}) = (E_o/c^2)\mathbf{U}$ — $\mathbf{P}\cdot\mathbf{P} = (E/c)^2 - \mathbf{p}\cdot\mathbf{p} = (E_o/c)^2$

4-WaveVector $\mathbf{K} = (\omega/c, \mathbf{k}) = (\omega_o/c^2)\mathbf{U}$ — $\mathbf{K}\cdot\mathbf{K} = (\omega/c)^2 - \mathbf{k}\cdot\mathbf{k} = (\omega_o/c)^2$

$\partial\cdot\mathbf{X} = (\partial_t/c, -\nabla)\cdot(ct, \mathbf{x}) = (\partial_t/c[ct] - (-\nabla\cdot\mathbf{x})) = 1-(-3) = 4$: Dimensionality of SpaceTime

$\mathbf{U}\cdot\partial = \gamma(c, \mathbf{u})\cdot(\partial_t/c, -\nabla) = \gamma(\partial_t + \mathbf{u}\cdot\nabla) = \gamma(d/dt) = d/d\tau$: Derivative wrt. ProperTime is Lorentz Scalar

$\partial[\mathbf{X}] = (\partial_t/c, -\nabla)(ct, \mathbf{x}) = (\partial_t/c[ct], -\nabla[\mathbf{x}]) = \mathrm{Diag}[1, -\mathbf{1}] = \eta^{\mu\nu}$: The Minkowski Metric

$\partial[\mathbf{K}] = (\partial_t/c, -\nabla)(\omega/c, \mathbf{k}) = (\partial_t/c[\omega/c], -\nabla[\mathbf{k}]) = [[\mathbf{0}]]$

$\mathbf{K}\cdot\mathbf{X} = (\omega/c, \mathbf{k})\cdot(ct, \mathbf{x}) = (\omega t - \mathbf{k}\cdot\mathbf{x}) = \varphi$: Phase of SR Wave

$\partial[\mathbf{K}\cdot\mathbf{X}] = \partial[\mathbf{K}]\cdot\mathbf{X} + \mathbf{K}\cdot\partial[\mathbf{X}] = \mathbf{K} = -\partial[\varphi]$: Neg 4-Gradient of Phase gives 4-WaveVector

$(\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}] = ((\partial_t/c)^2 - \nabla\cdot\nabla)(\omega t - \mathbf{k}\cdot\mathbf{x}) = 0$

$(\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}] = \partial\cdot(\partial[\mathbf{K}\cdot\mathbf{X}]) = \partial\cdot\mathbf{K} = 0$: Wave Continuity Equation, No sources or sinks

let f = ae^b($\mathbf{K}\cdot\mathbf{X}$): Standard mathematical plane-waves if { b = -i }

then $\partial[f] = (-i\mathbf{K})ae^{-i(\mathbf{K}\cdot\mathbf{X})} = (-i\mathbf{K})f$: $(\partial = -i\mathbf{K})$: Unitary Evolution, Operator Formalism

and $\partial\cdot\partial[f] = (-i)^2(\mathbf{K}\cdot\mathbf{K})f = -(\omega_o/c)^2 f$:

$(\partial\cdot\partial) = (\partial_t/c)^2 - \nabla\cdot\nabla = -(\omega_o/c)^2$: The Klein-Gordon Equation → RQM

Note that no QM Axioms are assumed: This is all just pure SR 4-vector (tensor) manipulation

SRQM:
Review of SR 4-Vector Mathematics

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Klein-Gordon Equation: $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = -(m_o c/\hbar)^2 = -(\omega_o/c)^2 = -(1/\lambda\!\!\!^{-}_C)^2$

Let $\mathbf{X}_T = (ct+c\Delta t,\mathbf{x})$, then $\partial[\mathbf{X}_T] = (\partial_t/c,-\nabla)(ct+c\Delta t,\mathbf{x}) = \mathrm{Diag}[1,-\mathbf{I}_{(3)}] = \partial[\mathbf{X}] = \eta^{\mu\nu}$
so $\partial[\mathbf{X}_T] = \partial[\mathbf{X}]$ and $\partial[\mathbf{K}] = [[\mathbf{0}]]$
let f = ae^-i($\mathbf{K}\cdot\mathbf{X}_T$), the time translated version
$(\partial\cdot\partial)[f]$
$\partial\cdot(\partial[f])$
$\partial\cdot(\partial[e^{-i(\mathbf{K}\cdot\mathbf{X}_T)}])$
$\partial\cdot(e^{-i(\mathbf{K}\cdot\mathbf{X}_T)}\partial[-i(\mathbf{K}\cdot\mathbf{X}_T)])$
$-i\partial\cdot(f\partial[\mathbf{K}\cdot\mathbf{X}_T])$
$-i\partial[f]\partial[\mathbf{K}\cdot\mathbf{X}_T])+\Psi(\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}_T])$
$(-i)^2 f(\partial[\mathbf{K}\cdot\mathbf{X}_T])^2 + 0$
$(-i)^2 f(\partial[\mathbf{K}]\cdot\mathbf{X}_T + \mathbf{K}\cdot\partial[\mathbf{X}_T])^2$
$(-i)^2 f(0+\mathbf{K}\cdot\partial[\mathbf{X}])^2$
$(-i)^2 f(\mathbf{K})^2$
$-(\mathbf{K}\cdot\mathbf{K})f$
$-(\omega_o/c)^2 f$

SRQM:
What does the Klein-Gordon Equation give us?
A lot of RQM!

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John B. Wilson
SciRealm@aol.com
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Relativistic Quantum Wave Equation: $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = -(m_o c/\hbar)^2 = (i m_o c/\hbar)^2 = -(\omega_o/c)^2$

The Klein-Gordon Eqn is itself the Relativistic Quantum Equation for spin=0 particles (4-Scalars)
Factoring the KG Eqn leads to the RQM Dirac Equation for spin=1/2 particles (4-Spinors)
Applying the KG Eqn to a 4-Vector field leads to the RQM Proca Equation for spin=1 particles (4-Vectors)

Taking the low-velocity-limit of the KG leads to the standard QM non-relativistic Schrödinger Eqn, for spin=0
Taking the low-velocity-limit of the Dirac leads to the standard QM non-relativistic Pauli Eqn, for spin=1/2

Setting RestMass $\{m_o \rightarrow 0\}$ leads to the RQM Free Wave Eqn., Weyl Eqn., and Free Maxwell (Standard EM) Eqn.

In all of these cases, the equations can be modified to work with various potentials by using more SR 4-Vectors, and more empirically found relations between them, e.g. the Minimal Coupling Relations: 4-TotalMomentum $\mathbf{P}_T = \mathbf{P} + q\mathbf{A}$, where **P** is the particle 4-Momentum, (q) is a charge, and **A** is a 4-VectorPotential, typically the 4-EMVectorPotential.

Also note that generating QM from RQM (via a low-energy limit) is much more natural than attempting to "relativize or generalize" a given NRQM equation. Facts assumed from a non-relativistic equation may or may not be applicable to a relativistic one, whereas the relativistic facts are still true in the low-velocity limiting-cases. This leads to the idea that QM is an approximation only of a more general RQM, just as SR is an approximation only of GR.

SRQM:
Relativistic Quantum Wave Eqns.

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John B. Wilson
SciRealm@aol.com
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Spin-(Statistics) **Bose-Einstein=n** **Fermi-Dirac=n/2**	**Relativistic Light-like** **Mass = 0**	**Relativistic Matter-like** **Mass > 0**	**Non-Relativistic Limit (|v|<<c)** **Mass >0**	**Field** **Representation**
0-(Boson)	**Free Wave** N-G Bosons $(\partial\cdot\partial)\Psi = 0$	**Klein-Gordon** Higgs Bosons, maybe Axions $(\partial\cdot\partial + (m_oc/\hbar)^2)\Psi = [\partial_\mu+im_oc/\hbar][\partial^\mu-im_oc/\hbar]\Psi = 0$ with minimal coupling $((i\hbar\partial_t - q\varphi)^2 - (m_oc^2)^2 - c^2(-i\hbar\nabla - qa)^2)\Psi = 0$?Axions? are KG with EM invariant src term $(\partial\cdot\partial + (m_{ao})^2)\Psi = -\kappa\mathbf{e}\cdot\mathbf{b} = -\kappa c\,\mathrm{Sqrt}[\mathrm{Det}[F^{\mu\nu}]]$ $L = (-\hbar^2/m_o)\partial^\mu\Psi^*\partial_\nu\Psi - m_oc^2\Psi^*\Psi$	**Schrödinger** Common NRQM Systems $(i\hbar\partial_t + [\hbar^2\nabla^2/2m_o - V])\Psi = 0$ with minimal coupling $(i\hbar\partial_t - q\varphi - [(\mathbf{p}-q\mathbf{a})^2]/2m_o)\Psi = 0$	Scalar (0-Tensor) $\Psi = \Psi[K_\mu X^\mu]$ $= \Psi[\Phi]$
1/2-(Fermion)	**Weyl** Idealized Matter Neutinos $(\boldsymbol{\sigma}\cdot\partial)\boldsymbol{\Psi} = 0$ factored to Right & Left Spinors $(\boldsymbol{\sigma}\cdot\partial)\boldsymbol{\Psi}_R = 0$, $(\bar{\boldsymbol{\sigma}}\cdot\partial)\boldsymbol{\Psi}_L = 0$ $L = i\boldsymbol{\Psi}^\dagger_R\sigma^\mu\partial_\mu\boldsymbol{\Psi}_R$, $L = i\boldsymbol{\Psi}^\dagger_L\bar{\sigma}^\mu\partial_\mu\boldsymbol{\Psi}_L$	**Dirac** Matter Leptons/Quarks $(i\gamma\cdot\partial - m_oc/\hbar)\boldsymbol{\Psi} = 0$ $(\gamma\cdot\partial + im_oc/\hbar)\boldsymbol{\Psi} = 0$ with minimal coupling $(i\gamma\cdot(\partial+iq\mathbf{A}) - m_oc/\hbar)\boldsymbol{\Psi} = 0$ $L = i\hbar c\bar{\boldsymbol{\Psi}}\boldsymbol{\gamma}^\mu\partial_\mu\boldsymbol{\Psi} - m_oc^2\bar{\boldsymbol{\Psi}}\boldsymbol{\Psi}$	**Pauli** Common NRQM Systems w Spin $(i\hbar\partial_t - [(\boldsymbol{\sigma}\cdot\mathbf{p})^2]/2m_o)\boldsymbol{\Psi} = 0$ with minimal coupling $(i\hbar\partial_t - q\varphi - [(\boldsymbol{\sigma}\cdot(\mathbf{p}-q\mathbf{a}))^2]/2m_o)\boldsymbol{\Psi} = 0$	Spinor $\boldsymbol{\Psi} = \boldsymbol{\Psi}[K_\mu X^\mu]$ $= \boldsymbol{\Psi}[\Phi]$
1-(Boson)	**Maxwell** Photons/Gluons $(\partial\cdot\partial)\mathbf{A} = 0$ free $(\partial\cdot\partial)\mathbf{A} = \mu_o\mathbf{J}$ w current src where $\partial\cdot\mathbf{A} = 0$ $(\partial\cdot\partial)\mathbf{A} = \mu_o e\bar{\Psi}\gamma^\nu\Psi$ QED	**Proca** Force Bosons $(\partial\cdot\partial + (m_oc/\hbar)^2)\mathbf{A} = 0$ where $\partial\cdot\mathbf{A} = 0$ $\partial^\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) + (m_oc/\hbar)^2 A^\nu = 0$		4-Vector (1-Tensor) $\mathbf{A} = A^\nu = A^\nu[K_\mu X^\mu]$ $= A^\nu[\Phi]$

SRQM:
Factoring the KG Equation → Dirac Eqn

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Klein-Gordon Equation: $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = -(m_oc/\hbar)^2$

Since the 4-vectors are related by constants, we can go back to the 4-Momentum description/representation:

$(\partial_t/c)^2 - \nabla\cdot\nabla = -(m_oc/\hbar)^2$
$(E/c)^2 - \mathbf{p}\cdot\mathbf{p} = (m_oc)^2$
$E^2 - c^2\mathbf{p}\cdot\mathbf{p} - (m_oc^2)^2 = 0$

Factoring: $[\, E - c\,\boldsymbol{\alpha}\cdot\mathbf{p} - \beta(m_oc^2)\,]\,[\, E + c\,\boldsymbol{\alpha}\cdot\mathbf{p} + \beta(m_oc^2)\,] = 0$

E & **p** are quantum operators,
α & β are matrices which must obey $\boldsymbol{\alpha}_i\beta = -\beta\boldsymbol{\alpha}_i$, $\boldsymbol{\alpha}_i\boldsymbol{\alpha}_j = -\boldsymbol{\alpha}_j\boldsymbol{\alpha}_i$, $\boldsymbol{\alpha}_i^2 = \beta^2 = \mathbf{I}$
The left hand term can be set to 0 by itself, giving...
$[\, E - c\,\boldsymbol{\alpha}\cdot\mathbf{p} - \beta(m_oc^2)\,] = 0$, which is the momentum-representation form of the Dirac equation

Remember: $P^\mu = (p^0,\mathbf{p}) = (E/c,\mathbf{p})$ and $\alpha^\mu = (\alpha^0,\boldsymbol{\alpha})$ where $\alpha^0 = I_{(2)}$

$[\, E - c\,\boldsymbol{\alpha}\cdot\mathbf{p} - \beta(m_oc^2)\,] = [\, c\alpha^0p^0 - c\,\boldsymbol{\alpha}\cdot\mathbf{p} - \beta(m_oc^2)\,] = [\, c\alpha^\mu P_\mu - \beta(m_oc^2)\,] = 0$
$[\, \alpha^\mu P_\mu - \beta(m_oc)\,] = [i\hbar\,\alpha^\mu\partial_\mu - \beta(m_oc)\,] = 0$
$\alpha^\mu\partial_\mu = -\,\beta(im_oc/\hbar)$

Transforming from Pauli Spinor (2 component) to Dirac Spinor (4 component) form:
Dirac Equation: $(\gamma^\mu\partial_\mu)[\psi] = -(im_oc/\hbar)\psi$

Thus, the Dirac Eqn is guaranteed by construction to be one solution of the KG Eqn

The KG Equation is at the heart of all the various relativistic wave equations, which differ based on mass and spin values, but all of them respect $E^2 - c^2\mathbf{p}\cdot\mathbf{p} - (m_oc^2)^2 = 0$

SRQM Study:
Lots of Relativistic Quantum Wave Equations
A lot of RQM!

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Relativistic Quantum Wave Equation: $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = -(m_o c/\hbar)^2 = (i m_o c/\hbar)^2 = -(\omega_o/c)^2$
$\partial\cdot\partial = -(m_o c/\hbar)^2$

The Klein-Gordon Eqn is itself the Relativistic Quantum Equation for spin=0 particles {Higgs} (4-Scalars)
Factoring the KG Eqn ("square root method") leads to the RQM Dirac Equation for spin=1/2 particles (4-Spinors)
Applying the KG Eqn to a SR 4-Vector field leads to the RQM Proca Equation for spin=1 particles (4-Vectors)

<u>Setting RestMass {$m_o \rightarrow 0$} leads to the:</u>
RQM Free Wave (4-Scalar massless)
RQM Weyl (4-Spinor massless)
Free Maxwell Eqns (4-Vector massless) = Standard EM

So, the same Relativistic Quantum Wave Equation is simply applied to different SR Tensorial Quantum Fields
See Mathematical_formulation_of_the_Standard_Model at Wikipedia:

4-Scalar (massive)	Higgs Field φ	$[\partial\cdot\partial = -(m_o c/\hbar)^2]\varphi$	Free Field Eqn→Klein-Gordon Eqn	$\partial\cdot\partial[\varphi] = -(m_o c/\hbar)^2\varphi$
4-Vector (massive)	Weak Field $Z^\mu, W^{\pm\mu}$	$[\partial\cdot\partial = -(m_o c/\hbar)^2]Z^\mu$	Free Field Eqn→Proca Eqn	$\partial\cdot\partial[Z^\mu] = -(m_o c/\hbar)^2 Z^\mu$
4-Vector (massless m_o=0)	Photon Field A^μ	$[\partial\cdot\partial = 0]A^\mu$	Free Field Eqn→EM Wave Eqn	$\partial\cdot\partial[A^\mu] = 0^\mu$
4-Spinor (massive)	Fermion Field ψ	$[\gamma\cdot\partial = -i m_o c/\hbar]\Psi$	Free Field Eqn→Dirac Eqn	$\gamma\cdot\partial[\Psi] = -(i m_o c/\hbar)\Psi$

*The Fermion Field is a special case, the Dirac Gamma Matrices γ^μ and 4-Spinor field Ψ work together to preserve Lorentz Invariance.

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SciRealm.org
John B. Wilson
SciRealm@aol.com
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Relativistic Quantum Wave Equation: $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = -(m_oc/\hbar)^2 = (im_oc/\hbar)^2 = -(\omega_o/c)^2$
$\partial\cdot\partial = -(m_oc/\hbar)^2$

$(\partial\cdot\partial)A^\nu = 0^\nu$: The Free Classical Maxwell EM Equation {no source, no spin effects}

$(\partial\cdot\partial)A^\nu = \mu_oJ^\nu$: The Classical Maxwell EM Equation {with 4-Current J source, no spin effects}

$(\partial\cdot\partial)A^\nu = q(\bar{\psi}\gamma^\nu\psi)$: The QED Maxwell EM Spin-1 Equation {with QED source, including spin effects}

So, the same Relativistic Quantum Wave Equation is simply applied to different SR Tensorial Quantum Fields
See Mathematical_formulation_of_the_Standard_Model at Wikipedia:

4-Scalar (massive)	Higgs Field φ	$[\partial\cdot\partial = -(m_oc/\hbar)^2]\varphi$	Free Field Eqn→Klein-Gordon Eqn	$\partial\cdot\partial[\varphi] = -(m_oc/\hbar)^2\varphi$
4-Vector (massive)	Weak Field $Z^\mu, W^{\pm\mu}$	$[\partial\cdot\partial = -(m_oc/\hbar)^2]Z^\mu$	Free Field Eqn→Proca Eqn	$\partial\cdot\partial[Z^\mu] = -(m_oc/\hbar)^2Z^\mu$
4-Vector (massless m_o=0)	Photon Field A^μ	$[\partial\cdot\partial = 0]A^\mu$	Free Field Eqn→EM Wave Eqn	$\partial\cdot\partial[A^\mu] = 0^\mu$
4-Spinor (massive)	Fermion Field ψ	$[\gamma\cdot\partial = -im_oc/\hbar]\Psi$	Free Field Eqn→Dirac Eqn	$\gamma\cdot\partial[\Psi] = -(im_oc/\hbar)\Psi$

*The Fermion Field is a special case, the Dirac Gamma Matrices γ^μ and 4-Spinor field Ψ work together to preserve Lorentz Invariance.

We can also do the same physics using Lagrangian Densities.

Proca Lagrangian Density $L = -(1/2)(\partial_\mu B^*_\nu - \partial_\nu B^*_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu) + (m_oc/\hbar)^2 B^*_\nu B^\nu$: with $B^\mu = (\varphi/c, \mathbf{a})[(ct, \mathbf{r})]$ is a generalized complex 4-(Vector)Potential
KG Lagrangian Density $L = -\eta^{\mu\nu}(\partial_\mu\psi^* - \partial_\nu\psi) - (m_oc/\hbar)^2\psi^*\psi$: with $\psi = \psi[R] = \psi[(ct, \mathbf{r})]$
Dirac Lagrangian Density $L = \bar{\psi}(\gamma_\mu P^\mu - m_oc/\hbar)\psi$: with ψ = a spinor $\psi[(ct, \mathbf{r})]$
QED Lagrangian Density $L = \bar{\psi}(i\hbar\gamma_\mu D^\mu - m_oc)\psi - (1/4)F_{\mu\nu}F^{\mu\nu}$: with $D^\mu = \partial^\mu + iqA^\mu + iqB^\mu$ and A^μ=EM field of the e^-, B^μ = external source EM field

SRQM Study:
Lots of Relativistic Quantum Wave Equations
A lot of RQM!

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

In relativistic quantum mechanics and quantum field theory, the Bargmann–Wigner equations describe free particles of arbitrary spin j, an integer for bosons (j = 1, 2, 3 ...) or half-integer for fermions (j = ½, ³⁄₂, ⁵⁄₂ ...). The solutions to the equations are wavefunctions, mathematically in the form of multi-component spinor fields.

Bargmann–Wigner equations: $(-\gamma^{\mu}P_{\mu} + mc)_{\alpha r\, \alpha' r}\, \Psi_{\alpha 1 \ldots \alpha' r \ldots \alpha 2j} = 0$

In relativistic quantum mechanics and quantum field theory, the Joos–Weinberg equation is a relativistic wave equations applicable to free particles of arbitrary spin j, an integer for bosons (j = 1, 2, 3 ...) or half-integer for fermions (j = ½, ³⁄₂, ⁵⁄₂ ...). The solutions to the equations are wavefunctions, mathematically in the form of multi-component spinor fields. The spin quantum number is usually denoted by s in quantum mechanics, however in this context j is more typical in the literature.

Joos–Weinberg equation: $[\gamma^{\mu 1 \mu 2 \ldots \mu 2j}\, P_{\mu 1}\, P_{\mu 2} \ldots P_{\mu 2j} + (mc)^{2j}]\, \Psi = 0$

The primary difference appears to be the expansion in either the wavefunctions for (BW) or the Dirac Gamma's for (JW)
For both of these: A state or quantum field in such a representation would satisfy no field equation except the Klein-Gordon equation.

Yet another form is the Duffin-Kemmer-Petiau Equation vs Dirac Equation
DKP Eqn {spin 0 or 1}: $(i\hbar\beta^{\alpha}\partial_{\alpha} - m_o c)\Psi = 0$, with β^{α} as the DKP matrices
Dirac Eqn (spin ½}: $(i\hbar\gamma^{\alpha}\partial_{\alpha} - m_o c)\Psi = 0$, with γ^{α} as the Dirac Gamma matrices

SRQM:
A few more SR 4-Vectors

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

SR 4-Vector	Definition	Unites
4-Position	$\mathbf{R} = (ct,\mathbf{r})$; alt. $\mathbf{X} = (ct,\mathbf{x})$	Time, Space
4-Velocity	$\mathbf{U} = \gamma(c,\mathbf{u})$	Gamma, Velocity
4-Momentum	$\mathbf{P} = (E/c,\mathbf{p}) = (mc,\mathbf{p})$	Energy:Mass, Momentum
4-WaveVector	$\mathbf{K} = (\omega/c,\mathbf{k}) = (\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$	Frequency, WaveNumber
4-Gradient	$\partial = (\partial_t/c,-\nabla)$	Temporal Partial, Space Partial
4-VectorPotential	$\mathbf{A} = (\varphi/c,\mathbf{a})$	Scalar Potential, Vector Potential
4-TotalMomentum	$\mathbf{P}_{tot} = (E/c+q\varphi/c,\mathbf{p}+q\mathbf{a})$	Energy-Momentum inc. EM fields
4-TotalWaveVector	$\mathbf{K}_{tot} = (\omega/c+(q/\hbar)\varphi/c,\mathbf{k}+(q/\hbar)\mathbf{a})$	Freq-WaveNum inc. EM fields
4-CurrentDensity	$\mathbf{J} = (c\rho,\mathbf{j}) = q\mathbf{J}_{prob}$	Charge Density, Current Density
4-ProbabiltyCurrentDensity can have complex values	$\mathbf{J}_{prob} = (c\rho_{prob},\mathbf{j}_{prob})$	QM Probability (Density, Current Density)

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

SRQM:
More SR 4-Vectors Explained

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

SR 4-Vector	Empirical Fact	What it means...
4-Position	$\mathbf{R} = (ct, \mathbf{r})$	SpaceTime as Single United Concept
4-Velocity	$\mathbf{U} = d\mathbf{R}/d\tau$	Velocity is Proper Time Derivative
4-Momentum	$\mathbf{P} = m_o\mathbf{U} = (E_o/c^2)\mathbf{U}$	Mass-Energy-Momentum Equivalence
4-WaveVector	$\mathbf{K} = \mathbf{P}/\hbar = (\omega_o/c^2)\mathbf{U}$	Wave-Particle Duality
4-Gradient	$\partial = -i\mathbf{K}$	Unitary Evolution of States Operator Formalism, Complex Waves
4-VectorPotential	$\mathbf{A} = (\varphi/c, \mathbf{a}) = (\varphi_o/c^2)\mathbf{U}$	Potential Fields...
4-TotalMomentum	$\mathbf{P}_{tot} = \mathbf{P} + q\mathbf{A}$	Energy-Momentum inc. Potential Fields
4-TotalWaveVector	$\mathbf{K}_{tot} = \mathbf{K} + (q/\hbar)\mathbf{A}$	Freq-WaveNum inc. Potential Fields
4-CurrentDensity	$\mathbf{J} = \rho_o\mathbf{U} = q\mathbf{J}_{prob}$ $\partial\cdot\mathbf{J} = 0$	ChargeDensity-CurrentDensity Equivalence CurrentDensity is conserved
4-Probability CurrentDensity	$\mathbf{J}_{prob} = (c\rho_{prob}, \mathbf{j}_{prob})$ $\partial\cdot\mathbf{J}_{prob} = 0$	QM Probability from SR Probability Worldlines are conserved

Minimal Coupling = Potential Interaction

Klein-Gordon Eqn → Schrödinger Eqn

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

$\mathbf{P}_T = \mathbf{P} + \mathbf{Q} = \mathbf{P} + q\mathbf{A}$	Minimal Coupling: Total = Dynamic + Charge_Coupled to 4-(EM)VectorPotential
$\mathbf{K} = i\partial$	Complex Plane-Waves
$\mathbf{P} = \hbar\mathbf{K}$	Einstein-de Broglie QM Relations
$\mathbf{P} = i\hbar\partial$	Schrödinger Relations

$\mathbf{P} = (E/c,\mathbf{p}) = \mathbf{P}_T - q\mathbf{A} = (E_T/c - q\varphi/c, \mathbf{p}_T - q\mathbf{a}) = \hbar\mathbf{K} = i\hbar\partial$

$\partial = (\partial_t/c, -\nabla) = \partial_T + (iq/\hbar)\mathbf{A} = (\partial_{tT}/c + (iq/\hbar)\varphi/c, -\nabla_T + (iq/\hbar)\mathbf{a}) = -i\mathbf{K} = (-i/\hbar)\mathbf{P}$

$\partial\cdot\partial = (\partial_t/c)^2 - \nabla^2 = -(m_o c/\hbar)^2$:	The Klein-Gordon RQM Wave Equation (relativistic QM)
$\mathbf{P}\cdot\mathbf{P} = (E/c)^2 - \mathbf{p}^2 = (m_o c)^2$:	Einstein Mass:Energy:Momentum Equivalence
$E^2 = (m_o c^2)^2 + c^2\mathbf{p}^2$:	Relativistic
$E \sim [\,(m_o c^2) + \mathbf{p}^2/2m_o\,]$:	Low velocity limit { $\lvert\mathbf{v}\rvert << c$ } from $(1+x)^n \sim [1 + nx + O(x^2)]$ for $\lvert x\rvert<<1$
$(E_T - q\varphi)^2 = (m_o c^2)^2 + c^2(\mathbf{p}_T - q\mathbf{a})^2$:	Relativistic with Minimal Coupling
$(E_T - q\varphi) \sim [\,(m_o c^2) + (\mathbf{p}_T - q\mathbf{a})^2/2m_o\,]$:	Low velocity with Minimal Coupling
$(i\hbar\partial_{tT} - q\varphi)^2 = (m_o c^2)^2 + c^2(-i\hbar\nabla_T - q\mathbf{a})^2$:	Relativistic with Minimal Coupling
$(i\hbar\partial_{tT} - q\varphi) \sim [\,(m_o c^2) + (-i\hbar\nabla_T - q\mathbf{a})^2/2m_o\,]$:	Low velocity with Minimal Coupling
$(i\hbar\partial_{tT}) \sim [\,q\varphi + (m_o c^2) + (i\hbar\nabla_T + q\mathbf{a})^2/2m_o\,]$:	Low velocity with Minimal Coupling
$(i\hbar\partial_{tT}) \sim [\,V + (i\hbar\nabla_T + q\mathbf{a})^2/2m_o\,]$:	$V = q\varphi + (m_o c^2)$
$(i\hbar\partial_{tT}) \sim [\,V - (\hbar\nabla_T)^2/2m_o\,]$:	Typically the 3-vector-potential $\mathbf{a} \sim 0$ in many situations

The better statement is that the Schrödinger Eqn is the limiting low-velocity case of the more general KG Eqn, not that the KG Eqn is the relativistic generalization of the Schrödinger Eqn

$(i\hbar\partial_{tT})\lvert\Psi\rangle \sim [\,V - (\hbar\nabla_T)^2/2m_o\,]\lvert\Psi\rangle$:	The Schrödinger NRQM Wave Equation (non-relativistic QM)

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$

$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$

= Lorentz Scalar Invariant

SRQM: Once one has a Relativistic Wave Eqn...

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Klein-Gordon Equation: $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = (-im_oc/\hbar)^2 = -(m_oc/\hbar)^2 = -(\omega_o/c)^2$

Once we have derived a RWE, what does it imply?

The KG Eqn. was derived from the physics of SR plus a few empirical facts. It is a 2nd order, linear, wave PDE that pertains to physical objects of reality from SR.

Just being a linear wave PDE implies all the mathematical techniques that have been discovered to solve such equations generally: Hilbert Space, Superpositions, <Bra|,|Ket> notation, wavevectors, wavefunctions, etc. These things are from mathematics in general, not only and specifically from an Axiom of QM.

Therefore, if one has a physical RWE, it implies the mathematics of waves, the formalism of the mathematics, and thus the mathematical Principles and Formalism of QM. Again, QM Axioms are not required – they emerge from the physics and math...

SRQM:
Once one has a Relativistic Wave Eqn…
Examine Photon Polarization

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

From the Wikipedia page on [Photon Polarization]

Photon polarization is the quantum mechanical description of the classical polarized sinusoidal plane electromagnetic wave. An individual photon can be described as having right or left circular polarization, or a superposition of the two. Equivalently, a photon can be described as having horizontal or vertical linear polarization, or a superposition of the two.

The description of photon polarization contains many of the physical concepts and much of the mathematical machinery of more involved quantum descriptions and forms a fundamental basis for an understanding of more complicated quantum phenomena. Much of the mathematical machinery of quantum mechanics, such as state vectors, probability amplitudes, unitary operators, and Hermitian operators, emerge naturally from the classical Maxwell's equations in the description. The quantum polarization state vector for the photon, for instance, is identical with the Jones vector, usually used to describe the polarization of a classical wave. Unitary operators emerge from the classical requirement of the conservation of energy of a classical wave propagating through lossless media that alter the polarization state of the wave. Hermitian operators then follow for infinitesimal transformations of a classical polarization state.

Many of the implications of the mathematical machinery are easily verified experimentally. In fact, many of the experiments can be performed with pairs of polarized sunglass-lenses.

The connection with quantum mechanics is made through the identification of a minimum packet size, called a photon, for energy in the electromagnetic field. The identification is based on the theories of Planck and the interpretation of those theories by Einstein. The correspondence principle then allows the identification of momentum and angular momentum (called spin), as well as energy, with the photon.

SRQM:
Principle of Superposition:
From the mathematics of waves

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Klein-Gordon Equation: $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = (-im_oc/\hbar)^2 = -(m_oc/\hbar)^2 = -(\omega_o/c)^2$

The Extended Superposition Principle for Linear Equations
==========
Suppose that the non-homogeneous equation, where L is linear, is solved by some particular u_p
Suppose that the associated homogeneous problem is solved by a sequence of u_i. i={0,1,2,...}
$L(u_p) = C$; $L(u_0) = 0$, $L(u_1) = 0$, $L(u_2) = 0$ ...
Then u_p plus any linear combination of the u_n satisfies the original non-homogeneous equation:
$L(u_p + \Sigma\, a_n\, u_n) = C$,
where a_n is a sequence of (possibly complex) constants and the sum is arbitrary.
==========

Note that there is no mention of partial differentiation. Indeed, it's true for any linear equation, algebraic or integro-partial differential-whatever.

QM superposition is not axiomatic, it emerges from the mathematics of the Linear PDE
The Klein-Gordon Equation is a 2nd-order LINEAR Equation.
This is the origin of superposition in QM.

SRQM: Klein-Gordon obeys Principle of Superposition

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Klein-Gordon Equation: $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = (-im_oc/\hbar)^2 = -(m_oc/\hbar)^2 = -(\omega_o/c)^2$

$\mathbf{K}\cdot\mathbf{K} = (\omega/c)^2 - \mathbf{k}\cdot\mathbf{k} = (\omega_o/c)^2$: The particular solution (w/ rest mass)
$\mathbf{K}_n\cdot\mathbf{K}_n = (\omega_n/c)^2 - \mathbf{k}_n\cdot\mathbf{k}_n = 0$: The homogenous solution for a (virtual photon?) microstate n
Note that $\mathbf{K}_n\cdot\mathbf{K}_n = 0$ is a null 4-vector (photonic)

Let Ψ_p = Ae^-i($\mathbf{K}\cdot\mathbf{X}$), then $\partial\cdot\partial[\Psi_p] = (-i)^2(\mathbf{K}\cdot\mathbf{K})\Psi_p = -(\omega_o/c)^2\Psi_p$
which is the Klein-Gordon Equation, the particular solution...

Let Ψ_n = A_ne^-i($\mathbf{K}_n\cdot\mathbf{X}$), then $\partial\cdot\partial[\Psi_n] = (-i)^2(\mathbf{K}_n\cdot\mathbf{K}_n)\Psi_n = (0)\Psi_n$
which is the Klein-Gordon Equation homogeneous solution for a microstate n

We may take $\Psi = \Psi_p + \Sigma_n \Psi_n$

Hence, the Principle of Superposition is not required as an QM Axiom, it follows from SR and our empirical facts which lead to the Klein-Gordon Equation. The Klein-Gordon equation is a linear wave PDE, which has overall solutions which can be the complex linear sums of individual solutions – i.e. it obeys the Principle of Superposition.
This is not an axiom – it is a general mathematical property of linear PDE's.
This property continues over as well to the limiting case { |**v**|<<c } of the Schrödinger Equation.

SRQM:
QM Hilbert Space:
From the mathematics of waves

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Klein-Gordon Equation: $\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = (-im_oc/\hbar)^2 = -(m_oc/\hbar)^2 = -(\omega_o/c)^2$

Hilbert Space (HS) representation:
if $|\Psi\rangle \varepsilon$ HS, then $c|\Psi\rangle \varepsilon$ HS, where c is complex number
if $|\Psi_1\rangle$ and $|\Psi_2\rangle \varepsilon$ HS, then $|\Psi_1\rangle+|\Psi_2\rangle \varepsilon$ HS
if $|\Psi\rangle = c_1|\Psi_1\rangle+c_2|\Psi_2\rangle$, then $\langle\Phi|\Psi\rangle = c_1\langle\Phi|\Psi_1\rangle+c_2\langle\Phi|\Psi_2\rangle$ and $\langle\Psi| = c_1^*\langle\Psi_1|+c_2^*\langle\Psi_2|$
$\langle\Phi|\Psi\rangle = \overline{\langle\Psi|\Phi\rangle}$
$\langle\Psi|\Psi\rangle >= 0$
if $\langle\Psi|\Psi\rangle = 0$, then $|\Psi\rangle = \mathbf{0}$
etc.

Hilbert spaces arise naturally and frequently in mathematics, physics, and engineering, typically as infinite-dimensional function spaces. They are indispensable tools in the theories of partial differential equations, Fourier analysis, signal processing, heat transfer, ergodic theory, and Quantum Mechanics.

The QM Hilbert Space emerges from the fact that the KG Equation is a linear wave PDE – Hilbert spaces as solutions to PDE's are a purely mathematical phenomenon – no QM Axiom is required.

Likewise, this introduces the <bra|,|ket> notation, wavevectors, wavefunctions, etc.

Note:

One can use Hilbert Space descriptions of Classical Mechanics using the Koopman-von Neumann formulation.
One can not use Hilbert Space descriptions of Quantum Mechanics by using the Phase Space formulation of QM.

SRQM Study:
Canonical Commutation Relation: Viewed from standard QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Standard QM Canonical Commutation Relation: $[x^j,p^k] = i\hbar\delta^{jk}$

The Standard QM Canonical Commutation Relation is simply an axiom in standard QM. It is just given, with no explanation. You just had to accept it.

I always found that unsatisfactory.

<u>There are at least 4 parts to it:</u>

Where does the commutation ([,]) come from?
Where does the imaginary constant (i) come from?
Where does the Dirac:reduced-Planck constant (ħ) come from?
Where does the Kronecker Delta (δ^{jk}) come from?

See the next page for SR enlightenment…
The SR Metric is the source of "quantization".

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Canonical QM Commutation Relation Derived from standard SR

4-Vector SRQM Interpretation of QM

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SciRealm@aol.com
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Let (f) be an arbitrary SR function
$\mathbf{X}[f] = \mathbf{X}f$, $\partial[f] = \partial[f]$
$\mathbf{X}$, function or not, has no effect on (f)
$\partial=\partial[\ ]$ is definitely an SR function:operator

$\mathbf{X}[\partial[f]] = \mathbf{X}\partial[f]$
$\partial[\mathbf{X}f] = \partial[\mathbf{X}]f + \mathbf{X}\partial[f]$
$\partial[\mathbf{X}f] - \mathbf{X}\partial[f] = \partial[\mathbf{X}]f$
$\partial[\mathbf{X}[f]] - \mathbf{X}[\partial[f]] = \partial[\mathbf{X}]f$
Recognize this as a commutation relation
$[\ \partial\ ,\ \mathbf{X}\ ]f = \partial[\mathbf{X}]f$

$[\ \partial\ ,\ \mathbf{X}\ ] = \partial[\mathbf{X}]$ Manifestly Invariant
$= \partial^\mu[X^\nu]$
$= (\partial_t/c,-\nabla)[(ct,\mathbf{x})]$
$= (\partial_t/c,-\partial_x,-\partial_y,-\partial_z)[(ct,x,y,z)]$
$= \mathrm{Diag}\{1,-1,-1,-1\} = \mathrm{Diag}[1,-\delta^{jk}]$
$= \eta^{\mu\nu}$ = Minkowski Metric

$[\partial^\mu,X^\nu] = \eta^{\mu\nu}$ Tensor form:true for all observers
$[i\hbar\partial^\mu,X^\nu] = i\hbar\eta^{\mu\nu}$ Also true from empirical constants (i),(ħ)
$[P^\mu,X^\nu] = i\hbar\eta^{\mu\nu}$ Empirical relation $P^\mu = i\hbar\partial^\mu$
$[p^k,x^j] = -i\hbar\delta^{kj}$ $[p^0,x^0] = [E/c,ct] = [E,t] = i\hbar$

$[x^j,p^k] = i\hbar\delta^{jk}$ Position:Momentum QM Commutation Relation

$[t,E] = -i\hbar(1)$ Time:Energy QM Commutation Relation

4-Displacement
$\Delta\mathbf{X}=(c\Delta t,\Delta\mathbf{x})$
$d\mathbf{X}=(cdt,d\mathbf{x})$
4-Position
$\mathbf{X}=(ct,\mathbf{x})$

$\partial\cdot\mathbf{X}=\partial_\mu X^\mu=4$ SpaceTime Dimension

Lorentz
$\partial_\nu[X^{\mu'}]=\partial X^{\mu'}/\partial X^\nu=\Lambda^{\mu'}{}_\nu$
Transform

$[\partial^\mu,X^\nu]=\partial^\mu[X^\nu]=\eta^{\mu\nu}$
$[\partial,\mathbf{X}]=\partial[\mathbf{X}]=\eta^{\mu\nu}$
$\rightarrow\mathrm{Diag}[1,-1,-1,-1]=\mathrm{Diag}[1,-\delta^{jk}]$
Minkowski Metric
Non-Zero Commutation Relation via natural SR 4-Gradient

4-Gradient
$\partial=(\partial_t/c,-\nabla)$

$\mathbf{U}\cdot\partial\,[..]$
$\gamma d/dt\,[..]$
$d/d\tau\,[..]$

ProperTime Derivative

Proper Time
$\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$
Derivative

$..[\mathbf{K}\cdot\mathbf{X}]$
$..[-\Phi_{phase}]$
(i)

Complex Plane-waves
$\mathbf{K}=i\partial$

$..[\mathbf{K}\cdot\mathbf{X}]$
$..[-\Phi_{phase}]$
(i)

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

ω_o/c^2

Wave Velocity
$v_{group}*v_{phase}=c^2$

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})$
$= -\partial[\Phi_{phase}]$

$i[\partial,\mathbf{X}]=[i\partial,\mathbf{X}]=[\mathbf{K},\mathbf{X}]=i\eta^{\mu\nu}$
Non-Zero Commutation Relation via SR 4-WaveVector

E_o/ω_o
(ħ)

Einstein
de Broglie
$\mathbf{P}=\hbar\mathbf{K}$

E_o/ω_o
(ħ)

$E=mc^2$

m_o
E_o/c^2

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$
$= -\partial[S_{action}]$

$[i\hbar\partial,\mathbf{X}]=[\hbar\mathbf{K},\mathbf{X}]=[\mathbf{P},\mathbf{X}]=i\hbar\eta^{\mu\nu}$
Non-Zero Commutation Relation via SR 4-Momentum

$\{\mathbf{P} = \hbar\mathbf{K}\}$ and $\{\mathbf{K} = i\partial\}$ are empirical SR relations

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(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
Quantum Principles

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

SRQM Study: Canonical Commutation Relation: Derived from SRQM

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Standard QM Canonical Commutation Relation:

$$[x^j,p^k] = i\hbar\delta^{jk}$$

As we have seen, this relation is generated from simple SR math.

$$[\,\partial\,,\mathbf{X}\,] = [\partial^\mu,X^\nu] = \partial[\mathbf{X}] = \partial^\mu[X^\nu] = (\partial_t/c,-\nabla)[(ct,\mathbf{x})] = (\partial_t/c,-\partial_x,-\partial_y,-\partial_z)[(ct,x,y,z)] = \mathrm{Diag}\{1,-1,-1,-1\} = \mathrm{Diag}[1,-\delta^{jk}] = \eta^{\mu\nu} = \text{Minkowski Metric}$$

$[\partial^\mu,X^\nu] = \eta^{\mu\nu}$: This, Minkowski SpaceTime, is where the [.. , ..] commutation comes from.
$[P^\mu,X^\nu] = i\hbar\eta^{\mu\nu}$: This is the more general 4D version, with the Standard QM version $[x^j,p^k] = i\hbar\delta^{jk}$ being just the spatial part.

The { i , ħ } are empirically found Lorentz Scalar Invariants and physical constants, relating these 4-Vectors.

One of the great misconceptions on modern physics is that since QM is about "tiny" things, that ALL things should be built up from it.
That paradigm of course works well for many things:
Compounds are built-up from smaller molecules.
Molecules are built-up from smaller elements:atoms.
Elements:atoms are built-up from smaller protons, neutrons, and electrons.
Protons and neutrons are built-up from smaller quarks.
And all experiments to-date show that electrons and quarks appear to be point-like, with wave-type properties giving extent.

So, one can mistakenly think that "SpaceTime" must be made up of smaller "quantum" stuff as well.
However, that is not what the math says. The "quantization" paradigm doesn't apply to SpaceTime itself, just to properties of <events>.
All of the "quantum"-sized things above, electrons and quarks, are material things, <events>, which move around "within" SpaceTime.
Their "quantization" comes about from the properties of the math and metric of SR.
The math does *NOT* say that SpaceTime itself is "quantized". It says that SR Minkowski SpaceTime is the source of "quantization".

SRQM Study: Canonical Commutation Relation: Derived from SRQM

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[Linear,Linear] Momentum QM Canonical Commutation Relation:

$[\partial^\mu,\partial^\nu]f = (\partial^\mu\partial^\nu - \partial^\nu\partial^\mu)f = 0^{\mu\nu}f$

$[\partial^\mu,\partial^\nu] = (\partial^\mu\partial^\nu - \partial^\nu\partial^\mu) = 0^{\mu\nu}$: Partials commute, or in this case, the 4-Gradients commute

$[i\hbar\partial^\mu,i\hbar\partial^\nu] = (i\hbar)^2(\partial^\mu\partial^\nu - \partial^\nu\partial^\mu) = (i\hbar)^2 0^{\mu\nu} = 0^{\mu\nu}$

$[P^\mu,P^\nu] = (i\hbar)^2(\partial^\mu\partial^\nu - \partial^\nu\partial^\mu) = (i\hbar)^2 0^{\mu\nu} = 0^{\mu\nu}$

$[P^\mu,P^\nu] = 0^{\mu\nu}$: see Poincaré Group:Algebra

{ i , ħ } do not "create" the actual [.. , ..] commutation bracket, 4-Vectors do.
{ i , ħ } are empirically found Scalar Invariants relating these 4-Vectors.
{ i , ħ } are physical constants which give correct dimensional units.

(i) Complex Plane-waves $\mathbf{K}=i\partial$

E_o/ω_o (ħ) Einstein de Broglie $\mathbf{P} = \hbar\mathbf{K}$

4-Gradient ∂^μ
$\partial=(\partial_t/c,-\nabla)=\partial/\partial X_\mu$

4-Position X^μ
$\mathbf{X}=(ct,\mathbf{r})\in$<Event>

4-Momentum P^μ
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})=m_o\mathbf{U}$

4-AngularMomentum
$M^{\alpha\beta} = X^\alpha P^\beta - X^\beta P^\alpha = \mathbf{X} \wedge \mathbf{P}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & -cn^x & -cn^y & -cn^z \\ x & +cn^x & 0 & +l^z & -l^y \\ y & +cn^y & -l^z & 0 & +l^x \\ z & +cn^z & +l^y & -l^x & 0 \end{array}$$

$$\begin{array}{cc} 0 & -cn^{0j} \\ +cn^{i0} & \varepsilon^{ij}{}_k l^k \end{array}$$

$$\begin{array}{cc} 0 & -c\mathbf{n} \\ +c\mathbf{n}^T & \mathbf{x}\wedge\mathbf{p} \end{array}$$

4-Tensor [SI Unit=J·s=Action]
Anti-symmetric (skew)

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(2,0)-Tensor $T^{\mu\nu}$
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[Angular,Linear] Momentum QM Canonical Commutation Relation:

Let $A^{\mu\nu} = \mathbf{X} \wedge \partial = X^\mu\partial^\nu - X^\nu\partial^\mu$ = Dimensionless version of 4-AngularMomentum, based on exterior product of 4-Position & 4-Gradient

$i\hbar A^{\mu\nu} = \mathbf{X} \wedge i\hbar\partial = X^\mu i\hbar\partial^\nu - X^\nu i\hbar\partial^\mu = \mathbf{X} \wedge \mathbf{P} = X^\mu P^\nu - X^\nu P^\mu = M^{\mu\nu}$ = 4-AngularMomentum, with { i , ħ } as Lorentz Scalar Invariants

$[A^{\mu\nu},\partial^\sigma]f = A^{\mu\nu}\partial^\sigma f - \partial^\sigma A^{\mu\nu}f = A^{\mu\nu}\partial^\sigma[f] - \partial^\sigma[A^{\mu\nu}]f - A^{\mu\nu}\partial^\sigma[f] = -\partial^\sigma[A^{\mu\nu}]f = -\partial^\sigma(X^\mu\partial^\nu - X^\nu\partial^\mu)f = \partial^\sigma(X^\nu\partial^\mu - X^\mu\partial^\nu)f = (\partial^\sigma X^\nu\partial^\mu - \partial^\sigma X^\mu\partial^\nu)f = (\eta^{\sigma\nu}\partial^\mu - \eta^{\sigma\mu}\partial^\nu)f$
$[A^{\mu\nu},\partial^\sigma]f = (\eta^{\sigma\nu}\partial^\mu - \eta^{\sigma\mu}\partial^\nu)f$
$[A^{\mu\nu},\partial^\sigma] = (\eta^{\sigma\nu}\partial^\mu - \eta^{\sigma\mu}\partial^\nu)$
$[A^{\mu\nu},i\hbar\partial^\sigma] = (\eta^{\sigma\nu}i\hbar\partial^\mu - \eta^{\sigma\mu}i\hbar\partial^\nu)$
$[A^{\mu\nu},P^\sigma] = (\eta^{\sigma\nu}P^\mu - \eta^{\sigma\mu}P^\nu)$
$[i\hbar A^{\mu\nu},P^\sigma] = i\hbar(\eta^{\sigma\nu}P^\mu - \eta^{\sigma\mu}P^\nu)$

$[M^{\mu\nu},P^\sigma] = i\hbar(\eta^{\sigma\nu}P^\mu - \eta^{\sigma\mu}P^\nu)$: see Poincaré Group:Algebra

{ i , ħ } do not "create" the actual [.. , ..] commutation bracket, 4-Vectors do.
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4-Tensor [SI Unit=J·s=Action]
Anti-symmetric (skew)

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ex.
$[M^{ab},P^{d}] = i\hbar(\eta^{db}P^{a} + \eta^{da}P^{b})$: Get the regular 3-angular-momenta via $L^{a} = ½\varepsilon^{a}{}_{jk}M^{jk}$
$[½\varepsilon^{c}{}_{ab}M^{ab},P^{d}] = ½\varepsilon^{c}{}_{ab}i\hbar(\eta^{db}P^{a} + \eta^{da}P^{b})$
$[L^{c},P^{d}] = ½\varepsilon^{c}{}_{ab}i\hbar(\eta^{db}P^{a} + \eta^{da}P^{b})$
$[L^{c},P^{b}] = ½\varepsilon^{c}{}_{ab}i\hbar(\eta^{bb}P^{a} + \eta^{ba}P^{b})$: chose first Minkowski to be non-zero, d→b
$[L^{c},P^{b}] = ½\varepsilon^{c}{}_{ab}i\hbar(-P^{a} + \{0\}P^{b})$
$[P^{b},L^{c}] = ½\varepsilon^{c}{}_{ab}i\hbar(P^{a})$

ex.
$[M^{ab},P^{d}] = i\hbar(\eta^{db}P^{a} + \eta^{da}P^{b})$: Get the regular 3-angular-momenta via $L^{a} = ½\varepsilon^{a}{}_{jk}M^{jk}$
$[M^{a0},P^{0}] = i\hbar(\eta^{00}P^{a} + \eta^{0a}P^{0})$
$[M^{a0},P^{0}] = i\hbar(\{1\}P^{a} + \{0\}P^{0})$
$[K^{a},P^{0}] = i\hbar(P^{a})$
$[K^{a},H/c] = i\hbar(P^{a})$
$[K^{a},H] = i\hbar c(P^{a})$
$[H,K^{a}] = -i\hbar c(P^{a})$

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$M^{\alpha\beta} = X^{\alpha}P^{\beta} - X^{\beta}P^{\alpha} = \mathbf{X} \wedge \mathbf{P}$

	t	x	y	z
t	0	$-cn^{x}$	$-cn^{y}$	$-cn^{z}$
x	$+cn^{x}$	0	$+l^{z}$	$-l^{y}$
y	$+cn^{y}$	$-l^{z}$	0	$+l^{x}$
z	$+cn^{z}$	$+l^{y}$	$-l^{x}$	0

0	$-cn^{0j}$
$+cn^{i0}$	$\varepsilon^{ij}{}_{k}l^{k}$

0	$-c\mathbf{n}$
$+c\mathbf{n}^{T}$	$\mathbf{x}\wedge\mathbf{p}$

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$i\hbar A^{\mu\nu} = \mathbf{X} \wedge i\hbar\partial = X^\mu i\hbar\partial^\nu - X^\nu i\hbar\partial^\mu = \mathbf{X} \wedge \mathbf{P} = X^\mu P^\nu - X^\nu P^\mu = M^{\mu\nu}$ = 4-AngularMomentum, with { i , ħ } as Lorentz Scalar Invariants

$[A^{\mu\nu},X^\sigma\partial^\rho] = (X^\mu\partial^\nu - X^\nu\partial^\mu)X^\sigma\partial^\rho - X^\sigma\partial^\rho(X^\mu\partial^\nu - X^\nu\partial^\mu) = X^\mu\partial^\nu X^\sigma\partial^\rho - X^\nu\partial^\mu X^\sigma\partial^\rho - X^\sigma\partial^\rho X^\mu\partial^\nu + X^\sigma\partial^\rho X^\nu\partial^\mu = X^\mu\eta^{\nu\sigma}\partial^\rho - X^\nu\eta^{\mu\sigma}\partial^\rho - X^\sigma\eta^{\rho\mu}\partial^\nu + X^\sigma\eta^{\rho\nu}\partial^\mu$
$[A^{\mu\nu},X^\sigma\partial^\rho] = \eta^{\nu\sigma}X^\mu\partial^\rho - \eta^{\mu\sigma}X^\nu\partial^\rho - \eta^{\rho\mu}X^\sigma\partial^\nu + \eta^{\rho\nu}X^\sigma\partial^\mu$

$[A^{\mu\nu},X^\rho\partial^\sigma] = \eta^{\nu\rho}X^\mu\partial^\sigma - \eta^{\mu\rho}X^\nu\partial^\sigma - \eta^{\sigma\mu}X^\rho\partial^\nu + \eta^{\sigma\nu}X^\rho\partial^\mu$

$[A^{\mu\nu},X^\sigma\partial^\rho - X^\rho\partial^\sigma] = (\eta^{\nu\sigma}X^\mu\partial^\rho - \eta^{\mu\sigma}X^\nu\partial^\rho - \eta^{\rho\mu}X^\sigma\partial^\nu + \eta^{\rho\nu}X^\sigma\partial^\mu) - (\eta^{\nu\rho}X^\mu\partial^\sigma - \eta^{\mu\rho}X^\nu\partial^\sigma - \eta^{\sigma\mu}X^\rho\partial^\nu + \eta^{\sigma\nu}X^\rho\partial^\mu)$
$[A^{\mu\nu},A^{\sigma\rho}] = (\eta^{\nu\sigma}X^\mu\partial^\rho - \eta^{\mu\sigma}X^\nu\partial^\rho - \eta^{\rho\mu}X^\sigma\partial^\nu + \eta^{\rho\nu}X^\sigma\partial^\mu - \eta^{\nu\rho}X^\mu\partial^\sigma + \eta^{\mu\rho}X^\nu\partial^\sigma + \eta^{\sigma\mu}X^\rho\partial^\nu - \eta^{\sigma\nu}X^\rho\partial^\mu)$
$[A^{\mu\nu},A^{\sigma\rho}] = (\eta^{\nu\sigma}X^\mu\partial^\rho - \eta^{\sigma\nu}X^\rho\partial^\mu + \eta^{\sigma\mu}X^\rho\partial^\nu - \eta^{\mu\sigma}X^\nu\partial^\rho + \eta^{\mu\rho}X^\nu\partial^\sigma - \eta^{\rho\mu}X^\sigma\partial^\nu + \eta^{\rho\nu}X^\sigma\partial^\mu - \eta^{\nu\rho}X^\mu\partial^\sigma)$
$[A^{\mu\nu},A^{\sigma\rho}] = (\eta^{\nu\sigma}A^{\mu\rho} + \eta^{\sigma\mu}A^{\rho\nu} + \eta^{\mu\rho}A^{\nu\sigma} + \eta^{\rho\nu}A^{\sigma\mu})$
$[A^{\mu\nu},i\hbar A^{\sigma\rho}] = (\eta^{\nu\sigma}i\hbar A^{\mu\rho} + \eta^{\sigma\mu}i\hbar A^{\rho\nu} + \eta^{\mu\rho}i\hbar A^{\nu\sigma} + \eta^{\rho\nu}i\hbar A^{\sigma\mu})$
$[A^{\mu\nu},M^{\sigma\rho}] = (\eta^{\nu\sigma}M^{\mu\rho} + \eta^{\sigma\mu}M^{\rho\nu} + \eta^{\mu\rho}M^{\nu\sigma} + \eta^{\rho\nu}M^{\sigma\mu})$
$[i\hbar A^{\mu\nu},M^{\sigma\rho}] = i\hbar(\eta^{\nu\sigma}M^{\mu\rho} + \eta^{\sigma\mu}M^{\rho\nu} + \eta^{\mu\rho}M^{\nu\sigma} + \eta^{\rho\nu}M^{\sigma\mu})$

$[M^{\mu\nu},M^{\sigma\rho}] = i\hbar(\eta^{\nu\sigma}M^{\mu\rho} + \eta^{\sigma\mu}M^{\rho\nu} + \eta^{\mu\rho}M^{\nu\sigma} + \eta^{\rho\nu}M^{\sigma\mu})$: see Poincaré Group:Algebra

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$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})=m_o\mathbf{U}$

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$M^{\alpha\beta} = X^\alpha P^\beta - X^\beta P^\alpha = \mathbf{X} \wedge \mathbf{P}$

$$\begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & -cn^x & -cn^y & -cn^z \\ x & +cn^x & 0 & +l^z & -l^y \\ y & +cn^y & -l^z & 0 & +l^x \\ z & +cn^z & +l^y & -l^x & 0 \end{array}$$

$$\begin{array}{cc} 0 & -cn^{0j} \\ +cn^{i0} & \varepsilon^{ij}{}_k l^k \end{array}$$

$$\begin{array}{cc} 0 & -c\mathbf{n} \\ +c\mathbf{n}^T & \mathbf{x}\wedge\mathbf{p} \end{array}$$

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Anti-symmetric (skew)

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Study:
Canonical Commutation Relation: Derived from SRQM

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Angular:Angular Momentum QM Canonical Commutation Relation:

X ^ P = $X^\mu P^\nu - X^\nu P^\mu = M^{\mu\nu}$ = 4-AngularMomentum, with { i , ħ } as Lorentz Scalar Invariants

$[M^{\mu\nu},M^{\sigma\rho}] = i\hbar(\eta^{\nu\sigma}M^{\mu\rho} + \eta^{\sigma\mu}M^{\rho\nu} + \eta^{\mu\rho}M^{\nu\sigma} + \eta^{\rho\nu}M^{\sigma\mu})$

$[M^{jk},M^{mn}] = i\hbar(\eta^{km}M^{jn} + \eta^{mj}M^{nk} + \eta^{jn}M^{km} + \eta^{nk}M^{mj})$: Look at just the spatial components

Get the regular 3-angular-momenta via $L^a = ½\varepsilon^a{}_{jk}M^{jk}$

$[½\varepsilon^a{}_{jk}M^{jk},½\varepsilon^b{}_{mn}M^{mn}] = i\hbar(½\varepsilon^a{}_{jk}½\varepsilon^b{}_{mn}\eta^{km}M^{jn} + ½\varepsilon^a{}_{jk}½\varepsilon^b{}_{mn}\eta^{mj}M^{nk} + ½\varepsilon^a{}_{jk}½\varepsilon^b{}_{mn}\eta^{jn}M^{km} + ½\varepsilon^a{}_{jk}½\varepsilon^b{}_{mn}\eta^{nk}M^{mj})$
$[L^a,L^b] = ¼i\hbar(\varepsilon^a{}_{jk}\varepsilon^b{}_{mn}\eta^{km}M^{jn} + \varepsilon^a{}_{jk}\varepsilon^b{}_{mn}\eta^{mj}M^{nk} + \varepsilon^a{}_{jk}\varepsilon^b{}_{mn}\eta^{jn}M^{km} + \varepsilon^a{}_{jk}\varepsilon^b{}_{mn}\eta^{nk}M^{mj})$
here we examine only the cases where the Minkowski Metric is non-zero, when $\eta^{i,j=i}$
paired indices on the same level are not summed...
$[L^a,L^b] = ¼i\hbar(\varepsilon^a{}_{jk}\varepsilon^b{}_{kn}\eta^{kk}M^{jn} + \varepsilon^a{}_{jk}\varepsilon^b{}_{jn}\eta^{jj}M^{nk} + \varepsilon^a{}_{jk}\varepsilon^b{}_{mj}\eta^{jj}M^{km} + \varepsilon^a{}_{jk}\varepsilon^b{}_{mk}\eta^{kk}M^{mj})$
$[L^a,L^b] = -¼i\hbar(\varepsilon^a{}_{jk}\varepsilon^b{}_{kn}M^{jn} + \varepsilon^a{}_{jk}\varepsilon^b{}_{jn}M^{nk} + \varepsilon^a{}_{jk}\varepsilon^b{}_{mj}M^{km} + \varepsilon^a{}_{jk}\varepsilon^b{}_{mk}M^{mj})$
$[L^a,L^b] = ¼i\hbar(\varepsilon^a{}_{jk}\varepsilon^b{}_{kn}M^{nj} + \varepsilon^a{}_{jk}\varepsilon^b{}_{jn}M^{kn} + \varepsilon^a{}_{jk}\varepsilon^b{}_{mj}M^{mk} + \varepsilon^a{}_{jk}\varepsilon^b{}_{mk}M^{jm})$
$[L^a,L^b] = ¼i\hbar(\varepsilon^a{}_{ec}\varepsilon^b{}_{cd}M^{de} + \varepsilon^a{}_{cd}\varepsilon^b{}_{ce}M^{de} + \varepsilon^a{}_{ce}\varepsilon^b{}_{dc}M^{de} + \varepsilon^a{}_{dc}\varepsilon^b{}_{ec}M^{de})$
$[L^a,L^b] = ¼i\hbar(\varepsilon^a{}_{ec}\varepsilon^b{}_{cd} + \varepsilon^a{}_{cd}\varepsilon^b{}_{ce} + \varepsilon^a{}_{ce}\varepsilon^b{}_{dc} + \varepsilon^a{}_{dc}\varepsilon^b{}_{ec})M^{de}$
$[L^a,L^b] = ¼i\hbar(\varepsilon^a{}_{ec}\varepsilon^b{}_{cd} + \varepsilon^a{}_{cd}\varepsilon^b{}_{ce} + \varepsilon^a{}_{ce}\varepsilon^b{}_{dc} + \varepsilon^a{}_{dc}\varepsilon^b{}_{ec})½\varepsilon^{de}{}_cL^c$
$[L^a,L^b] = ¼i\hbar(\varepsilon^a{}_{ec}\delta^{be} + \varepsilon^a{}_{cd}\delta^{bd} + \varepsilon^a{}_{ce}\delta^{be} + \varepsilon^a{}_{dc}\delta^{bd})L^c$
$[L^a,L^b] = ¼i\hbar(\varepsilon^{ab}{}_c + \varepsilon^{ab}{}_c + \varepsilon^{ab}{}_c + \varepsilon^{ab}{}_c)L^c$

$[L^a,L^b] = i\hbar(\varepsilon^{ab}{}_cL^c)$: Standard 3-angular-momentum commutation

ex.
$[M^{jj},M^{mn}] = i\hbar(\eta^{jm}M^{jn} + \eta^{mj}M^{nj} + \eta^{jn}M^{jm} + \eta^{nj}M^{mj})$
$[M^{jj},M^{mn}] = i\hbar(\eta^{jm}M^{jn} - \eta^{mj}M^{jn} + \eta^{jn}M^{jm} - \eta^{nj}M^{jm})$
$[M^{jj},M^{mn}] = 0$
for the case of either 3-angular-momentum index matching itself. Not surprising since the 4-AngularMomentum is an Antisymmetric Tensor with zeroes along the diagonal.

ex.
$[M^{yz},M^{zx}] = i\hbar(\eta^{zz}M^{yx} + \eta^{zy}M^{xz} + \eta^{yx}M^{zz} + \eta^{xz}M^{zy})$
$[M^{yz},M^{zx}] = i\hbar(\eta^{zz}M^{yx})$
$[M^{yz},M^{zx}] = i\hbar(-M^{yx})$
$[L^x,L^y] = i\hbar(L^z)$

(i) Complex Plane-waves **K**=i∂

E_o/ω_o (ħ) Einstein de Broglie **P** =ħ**K**

4-Gradient ∂^μ
$\partial=(\partial_t/c,-\nabla)=\partial/\partial X_\mu$

4-Position X^μ
X=(ct,**r**)∈<Event>

4-Momentum P^μ
P=(mc,**p**)=(E/c,**p**)=m_o**U**

4-AngularMomentum
$M^{\alpha\beta} = X^\alpha P^\beta - X^\beta P^\alpha$ = **X ^ P**

	t	x	y	z
t	0	$-cn^x$	$-cn^y$	$-cn^z$
x	$+cn^x$	0	$+l^z$	$-l^y$
y	$+cn^y$	$-l^z$	0	$+l^x$
z	$+cn^z$	$+l^y$	$-l^x$	0

0	$-cn^{0j}$
$+cn^{i0}$	$\varepsilon^{ij}{}_kl^k$

0	-cn
$+cn^T$	**x^p**

4-Tensor [SI Unit=J·s=Action]
Anti-symmetric (skew)

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(1,0)-Tensor V^μ = **V** = (v^0,**v**)
SR 4-CoVector:OneForm
(0,1)-Tensor V_μ = (v_0,-**v**)

SR 4-Scalar
(0,0)-Tensor S or S_o
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SRQM Study:
4-Position and 4-Gradient

4-Vector SRQM Interpretation of QM

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Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$

4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=R^\mu=(ct,\mathbf{r})$

Invariant Calculus
$d\mathbf{R}\cdot\partial = (cdt,d\mathbf{r})\cdot(\partial_t/c,-\nabla)$
$dR^\mu\eta_{\mu\nu}(\partial^\nu) = dR^\mu(\partial_\mu) = dR^\mu(\partial/\partial R^\mu) = (dt\partial_t + d\mathbf{r}\cdot\nabla)$
$=dt(\partial/\partial t)+dx(\partial/\partial x)+dy(\partial/\partial y)+dz(\partial/\partial z)$
Total Derivative Chain Rule

Invariant d'Alembertian Wave Equation
$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla=(\partial_\tau/c)^2$

4-Gradient
$\partial=\partial^\mu=(\partial_t/c,-\nabla)$
$=(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=\partial/\partial R_\mu$

SR: Minkowski Metric
$\partial[\mathbf{R}] = \partial^\mu R^\nu = \eta^{\mu\nu}$

$\rightarrow \mathrm{Diag}[1,-1,-1,-1]$
$= \mathrm{Diag}[1,-I_{(3)}]$
$= \mathrm{Diag}[1,-\delta^{jk}]$
{in Cartesian form}
"Particle Physics" Convention
$\{\eta_{\mu\mu}\} = 1/\{\eta^{\mu\mu}\}$
$\mathrm{Tr}[\eta^{\mu\nu}] = 4$
$\eta_\mu{}^\nu = \delta_\mu{}^\nu$

SR: Lorentz Transform
$\partial_\nu[R^{\mu'}] = \partial R^{\mu'}/\partial R^\nu = \Lambda^{\mu'}{}_\nu$

$\Lambda^\mu{}_\alpha(\Lambda^{-1})^\alpha{}_\nu = \eta^\mu{}_\nu = \delta^\mu{}_\nu$
$\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta = \eta_{\alpha\beta}$
$(\mathrm{Det}[\Lambda])^2 = 1$
$\mathrm{Det}[\Lambda] = \pm 1$
$\Lambda^\mu{}_\nu = (\Lambda^{-1})_\nu{}^\mu$
$\Lambda_{\mu\nu}\Lambda^{\mu\nu} = 4$

Rotations
Boosts
CPT

SpaceTime
$\partial\cdot\mathbf{R} = \partial_\mu R^\mu = 4$
Dimension

SRQM: Tensor Zero Exterior Product
$\partial\wedge\mathbf{R} = \partial^\mu R^\nu - \partial^\nu R^\mu$
$= \eta^{\mu\nu} - \eta^{\nu\mu} = 0^{\mu\nu}$

SRQM: Non-Zero Commutation
$[\partial,\mathbf{R}] = [\partial^\mu,R^\nu]$
$=\partial^\mu R^\nu - R^\nu\partial^\mu$
$= \eta^{\mu\nu}$

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM:
Heisenberg Uncertainty Principle: Viewed from SRQM

4-Vector SRQM Interpretation of QM

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SciRealm@aol.com
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Heisenberg Uncertainty { $\sigma^2_A\sigma^2_B$ } >= (1/2)|<[A,B]>| }
arises from the non-commuting nature of certain operators.

The commutator is [A,B] = AB-BA, where A & B are functional "measurement" operators.
The Operator Formalism arose naturally from our SR → QM path: [∂ = -i**K**].

The Generalized Uncertainty Relation: $\sigma_f^2\sigma_g^2 = (\Delta F) * (\Delta G) >= (1/2)|\langle i[F,G] \rangle|$

The uncertainty relation is a very general mathematical property, which applies to both classical or quantum systems. From Wikipedia: Photon Polarization: "This is a purely mathematical result. No reference to a physical quantity or principle is required."

The Cauchy–Schwarz inequality asserts that (for all vectors f and g of an inner product space, with either real or complex numbers):
$\sigma_f^2\sigma_g^2 = [\langle f | f \rangle \cdot \langle g | g \rangle] >= |\langle f | g \rangle|^2$

But first, let's back up a bit; Using standard complex number math, we have:
$z = a + ib$
$z^* = a - ib$
$Re(z) = a = (z + z^*)/(2)$
$Im(z) = b = (z - z^*)/(2i)$
$z^*z = |z|^2 = a^2 + b^2 = [Re(z)]^2 + [Im(z)]^2 = [(z + z^*)/(2)]^2 + [(z - z^*)/(2i)]^2$
or
$|z|^2 = [(z + z^*)/(2)]^2 + [(z - z^*)/(2i)]^2$

Now, generically, based on the rules of a complex inner product space we can arbitrarily assign:
$z = \langle f | g \rangle$, $z^* = \langle g | f \rangle$

Which allows us to write:
$|\langle f | g \rangle|^2 = [(\langle f | g \rangle + \langle g | f \rangle)/(2)]^2 + [(\langle f | g \rangle - \langle g | f \rangle)/(2i)]^2$

We can also note that:
$| f \rangle = F| \Psi \rangle$ and $| g \rangle = G| \Psi \rangle$

Thus,
$|\langle f | g \rangle|^2 = [(\langle \Psi |F^* G| \Psi \rangle + \langle \Psi |G^* F| \Psi \rangle)/(2)]^2 + [(\langle \Psi |F^* G| \Psi \rangle - \langle \Psi |G^* F| \Psi \rangle)/(2i)]^2$

For Hermetian Operators…
$F^* = +F$, $G^* = +G$

For Anti-Hermetian (Skew-Hermetian) Operators…
$F^* = -F$, $G^* = -G$

Assuming that F and G are either both Hermetian, or both anti-Hermetian…
$|\langle f | g \rangle|^2 = [(\langle \Psi |(\pm)FG| \Psi \rangle + \langle \Psi |(\pm)GF| \Psi \rangle)/(2)]^2 + [(\langle \Psi |(\pm)FG| \Psi \rangle - \langle \Psi |(\pm)GF| \Psi \rangle)/(2i)]^2$
$|\langle f | g \rangle|^2 = [(\pm)(\langle \Psi |FG| \Psi \rangle + \langle \Psi |GF| \Psi \rangle)/(2)]^2 + [(\pm)(\langle \Psi |FG| \Psi > - \langle \Psi |GF| \Psi \rangle)/(2i)]^2$

We can write this in commutator and anti-commutator notation…
$|\langle f | g \rangle|^2 = [(\pm)(\langle \Psi |\{F,G\}| \Psi \rangle)/(2)]^2 + [(\pm)(\langle \Psi |[F,G]| \Psi \rangle)/(2i)]^2$

Due to the squares, the (±)'s go away, and we can also multiply the commutator by an (i^2)

$|\langle f | g \rangle|^2 = [(\langle \Psi |\{F,G\}| \Psi \rangle)/2]^2 + [(\langle \Psi |i[F,G]| \Psi \rangle)/2]^2$

$|\langle f | g \rangle|^2 = [(\langle \{F,G\} \rangle)/2]^2 + [(\langle i[F,G] \rangle)/2]^2$

The Cauchy–Schwarz inequality again…
$\sigma_f^2\sigma_g^2 = [\langle f | f \rangle \cdot \langle g | g \rangle] >= |\langle f | g \rangle|^2 = [(\langle \{F,G\} \rangle)/2]^2 + [(\langle i[F,G] \rangle)/2]^2$

Taking the root:
$\sigma_f^2\sigma_g^2 >= (1/2)|\langle i[F,G] \rangle|$

Which is what we had for the generalized Uncertainty Relation.

Note This is not a QM axiom - This is just pure math. At this stage we already see the hints of commutation and anti-commutation.
It is true generally, whether applying to a physical or purely mathematical situation.

SRQM:
Heisenberg Uncertainty Principle: Simultaneous vs Sequential

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Heisenberg Uncertainty { $\sigma^2_A\sigma^2_B >= (1/2)|<[A,B]>|$ } arises from the non-commuting nature of certain operators.
$[\partial^\mu,X^\nu] = \partial[\mathbf{X}] = \eta^{\mu\nu}$ = Minkowski Metric
$[P^\mu,X^\nu] = [i\hbar\partial^\mu,X^\nu] = i\hbar[\partial^\mu,X^\nu] = i\hbar\eta^{\mu\nu}$

Consider the following:
Operator A acts on System $|\Psi>$ at SR Event A: $A|\Psi> \rightarrow |\Psi'>$
Operator B acts on System $|\Psi'>$ at SR Event B: $B|\Psi'> \rightarrow |\Psi''>$
or $BA|\Psi> = B|\Psi'> = |\Psi''>$

If measurement Events A & B are space-like separated, then there are observers who can see {A before B, A simultaneous with B, A after B}, which of course does not match the quantum description of how Operators act on Kets

If Events A & B are time-like separated, then all observers will always see A before B. This does match how the operators act on Kets, and also matches how $|\Psi>$ would be evolving along its worldline, starting out as $|\Psi>$, getting hit with operator A at Event A to become $|\Psi'>$, then getting hit with operator B at Event B to become $|\Psi''>$.

The Uncertainty Relation here does NOT refer to simultaneous (space-like separated) measurements, it refers to sequential (time-like separated) measurements. This removes the need for ideas about the particles not having simultaneous properties. There are simply no "simultaneous measurements" of non-zero commuting properties on an individual system, a single worldline – they are sequential, and the first measurement places the system in such a state that the outcome of the second measurement will be altered wrt. if the order of the operations had been reversed.

SRQM:
Pauli Exclusion Principle:
Requires SR for the detailed explanation

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

The Pauli Exclusion Principle is a result of the empirical fact that nature uses identical (indistinguishable) particles, and this combined with the Spin-Statistics theorem from SR, leads to an exclusion principle for fermions (anti-symmetric, Fermi-Dirac statistics) and an aggregation principle for bosons (symmetric, Bose-Einstein statistics). The Spin-Statistics Theorem is related as well to the CPT Theorem.

For large numbers and/or mixed states these both tend to the Maxwell-Boltzmann statistics. In the $\{kT >> (\varepsilon_i - \mu)\}$ limit, Bose-Einstein reduces to Rayleigh-Jeans. The commutation relations here are based on space-like separation particle exchanges. Exchange operator P, $P^2 = +1$, Since two exchanges bring one back to the original state. P thus has two eigenvalues (±1) and two eigenvectors { |Symm> , |AntiSymm> }

P|Symm> = +|Symm>

P|AntiSymm> = -|AntiSymm>

Spin-Symmetry	Particle Type	Quantum Statistics	Classical { $kT >> (\varepsilon_i - \mu)$ }
spin:(0,1,...,N) bosons symmetric	Indistinguishable, Commutation relation [a,b] = ab-ba = -[b,a] = constant (ab = ba) if commutes	Bose-Einstein: $n_i = g_i / [\, e^{(\varepsilon_i - \mu)/kT} - 1 \,]$ aggregation principle (condensation)	Rayleigh-Jeans: from $e^x \sim (1 + x + ...)$ $n_i = g_i / [\, (\varepsilon_i - \mu)/kT \,]$
		↓ Limit as $e^{(\varepsilon_i - \mu)/kT} >> 1$ ↓	
Multi-particle Mixed	Distinguishable, or high temp, or low density	Maxwell-Boltzmann: $n_i = g_i / [\, e^{(\varepsilon_i - \mu)/kT} + 0 \,]$	Maxwell-Boltzmann: $n_i = g_i / [\, e^{(\varepsilon_i - \mu)/kT} \,]$
		↑ Limit as $e^{(\varepsilon_i - \mu)/kT} >> 1$ ↑	
spin:(1/2,3/2,...,N/2) fermions anti-symmetric	Indistinguishable, Anti-commutation relation {a,b} = ab+ba = +{b,a} = constant (ab = - ba) if anti-commutes	Fermi-Dirac: $n_i = g_i / [\, e^{(\varepsilon_i - \mu)/kT} + 1 \,]$ exclusion principle	

SRQM: 4-Vectors & Minkowski Space Review Complex 4-Vectors

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John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Complex 4-vectors are simply 4-Vectors where the components may be complex-valued

$\mathbf{A} = A^\mu = (a^0,\mathbf{a}) = (a^0,a^1,a^2,a^3) \rightarrow (a^t,a^x,a^y,a^z)$
$\mathbf{B} = B^\mu = (b^0,\mathbf{b}) = (b^0,b^1,b^2,b^3) \rightarrow (b^t,b^x,b^y,b^z)$

Examples of 4-Vectors with complex components are the 4-Polarization and the 4-ProbabilityCurrentDensity

Minkowski Metric $g^{\mu\nu} \rightarrow \eta^{\mu\nu} = \eta_{\mu\nu} \rightarrow \mathrm{Diag}[1,-1,-1,-1] = \mathrm{Diag}[1,-\mathbf{I}_{(3)}]$,
which is the {curvature~0 limit = low-mass limit} of the GR metric $g^{\mu\nu}$.

Applying the Metric to raise or lower an index also applies a complex-conjugation *

Scalar Product = Lorentz Invariant → Same value for all inertial observers
$\mathbf{A}\cdot\mathbf{B} = \eta_{\mu\nu} A^\mu B^\nu = A_\nu^* B^\nu = A^\mu B_\mu^* = (a^{0*}b^0 - \mathbf{a}^*\cdot\mathbf{b})$ using the Einstein summation convention

This reverts to the usual rules for real components
However, it does imply that $\mathbf{A}\cdot\mathbf{B} = \overline{\mathbf{B}\cdot\mathbf{A}}$

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM:
CPT Theorem
Phase Connection, Lorentz Invariance

4-Vector SRQM Interpretation of QM

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The Phase is a Lorentz Scalar Invariant – all observers must agree on its value.
$\mathbf{K}\cdot\mathbf{X} = (\omega/c,\mathbf{k})\cdot(ct,\mathbf{x}) = (\omega t - \mathbf{k}\cdot\mathbf{x}) = -\Phi$: Phase of SR Wave

We take the point of view of an observer operating on a particle at 4-Position **X**, which has an initial 4-WaveVector **K**. The 4-Position **X** of the particle, the operation's event, will not change: we are applying the various operations only to the particle's 4-Momentum **K**.

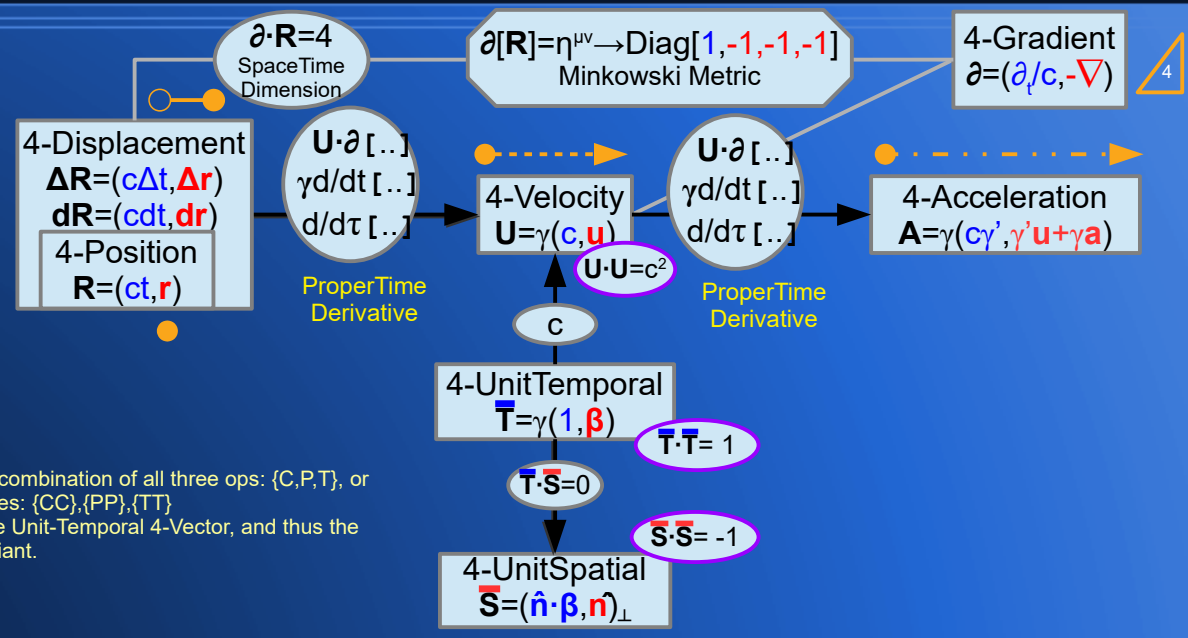
Note that for matter particles $\mathbf{K} = (\omega_o/c)\mathbf{T}$,
where **T** is the Unit-Temporal 4-Vector $\mathbf{T} = \gamma(1,\boldsymbol{\beta})$,
which defines the particle's worldline at each point.
The gamma factor (γ) will be unaffected in the following operations, since it uses the square of $\boldsymbol{\beta}$: $\gamma = 1/\mathrm{Sqrt}(1-\boldsymbol{\beta}\cdot\boldsymbol{\beta})$.
For photonic particles, $\mathbf{K} = (\omega/c)\mathbf{N}$,
where **N** is the "Unit"-Null 4-Vector $\mathbf{N} = (1,\mathbf{n})$ and **n** is a unit-spatial 3-vector.
All operations listed below work similarly on the Null 4-Vector.

Do a Time Reversal Operation: T
The particle's temporal direction is reversed & complex-conjugated:
$\mathbf{T}_T = -\mathbf{T}^* = \gamma(-1,\boldsymbol{\beta})^*$

Do a Parity Operation (Space Reflection): P
Only the spatial directions are reversed:
$\mathbf{T}_P = \gamma(1,-\boldsymbol{\beta})$

Do a Charge Conjugation Operation: C
Charge Conjugation actually changes all internal quantum #'s: charge, lepton #, etc.
Feynman showed this is the equivalent of a world-line reversal & complex-conjugation:
$\mathbf{T}_C = \gamma(-1,-\boldsymbol{\beta})^*$

Pairwise combinations:
$\mathbf{T}_{TP} = \mathbf{T}_{PT} = \mathbf{T}_C = \gamma(-1,-\boldsymbol{\beta})^*$
$\mathbf{T}_{TC} = \mathbf{T}_{CT} = \mathbf{T}_P = \gamma(1,-\boldsymbol{\beta})$
$\mathbf{T}_{PC} = \mathbf{T}_{CP} = \mathbf{T}_T = \gamma(-1,\boldsymbol{\beta})^*$, a CP event is mathematically the same as a T event
$\mathbf{T}_{CPT} = \mathbf{T} = \gamma(1,\boldsymbol{\beta})$ $\quad \mathbf{T}_{CC} = \mathbf{T} = \gamma(1,\boldsymbol{\beta})$ $\quad \mathbf{T}_{PP} = \mathbf{T} = \gamma(1,\boldsymbol{\beta})$ $\quad \mathbf{T}_{TT} = \mathbf{T} = \gamma(1,\boldsymbol{\beta})$



It is only the combination of all three ops: {C,P,T}, or pairs of singles: {CC},{PP},{TT} that leave the Unit-Temporal 4-Vector, and thus the Phase, Invariant.

Matter-like
$\mathbf{T} = \gamma(1,\boldsymbol{\beta})$
$\mathbf{T}\cdot\mathbf{T} = \gamma(1,\boldsymbol{\beta})^*\cdot\gamma(1,\boldsymbol{\beta}) = \gamma^2(1^2 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}) = +1$: It's a temporal 4-vector

$\mathbf{T}_C\cdot\mathbf{T}_C = \gamma(-1,-\boldsymbol{\beta})\cdot\gamma(-1,-\boldsymbol{\beta})^* = \gamma^2((-1)^2 - (-\boldsymbol{\beta})\cdot(-\boldsymbol{\beta})) = \gamma^2(1^2 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}) = 1$
$\mathbf{T}_P\cdot\mathbf{T}_P = \gamma(1,-\boldsymbol{\beta})^*\cdot\gamma(1,-\boldsymbol{\beta}) = \gamma^2(1^2 - (-\boldsymbol{\beta})\cdot(-\boldsymbol{\beta})) = \gamma^2(1^2 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}) = 1$
$\mathbf{T}_T\cdot\mathbf{T}_T = \gamma(-1,\boldsymbol{\beta})\cdot\gamma(-1,\boldsymbol{\beta})^* = \gamma^2((-1)^2 - (\boldsymbol{\beta})\cdot(\boldsymbol{\beta})) = \gamma^2(1^2 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}) = 1$
They all remain temporal 4-vectors

$\mathbf{T}_{CPT} = \mathbf{T} = \gamma(1,\boldsymbol{\beta})$
$\mathbf{T}_{CPT}\cdot\mathbf{T}_{CPT} = \mathbf{T}\cdot\mathbf{T} = 1$

Light-like/Photonic
$\mathbf{N} = (1,\mathbf{n})$
$\mathbf{N}\cdot\mathbf{N} = (1,\mathbf{n})^*\cdot(1,\mathbf{n}) = (1^2 - \mathbf{n}\cdot\mathbf{n}) = (1-1) = 0$: It's a null 4-vector

$\mathbf{N}_C\cdot\mathbf{N}_C = (-1,-\mathbf{n})\cdot(-1,-\mathbf{n})^* = ((-1)^2 - (-\mathbf{n})\cdot(-\mathbf{n})) = (1^2 - \mathbf{n}\cdot\mathbf{n}) = (1-1) = 0$
$\mathbf{N}_P\cdot\mathbf{N}_P = (1,-\mathbf{n})^*\cdot(1,-\mathbf{n}) = (1^2 - (-\mathbf{n})\cdot(-\mathbf{n})) = (1^2 - \mathbf{n}\cdot\mathbf{n}) = (1-1) = 0$
$\mathbf{N}_T\cdot\mathbf{N}_T = (-1,\mathbf{n})\cdot(-1,\mathbf{n})^* = ((-1)^2 - (\mathbf{n})\cdot(\mathbf{n})) = (1^2 - \mathbf{n}\cdot\mathbf{n}) = (1-1) = 0$
They all remain null 4-vectors

$\mathbf{N}_{CPT} = \mathbf{N} = (1,\mathbf{n})$
$\mathbf{N}_{CPT}\cdot\mathbf{N}_{CPT} = \mathbf{N}\cdot\mathbf{N} = 0$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM: CPT Theorem (Charge) vs (Parity) vs (Time)

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

4-Vector
A=A^ν=(a^0,**a**)

No mixing

$Tr[\eta^{\mu'}{}_\nu]$= +4

Lorentz Identity Transform
$\Lambda^{\mu'}{}_\nu \rightarrow \eta^{\mu'}{}_\nu = I_{(4)}$
=
$\begin{bmatrix} 1 & 0 \\ 0 & \delta^i_j \end{bmatrix}$

$Det[\eta^{\mu'}{}_\nu]$ = +1

Identical 4-Vector
A'=$A^{\mu'}=\eta^{\mu'}{}_\nu A^\nu$=($a^{0'}$,**a'**)
=(a^0,**a**)=**A**

Lorentz Identity Transform
$\Lambda^{\mu'}{}_\nu \rightarrow \eta^{\mu'}{}_\nu$

4-Vector
B=B^ν=(b^0,**b**)

Space

$Tr[P^{\mu'}{}_\nu]$= -2

Lorentz Parity-Inversion Transform
$\Lambda^{\mu'}{}_\nu \rightarrow P^{\mu'}{}_\nu$
=
$\begin{bmatrix} 1 & 0 \\ 0 & -\delta^i_j \end{bmatrix}$

$Det[P^{\mu'}{}_\nu]$ = -1

Parity-Inverted 4-Vector
A'=$A^{\mu'}=P^{\mu'}{}_\nu A^\nu$=($a^{0'}$,**a'**)
=(a^0,-**a**)

Lorentz Parity-Inversion Transform
$\Lambda^{\mu'}{}_\nu \rightarrow P^{\mu'}{}_\nu$

Time

$Tr[T^{\mu'}{}_\nu]$= +2*

Lorentz Time-Reversal Transform
$\Lambda^{\mu'}{}_\nu \rightarrow T^{\mu'}{}_\nu$
=
$\begin{bmatrix} -1 & 0 \\ 0 & \delta^i_j \end{bmatrix}$ *

$Det[T^{\mu'}{}_\nu]$ = -1

Time-Reversed 4-Vector
A'=$A^{\mu'}=T^{\mu'}{}_\nu A^\nu$=($a^{0'}$,**a'**)
=(-a^0,**a**)*

Lorentz Time-Reversal Transform
$\Lambda^{\mu'}{}_\nu \rightarrow T^{\mu'}{}_\nu$

Charge

$Tr[C^{\mu'}{}_\nu]$= -4*

Lorentz Charge-Conjugation Transform
$\Lambda^{\mu'}{}_\nu \rightarrow C^{\mu'}{}_\nu$
=
$\begin{bmatrix} -1 & 0 \\ 0 & -\delta^i_j \end{bmatrix}$ *

$Det[C^{\mu'}{}_\nu]$ = +1

Charge-Conjugated 4-Vector
A'=$A^{\mu'}=C^{\mu'}{}_\nu A^\nu$=($a^{0'}$,**a'**)
=(-a^0,-**a**)*

Lorentz Charge-Conjugation Transform
$\Lambda^{\mu'}{}_\nu \rightarrow C^{\mu'}{}_\nu$

After (CPT)

Original 4-Vector
B=B^ν=(b^0,**b**)

(CPT)
=(CTP)
=(PCT)
=(PTC)
=(TCP)
=(TPC)

After (PP) or (TT) or (CC)

Original 4-Vector
A=A^ν=(a^0,**a**)

Classical SR Time-Reversal neglects spin and charge.
SRQM includes these effects.
Then one gets (CC),(PP),(TT), & (CPT) transforms all leading back to the Identity ($I_{(4)}$).

Identity and Space-Parity are Unitary
Time-Reversal and Charge-Conjugation are Anti-Unitary.

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor V^μ = **V** = (v^0,**v**)
SR 4-CoVector:OneForm
(0,1)-Tensor V_μ = (v_0,-**v**)

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$Trace[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
V·V = $V^\mu\eta_{\mu\nu}V^\nu$ = [$(v^0)^2$ - **v·v**] = $(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Transforms: Venn Diagram

Poincaré = Lorentz + Translations

(10) (6) (4)

4-Vector SRQM Interpretation of QM

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John B. Wilson
SciRealm@aol.com
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Transformations

(# of independent parameters = # continuous symmetries = # Lie Dimensions)

Poincaré Transformation Group aka. Inhomogeneous Lorentz Transformation

Lie group of all affine isometries of SR:Minkowski Time·Space (preserve quadratic form $\eta_{\mu\nu}$)

General Linear,Affine Transform $X^{\mu'} = \Lambda^{\mu'}{}_{\nu}X^{\nu} + \Delta X^{\mu'}$ with $\mathrm{Det}[\Lambda^{\mu'}{}_{\nu}] = \pm 1$

(6+4=10)

Lorentz Transform $\Lambda^{\mu'}{}_{\nu}$

(3+3=6) 4-Tensor {mixed type-(1,1)}

Discrete

- Time-reversal $\Lambda^{\mu'}{}_{\nu} \to T^{\mu'}{}_{\nu}$ (0) $t \to -t^*$ time parity anti-unitary
- SpatialFlipCombos $\Lambda^{\mu'}{}_{\nu} \to F^{\mu'}{}_{\nu}$ (0) {x|y|z} → -{x|y|z} unitary
- Parity-Inversion $\Lambda^{\mu'}{}_{\nu} \to P^{\mu'}{}_{\nu}$ (0) **r → -r** space parity unitary
- Identity $I_{(4)}$ $\Lambda^{\mu'}{}_{\nu} \to \eta^{\mu'}{}_{\nu}=\delta^{\mu'}{}_{\nu}$ (0) no mixing unitary
- Charge-Conjugation $\Lambda^{\mu'}{}_{\nu} \to C^{\mu'}{}_{\nu}$ (0) **R → -R***, q → -q charge parity anti-unitary

Continuous

- Rotation $\Lambda^{\mu'}{}_{\nu} \to R^{\mu'}{}_{\nu}$ (3) x:y | x:z | y:z
- Boost $\Lambda^{\mu'}{}_{\nu} \to B^{\mu'}{}_{\nu}$ (3) t:x | t:y | t:z

CPT Symmetry {Charge} {Partiy} {Time}

Isotropy {same all directions}

Translation Transform $\Delta X^{\mu'}$

(1+3=4) 4-Vector

Discrete

- 4-Zero $\Delta X^{\mu'} \to (0,\mathbf{0})$ (0) no motion

Continuous

- Temporal $\Delta X^{\mu'} \to (c\Delta t,\mathbf{0})$ (1) Δt
- Spatial $\Delta X^{\mu'} \to (0,\mathbf{\Delta x})$ (3) Δx | Δy | Δz

Homogeneity {same all points}

	M^{01}	M^{02}	M^{03}
M^{10}		M^{12}	M^{13}
M^{20}	M^{21}		M^{23}
M^{30}	M^{31}	M^{32}	

P^0
P^1
P^2
P^3

4-AngularMomentum $M^{\mu\nu} = X^{\mu} \wedge P^{\nu} = X^{\mu}P^{\nu} - X^{\nu}P^{\mu}$
= Generator of Lorentz Transformations (6)
= { $\Lambda^{\mu'}{}_{\nu} \to R^{\mu'}{}_{\nu}$ Rotations (3) + $\Lambda^{\mu'}{}_{\nu} \to B^{\mu'}{}_{\nu}$ Boosts (3) }

4-LinearMomentum P^{μ}
= Generator of Translation Transformations (4)
= { $\Delta X^{\mu'} \to (c\Delta t,\mathbf{0})$ Time (1) + $\Delta X^{\mu'} \to (0,\mathbf{\Delta x})$ Space (3) }

$\mathrm{Det}[\Lambda^{\mu'}{}_{\nu}] = +1$ for Proper Lorentz Transforms
$\mathrm{Det}[\Lambda^{\mu'}{}_{\nu}] = -1$ for Improper Lorentz Transforms

Lorentz Matrices can be generated by a matrix M with Tr[M]=0 which gives:
{ $\Lambda = e \wedge M = e \wedge (+\boldsymbol{\theta}\cdot\mathbf{J} - \boldsymbol{\zeta}\cdot\mathbf{K})$ }
{ $\Lambda^T = (e \wedge M)^T = e \wedge M^T$ }
{ $\Lambda^{-1} = (e \wedge M)^{-1} = e \wedge -M$ }

$\mathrm{Tr}[\Lambda^{\mu}{}_{\nu}]=\{-\infty..+\infty\}$ =Lorentz Transform Type

SR:Lorentz Transform

$\partial_{\nu}[R^{\mu'}] = \partial R^{\mu'}/\partial R^{\nu} = \Lambda^{\mu'}{}_{\nu}$

$\Lambda^{\mu}{}_{\nu} = (\Lambda^{-1})_{\nu}{}^{\mu} : \Lambda^{\mu}{}_{\alpha}(\Lambda^{-1})^{\alpha}{}_{\nu} = \eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$

$\eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$

$\mathrm{Det}[\Lambda^{\mu}{}_{\nu}]=\pm 1$ $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4$

$M = +\boldsymbol{\theta}\cdot\mathbf{J} - \boldsymbol{\zeta}\cdot\mathbf{K}$
$B[\boldsymbol{\zeta}] = e\wedge(-\boldsymbol{\zeta}\cdot\mathbf{K})$
$R[\boldsymbol{\theta}] = e\wedge(+\boldsymbol{\theta}\cdot\mathbf{J})$
$\Lambda = e \wedge M = e \wedge (+\boldsymbol{\theta}\cdot\mathbf{J} - \boldsymbol{\zeta}\cdot\mathbf{K})$

Rotations $J_i = -\varepsilon_{imn}M^{mn}/2$, Boosts $K_i = M_{i0}$

[(**R**→ -**R***)] or [(t→ -t*) & (**r**→ -**r**)] imply q→ -q
Feynman-Stueckelberg Interpretation
Amusingly, Inhomogeneous Lorentz adds homogeneity.

Hermitian Generators
Noether's Theorem - Continuity

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

The Hermitian Generators that lead to translations and rotations via unitary operators in QM...

These all ultimately come from the Poincaré Invariance → Lorentz Invariance that is at the heart of SR and Minkowski Space.

Infintesimal Unitary Transformation
$\hat{\mathbf{U}}_\varepsilon(\hat{\mathbf{G}}) = \mathbf{I} + i\varepsilon\hat{\mathbf{G}}$

Finite Unitary Transformation
$\hat{\mathbf{U}}_\alpha(\hat{\mathbf{G}}) = e\text{^}(i\alpha\hat{\mathbf{G}})$

let $\hat{\mathbf{G}} = \mathbf{P}/\hbar = \mathbf{K}$
let $\alpha=\mathbf{\Delta x}$

$\hat{\mathbf{U}}_{\Delta x}(\mathbf{P}/\hbar)\Psi(\mathbf{X}) = e\text{^}(i\mathbf{\Delta x}\cdot\mathbf{P}/\hbar)\Psi(\mathbf{X}) = e\text{^}(-\mathbf{\Delta x}\cdot\partial)\Psi(\mathbf{X}) = \Psi(\mathbf{X} - \mathbf{\Delta x})$

Time component: $\hat{\mathbf{U}}_{\Delta ct}(\mathbf{P}/\hbar)\Psi(ct) = e\text{^}(i\Delta tE/\hbar)\Psi(ct) = e\text{^}(-\Delta t\ \partial_t)\Psi(ct) = \Psi(ct - c\Delta t) = c\Psi(t - \Delta t)$
Space component: $\hat{\mathbf{U}}_{\Delta x}(\mathbf{p}/\hbar)\Psi(\mathbf{x}) = e\text{^}(i\mathbf{\Delta x}\cdot\mathbf{p}/\hbar)\Psi(\mathbf{x}) = e\text{^}(\mathbf{\Delta x}\cdot\nabla)\Psi(\mathbf{x}) = \Psi(\mathbf{x} + \mathbf{\Delta x})$

By Noether's Theorem, this leads to $\partial\cdot\mathbf{K} = 0$

We had already calculated
$(\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}] = ((\partial_t/c)^2 - \nabla\cdot\nabla)(\omega t - \mathbf{k}\cdot\mathbf{x}) = 0$
$(\partial\cdot\partial)[\mathbf{K}\cdot\mathbf{X}] = \partial\cdot(\partial[\mathbf{K}\cdot\mathbf{X}]) = \partial\cdot\mathbf{K} = 0$

Poincaré Invariance also gives the Casimir invariants of mass and spin, and ultimately leads to the spin-statistics theorem of RQM.

SRQM:
QM Correspondence Principle: Analogous to the GR and SR limits

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Basically, the old school QM Correspondence Principle says that QM should give the same results as classical physics in the realm of large quantum systems, i.e. where macroscopic behavior overwhelms quantum effects. Perhaps a better way to state it is when the change of system by a single quantum has a negligible effect on the overall state.

There is a way to derive this limit, by using Hamilton-Jacobi Theory:
$(i\hbar\partial_{tT})|\Psi> \sim [\ V - (\hbar\nabla_T)^2/2m_o\ ]|\Psi>$: The Schrödinger NRQM Equation for a point particle (non-relativistic QM)

Examine solutions of form $\Psi = \Psi_o e^{(i\Phi)} = \Psi_o e^{(iS/\hbar)}$, where S is the QM Action
$\partial_t[\Psi] = (i/\hbar)\Psi\partial_t[S]$ and $\partial_x[\Psi] = (i/\hbar)\Psi\partial_x[S]$ and $\nabla^2[\Psi] = (i/\hbar)\Psi\nabla^2[S] - (\Psi/\hbar^2)(\nabla[S])^2$

$(i\hbar)(i/\hbar)\Psi\partial_t[S] = V\Psi - (\hbar^2/2m_o)((i/\hbar)\Psi\nabla^2[S] - (\Psi/\hbar^2)(\nabla[S])^2)$

$(i)(i)\Psi\partial_t[S] = V\Psi - ((i\hbar/2m_o)\Psi\nabla^2[S] - (\Psi/2m_o)(\nabla[S])^2)$

$\partial_t[S] = -V + (i\hbar/2m_o)\nabla^2[S] - (1/2m_o)(\nabla[S])^2$

$\partial_t[S] + [V+(1/2m_o)(\nabla[S])^2\ ] = (i\hbar/2m_o)\nabla^2[S]$: Quantum Single Particle Hamilton-Jacobi
$\partial_t[S] + [V+(1/2m_o)(\nabla[S])^2\ ] = 0$: Classical Single Particle Hamilton-Jacobi

Thus, the classical limiting case is:
$\nabla^2[\Phi] << (\nabla[\Phi])^2$
$\hbar\nabla^2[S] << (\nabla[S])^2$
$\hbar\nabla\cdot\mathbf{p} << (\mathbf{p}\cdot\mathbf{p})$ $\qquad$ $\nabla\cdot\mathbf{k} << (\mathbf{k}\cdot\mathbf{k})$
$(p\lambda\!\!\!^{-})\nabla\cdot\mathbf{p} << (\mathbf{p}\cdot\mathbf{p})$

QM Correspondence Principle:
Analogous to the GR and SR limits

SciRealm.org
John B. Wilson
SciRealm@aol.com
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$\partial_t[S] + [V+(1/2m_o)(\nabla[S])^2] = (i\hbar/2m_o)\nabla^2[S]$: Quantum Single Particle Hamilton-Jacobi
$\partial_t[S] + [V+(1/2m_o)(\nabla[S])^2] = 0$: Classical Single Particle Hamilton-Jacobi

Thus, the quantum→classical limiting-case is: {all equivalent representations}

$\hbar\nabla^2[S_{action}] << (\nabla[S_{action}])^2$ $\quad$ $\nabla^2[\Phi_{phase}] << (\nabla[\Phi_{phase}])^2$

$\hbar\nabla\cdot\nabla[S_{action}] << (\nabla[S_{action}])^2$ $\quad$ $\nabla\cdot\nabla[\Phi_{phase}] << (\nabla[\Phi_{phase}])^2$

$\hbar\nabla\cdot\mathbf{p} << (\mathbf{p}\cdot\mathbf{p})$ $\quad$ $\nabla\cdot\mathbf{k} << (\mathbf{k}\cdot\mathbf{k})$

$(p\cancel{A})\nabla\cdot\mathbf{p} << (\mathbf{p}\cdot\mathbf{p})$

This page needs some work. Source was from Goldstein

with

$\mathbf{P} = (E/c,\mathbf{p}) = -\partial[S_{action}] = -(\partial_t/c,-\nabla)[S_{action}] = (-\partial_t/c,\nabla)[S_{action}]$

$\mathbf{K} = (\omega/c,\mathbf{k}) = -\partial[\Phi_{phase}] = -(\partial_t/c,-\nabla)[\Phi_{phase}] = (-\partial_t/c,\nabla)[\Phi_{phase}]$

It is analogous to GR → SR in limit of low curvature (low mass), or SR → CM in limit of low velocity { |v|<<c }.
It still applies, but is now understood as the same type of limiting-case as these others.

Note The commonly seen form of (c→∞,ħ→0) as limits are incorrect!
c and ħ are universal constants – they never change.
If c→∞, then photons (light-waves) would have infinite energy { E = pc }. This is not true classically.
If ħ→0, then photons (light-waves) would have zero energy { E = ħω }. This is not true classically.
Always better to write the SR Classical limit as { |**v**|<<c }, the QM Classical limit as { $\nabla^2[\Phi_{phase}] << (\nabla[\Phi_{phase}])^2$ }

Again, it is more natural to find a limiting-case of a more general system than to try to unite two separate theories which may or may not ultimately be compatible. From logic, there is always the possibility to have a paradox result from combination of arbitrary axioms, whereas deductions from a single true axiom will always give true results.

SRQM: 4-Vector Quantum Probability

Conservation of ProbabilityDensity

A Tensor Study of Physical 4-Vectors

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Conservation of Probability : Probability Current : Charge Current
Consider the following purely mathematical argument
(based on Green's Vector Identity):

$\partial\cdot(f\, \partial[g] - \partial[f]\, g) = f\, \partial\cdot\partial[g] - \partial\cdot\partial[f]\, g$
with (f) and (g) as SR Lorentz Scalar functions

Proof:
$\partial\cdot(f\, \partial[g] - \partial[f]\, g)$
$= \partial\cdot(f\, \partial[g]) - \partial\cdot(\partial[f]\, g)$
$= (f\, \partial\cdot\partial[g] + \partial[f]\cdot\partial[g]) - (\partial[f]\cdot\partial[g] + \partial\cdot\partial[f]\, g)$
$= f\, \partial\cdot\partial[g] - \partial\cdot\partial[f]\, g$

We can also multiply this by a Lorentz Invariant Scalar Constant s
$s\, (f\, \partial\cdot\partial[g] - \partial\cdot\partial[f]\, g) = s\, \partial\cdot(f\, \partial[g] - \partial[f]\, g) = \partial\cdot s(f\, \partial[g] - \partial[f]\, g)$

Ok, so we have the math that we need…

Now, on to the physics… Start with the Klein-Gordon Eqn.
$\partial\cdot\partial = (-im_o c/\hbar)^2 = -(m_o c/\hbar)^2$
$\partial\cdot\partial + (m_o c/\hbar)^2 = 0$

Let it act on SR Lorentz Invariant function g
$\partial\cdot\partial[g] + (m_o c/\hbar)^2[g] = 0\, [g]$
Then pre-multiply by f
$[f]\partial\cdot\partial[g] + [f]\, (m_o c/\hbar)^2[g] = [f]\, 0\, [g]$
$[f]\partial\cdot\partial[g] + (m_o c/\hbar)^2[f][g] = 0$

Do similarly with SR Lorentz Invariant function f
$\partial\cdot\partial[f] + (m_o c/\hbar)^2[f] = 0\, [f]$
Then post-multiply by g
$\partial\cdot\partial[f][g] + (m_o c/\hbar)^2[f][g] = 0\, [f][g]$
$\partial\cdot\partial[f][g] + (m_o c/\hbar)^2[f][g] = 0$

Now, subtract the two equations
$\{[f]\, \partial\cdot\partial[g] + (m_o c/\hbar)^2[f][g] = 0\} - \{ \partial\cdot\partial[f][g] + (m_o c/\hbar)^2[f][g] = 0\}$
$[f]\, \partial\cdot\partial[g] + (m_o c/\hbar)^2[f][g] - \partial\cdot\partial[f][g\,] - (m_o c/\hbar)^2[f][g] = 0$
$[f]\, \partial\cdot\partial[g] - \partial\cdot\partial[f][g] = 0$

And as we noted from the mathematical Green's Vector identity at the start…
$[f]\, \partial\cdot\partial[g] - \partial\cdot\partial[f][g] = \partial\cdot(f\, \partial[g] - \partial[f]\, g) = 0$

Therefore,
$s\, \partial\cdot(f\, \partial[g] - \partial[f]\, g) = 0$
$\partial\cdot s(f\, \partial[g] - \partial[f]\, g) = 0$

Thus, there is a conserved current 4-Vector, $\mathbf{J}_{prob} = s(f\, \partial[g] - \partial[f]\, g)$, for which $\partial\cdot\mathbf{J}_{prob} = 0$,
and which also solves the Klein-Gordon equation.

Let's choose as before ($\partial = -i\mathbf{K}$) with a plane wave function f = ae^-i($\mathbf{K}\cdot\mathbf{X}$) = ψ,
and choose g = f* = ae^i($\mathbf{K}\cdot\mathbf{X}$) = ψ* as its complex conjugate.

At this point, I am going to choose $s = (i\hbar/2m_o)$, which is Lorentz Scalar Invariant, in order to make the probability have dimensionless units and be normalized to unity in the rest case.

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

4-Vector Quantum Probability

4-ProbabilityFlux, Klein-Gordon RQM Eqn

4-Vector SRQM Interpretation of QM

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SciRealm@aol.com
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4-ProbabilityCurrentDensity, a.k.a. 4-ProbabilityFlux

$\mathbf{J}_{prob} = (c\rho_{prob}, \mathbf{j}_{prob}) = (i\hbar/2m_o)(\psi^* \partial[\psi] - \partial[\psi^*]\psi) = (\rho_{probo})\mathbf{U} = (\rho_{probo})\gamma(c,\mathbf{u}) = (\gamma\rho_{probo})(c,\mathbf{u}) = (\rho_{prob})(c,\mathbf{u})$

with 4-Divergence of Probability { $\partial\cdot\mathbf{J}_{prob} = 0$ } by construction via Green's Vector Identity and the Klein-Gordon RQM Eqn.

The reason for $s = (i\hbar/2m_o) = (ic^2/2\omega_o)$ becomes more clear by examining our diagram:

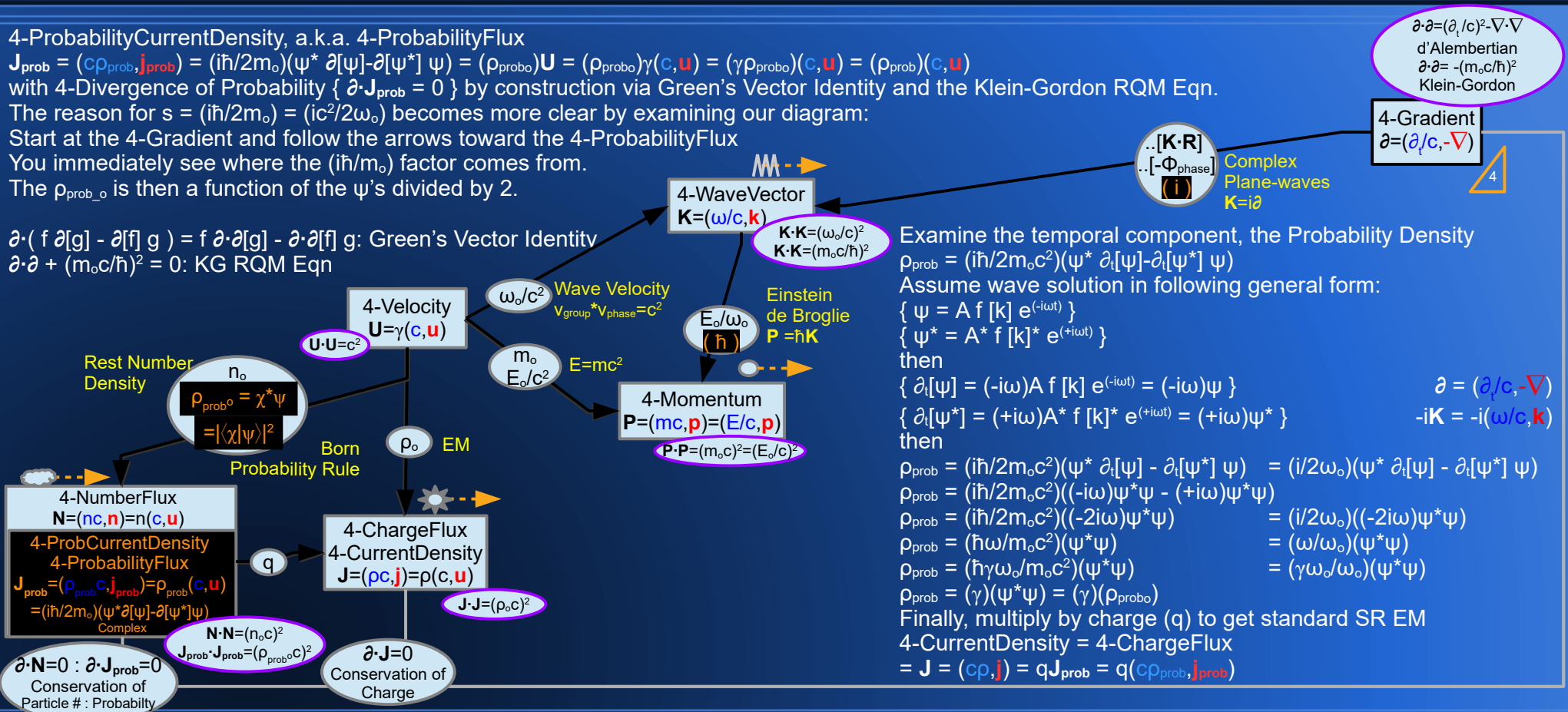
Start at the 4-Gradient and follow the arrows toward the 4-ProbabilityFlux

You immediately see where the $(i\hbar/m_o)$ factor comes from.

The ρ_{prob_o} is then a function of the ψ's divided by 2.

$\partial\cdot(f\,\partial[g] - \partial[f]\,g) = f\,\partial\cdot\partial[g] - \partial\cdot\partial[f]\,g$: Green's Vector Identity

$\partial\cdot\partial + (m_oc/\hbar)^2 = 0$: KG RQM Eqn



Examine the temporal component, the Probability Density

$\rho_{prob} = (i\hbar/2m_oc^2)(\psi^* \partial_t[\psi] - \partial_t[\psi^*]\psi)$

Assume wave solution in following general form:

$\{ \psi = A\,f[k]\,e^{(-i\omega t)} \}$

$\{ \psi^* = A^*\,f[k]^*\,e^{(+i\omega t)} \}$

then

$\{ \partial_t[\psi] = (-i\omega)A\,f[k]\,e^{(-i\omega t)} = (-i\omega)\psi \}$ $\qquad \partial = (\partial_t/c, -\nabla)$

$\{ \partial_t[\psi^*] = (+i\omega)A^*\,f[k]^*\,e^{(+i\omega t)} = (+i\omega)\psi^* \}$ $\qquad -i\mathbf{K} = -i(\omega/c, \mathbf{k})$

then

$\rho_{prob} = (i\hbar/2m_oc^2)(\psi^* \partial_t[\psi] - \partial_t[\psi^*]\psi) = (i/2\omega_o)(\psi^* \partial_t[\psi] - \partial_t[\psi^*]\psi)$

$\rho_{prob} = (i\hbar/2m_oc^2)((-i\omega)\psi^*\psi - (+i\omega)\psi^*\psi)$

$\rho_{prob} = (i\hbar/2m_oc^2)((-2i\omega)\psi^*\psi) = (i/2\omega_o)((-2i\omega)\psi^*\psi)$

$\rho_{prob} = (\hbar\omega/m_oc^2)(\psi^*\psi) = (\omega/\omega_o)(\psi^*\psi)$

$\rho_{prob} = (\hbar\gamma\omega_o/m_oc^2)(\psi^*\psi) = (\gamma\omega_o/\omega_o)(\psi^*\psi)$

$\rho_{prob} = (\gamma)(\psi^*\psi) = (\gamma)(\rho_{probo})$

Finally, multiply by charge (q) to get standard SR EM

4-CurrentDensity = 4-ChargeFlux

$= \mathbf{J} = (c\rho, \mathbf{j}) = q\mathbf{J}_{prob} = q(c\rho_{prob}, \mathbf{j}_{prob})$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
Quantum Principles

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$

$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$

= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

4-Vector Quantum Probability

4-ProbabilityFlux, Klein-Gordon RQM Eqn with Minimal Coupling

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

4-ProbabilityCurrentDensity, a.k.a. 4-ProbabilityFlux

$\mathbf{J}_{prob} = (c\rho_{prob}, \mathbf{j}_{prob}) = (i\hbar/2m_o)(\psi^*\partial[\psi]-\partial[\psi^*]\psi) = (\rho_{probo})\mathbf{U} = (\rho_{probo})\gamma(c,\mathbf{u}) = (\gamma\rho_{probo})(c,\mathbf{u}) = (\rho_{prob})(c,\mathbf{u})$

with 4-Divergence of Probability { $\partial\cdot\mathbf{J}_{prob} = 0$ } by construction via Green's Vector Identity and the Klein-Gordon RQM Eqn.

If we include minimal coupling:

$\mathbf{J}_{prob} = (c\rho_{prob}, \mathbf{j}_{prob}) = (i\hbar/2m_o)(\psi^*\partial[\psi]-\partial[\psi^*]\psi) + (q/m_o)(\psi^*\psi)\mathbf{A}$

Start at **A** on the chart

Follow past (q) factor to get to $\mathbf{Q} = q\mathbf{A}$

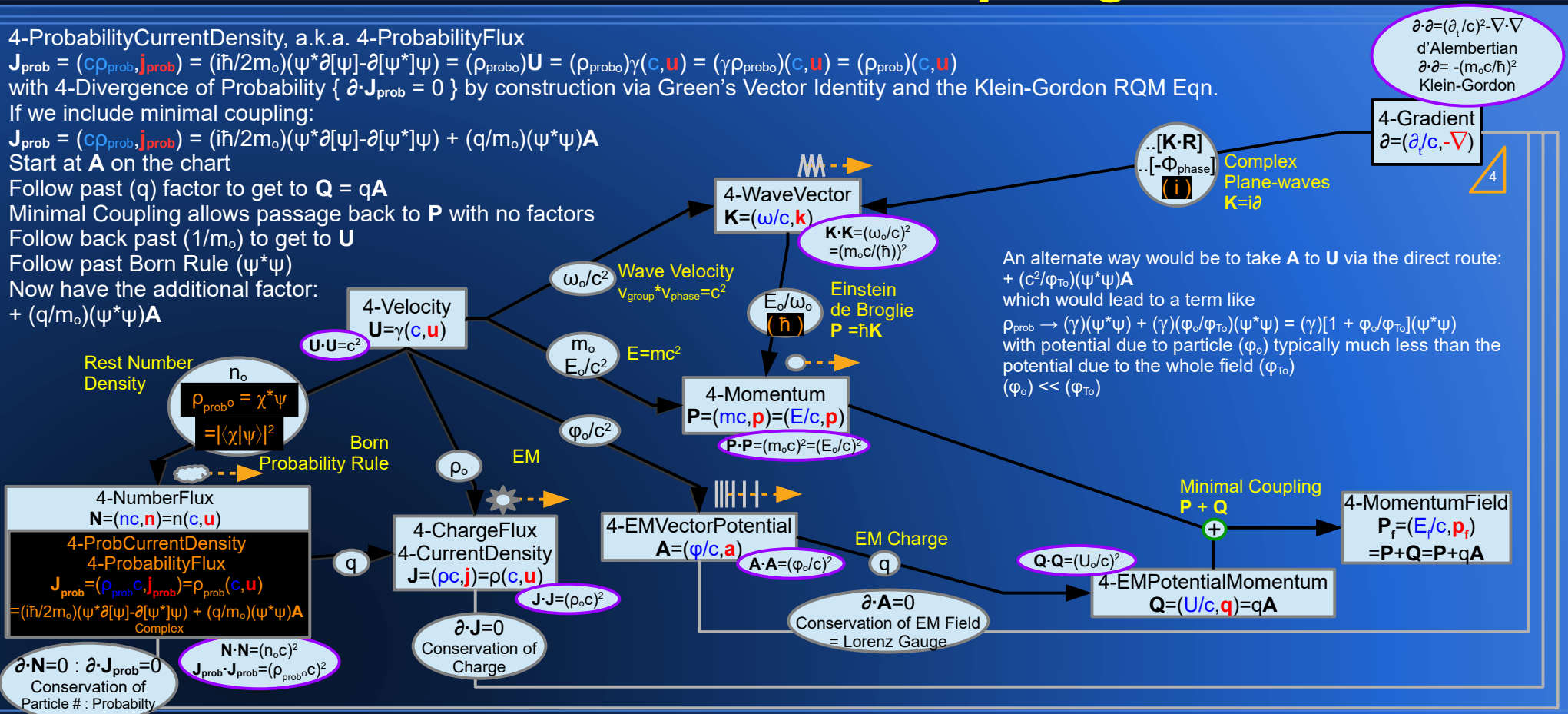
Minimal Coupling allows passage back to **P** with no factors

Follow back past $(1/m_o)$ to get to **U**

Follow past Born Rule $(\psi^*\psi)$

Now have the additional factor:

$+ (q/m_o)(\psi^*\psi)\mathbf{A}$



An alternate way would be to take **A** to **U** via the direct route:

$+ (c^2/\varphi_{To})(\psi^*\psi)\mathbf{A}$

which would lead to a term like

$\rho_{prob} \rightarrow (\gamma)(\psi^*\psi) + (\gamma)(\varphi_o/\varphi_{To})(\psi^*\psi) = (\gamma)[1 + \varphi_o/\varphi_{To}](\psi^*\psi)$

with potential due to particle (φ_o) typically much less than the potential due to the whole field (φ_{To})

$(\varphi_o) << (\varphi_{To})$

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4-Vector Quantum Probability
Newtonian Limit

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4-ProbabilityCurrentDensity $\mathbf{J}_{prob} = (c\rho_{prob}, \mathbf{j}_{prob}) = (i\hbar/2m_o)(\psi^*\partial[\psi]-\partial[\psi^*]\psi) + (q/m_o)(\psi^*\psi)\mathbf{A}$

Examine the temporal component:
$\rho_{prob} = (i\hbar/2m_oc^2)(\psi^* \partial_t[\psi]-\partial_t[\psi^*]\psi) + (q/m_o)(\psi^*\psi)(\varphi/c^2)$
$\rho_{prob} \rightarrow (\gamma)(\psi^*\psi) + (\gamma)(q\varphi_o/m_oc^2)(\psi^*\psi) = (\gamma)[1 + q\varphi_o/E_o](\psi^*\psi)$

Typically, the particle EM potential energy ($q\varphi_o$) is much less than the particle rest energy (E_o), else it could generate new particles.
So, take ($q\varphi_o << E_o$), which gives the EM factor $(q\varphi_o/E_o) \sim 0$

Now, taking the low-velocity limit ($\gamma \rightarrow 1$), $\rho_{prob} = \gamma[1 + \sim 0](\psi^*\psi)$, $\rho_{prob} \rightarrow (\psi^*\psi) = (\rho_{probo})$ for $|\mathbf{v}|<<c$

The Standard Born Probability Interpretation, $(\psi^*\psi) = (\rho_{prob})$, only applies in the low-potential-energy & low-velocity limit

This is why the {non-positive-definite} probabilities and {|probabilities| > 1} in the RQM Klein-Gordon equation gave physicists fits, and is the reason why one must regard the probabilities as charge conservation instead.

The original definition from SR is Continuity of Worldlines, $\partial\cdot\mathbf{J}_{prob} = 0$, for which all is good and well in the RQM version.
The definition says there are no external sources or sinks of probability = conservation of probability.

The Born idea that $(\rho_{prob}) \rightarrow \text{Sum}[(\psi^*\psi)] = 1$ is just the Low-Velocity QM limit.
Only the non-EM rest version $(\rho_{probo}) = \text{Sum}[(\psi^*\psi)] = 1$ is true.
It is not a fundamental axiom, it is an emergent property which is valid only in the NRQM limit

We now multiply by charge (q) to instead get a
4-"Charge"CurrentDensity $\mathbf{J} = (c\rho, \mathbf{j}) = q\mathbf{J}_{prob} = q(c\rho_{prob}, \mathbf{j}_{prob})$, which is the standard SR EM 4-CurrentDensity

(+,-,-,-) SR → QM Physics

4

A Tensor Study of Physical 4-Vectors

SRQM 4-Vector Study: The QM Compton Effect Compton Scattering

4-Vector SRQM Interpretation of QM

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Electron e⁻ initial | Photon γ initial

4-Momentum of e⁻
$\mathbf{P}_{ei}=(m_{ei}c,\mathbf{p}_{ei})=(E_{ei}/c,\mathbf{p}_{ei})$

4-WaveVector of γ
$\mathbf{K}_{pi}=(\omega_{pi}/c,\mathbf{k}_{pi})$

$E/\omega = E_o/\omega_o = (\hbar)$

+

4-Momentum of γ
$\mathbf{P}_{pi}=(m_{pi}c,\mathbf{p}_{pi})=(E_{pi}/c,\mathbf{p}_{pi})=(\hbar)\mathbf{K}_{pi}$

4-TotalMomentum of e⁻+γ
$\mathbf{P}_{ti}=(E_T/c,\mathbf{p}_T)=(H/c,\mathbf{p}_T)$
$=\mathbf{P}_{ei}+\mathbf{P}_{pi}$

=

4-TotalMomentum of e⁻+γ
$\mathbf{P}_{tf}=(E_T/c,\mathbf{p}_T)=(H/c,\mathbf{p}_T)$
$=\mathbf{P}_{ef}+\mathbf{P}_{pf}$

+

4-Momentum of γ
$\mathbf{P}_{pf}=(m_{pf}c,\mathbf{p}_{pf})=(E_{pf}/c,\mathbf{p}_{pf})=(\hbar)\mathbf{K}_{pf}$

$\omega/E = \omega_o/E_o = (1/\hbar)$

4-Momentum of e⁻
$\mathbf{P}_{ef}=(m_{ef}c,\mathbf{p}_{ef})=(E_{ef}/c,\mathbf{p}_{ef})$

4-WaveVector of γ
$\mathbf{K}_{pf}=(\omega_{pf}/c,\mathbf{k}_{pf})$

Electron e⁻ final | Photon γ final

Compton Scattering Derivation : Compton Effect

$\mathbf{P}\cdot\mathbf{P} = (m_oc)^2$ generally → 0 for photons (m_o=0)

$\mathbf{P}_{phot1}\cdot\mathbf{P}_{phot2} = \hbar^2\mathbf{K}_1\cdot\mathbf{K}_2 = (\hbar^2\omega_1\omega_2/c^2)(1-\hat{\mathbf{n}}_1\cdot\hat{\mathbf{n}}_2) = (\hbar^2\omega_1\omega_2/c^2)(1-\cos[\emptyset])$

$\mathbf{P}_{phot}\cdot\mathbf{P}_{mass} = \hbar\mathbf{K}\cdot\mathbf{P} = (\hbar\omega/c)(1,\hat{\mathbf{n}})\cdot(E/c,\mathbf{p}) = (\hbar\omega/c)(E/c - \hat{\mathbf{n}}\cdot\mathbf{p}) = (\hbar\omega E_o/c^2) = (\hbar\omega m_o)$

$\mathbf{P}_{phot} + \mathbf{P}_{mass} = \mathbf{P'}_{phot} + \mathbf{P'}_{mass}$:4-MomentumConservation in Photon·Mass Interaction

===

$\mathbf{P}_{phot} + \mathbf{P}_{mass} - \mathbf{P'}_{phot} = \mathbf{P'}_{mass}$:rearrange

$(\mathbf{P}_{phot} + \mathbf{P}_{mass} - \mathbf{P'}_{phot})^2 = (\mathbf{P'}_{mass})^2$:square to get scalars

$(\mathbf{P}_{phot}\cdot\mathbf{P}_{phot} + 2\mathbf{P}_{phot}\cdot\mathbf{P}_{mass} - 2\mathbf{P}_{phot}\cdot\mathbf{P'}_{phot} + \mathbf{P}_{mass}\cdot\mathbf{P}_{mass} - 2\mathbf{P}_{mass}\cdot\mathbf{P'}_{phot} + \mathbf{P'}_{phot}\cdot\mathbf{P'}_{phot}) = (\mathbf{P'}_{mass})^2$

$(0 + 2\mathbf{P}_{phot}\cdot\mathbf{P}_{mass} - 2\mathbf{P}_{phot}\cdot\mathbf{P'}_{phot} + (m_oc)^2 - 2\mathbf{P}_{mass}\cdot\mathbf{P'}_{phot} + 0) = (m_oc)^2$

$\mathbf{P}_{phot}\cdot\mathbf{P}_{mass} - \mathbf{P}_{mass}\cdot\mathbf{P'}_{phot} = \mathbf{P}_{phot}\cdot\mathbf{P'}_{phot}$

$(\hbar\omega m_o) - (\hbar\omega' m_o) = (\hbar^2\omega\omega'/c^2)(1 - \cos[\emptyset])$

$(\omega-\omega')/(\omega\omega') = (\hbar/m_oc^2)(1 - \cos[\emptyset])$

$(1/\omega' - 1/\omega) = (\hbar/m_oc^2)(1 - \cos[\emptyset])$

$\Delta\lambda\!\!\!^{-} = (\lambda\!\!\!^{-}' - \lambda\!\!\!^{-}) = (\hbar/m_oc)(1 - \cos[\emptyset]) = \lambda\!\!\!^{-}_C(1 - \cos[\emptyset])$

The Compton Effect:Compton Scattering

with

$\lambda\!\!\!^{-}_C = \lambda_C/2\pi = (\hbar/m_oc)$ = Reduced Compton Wavelength

$\lambda_C = (h/m_oc)$ = Compton Wavelength (not a rest-wavelength, but the wavelength of a photon with the energy equivalent to a massive particle of rest-mass m_o)

Calculates the wavelength shift of a photon scattering from an electron (*ignoring spin*)

Proves that light does not have a "wave-only" description, photon 4-Momentum required

$E/\omega = \gamma E_o/\gamma\omega_o = E_o/\omega_o = \hbar$ $\quad \mathbf{K}_{photon} = (\omega/c)(1,\mathbf{n})$ = null $\quad \{\omega\lambda\!\!\!^{-} = v\lambda = c\}$ for photons

$\partial^\mu[R^\nu]=\eta^{\mu\nu}$ Minkowski Metric

4-Gradient
$\partial=(\partial_t/c,-\nabla)$

SpaceTime
$\partial\cdot\mathbf{R}$=4 Dimension

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{U}\cdot\partial[..]$
$d/d\tau[..]$

Wave Velocity
$v_{group}*v_{phase}=c^2$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

ω_o/c^2

$\mathbf{K}\cdot\mathbf{K}=(\omega_o/c)^2$
$=(m_oc/(\hbar))^2$

Energy:Mass
$E = mc^2$

m_o
E_o/c^2

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(1/c\mathbf{T},\hat{\mathbf{n}}/\lambda\!\!\!^{-})$

$\mathbf{P}\cdot\mathbf{P}=(m_oc)^2$
$=(E_o/c)^2$

$\{\omega_o=0\} \leftrightarrow \{\mathbf{K}\cdot\mathbf{U}=0\} \leftrightarrow \{\mathbf{K}$ is null$\}$

E_o/ω_o
$(\hbar)$

Einstein
de Broglie
$\mathbf{P} = \hbar\mathbf{K}$

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

$\{m_o=0\} \leftrightarrow \{\mathbf{P}\cdot\mathbf{U}=0\} \leftrightarrow \{\mathbf{P}$ is null$\}$

$\sum_n[..]$

Conservation of
4-TotalMomentum

4-TotalMomentum
$\mathbf{P}_T=(E_T/c,\mathbf{p}_T)=(H/c,\mathbf{p}_T)$

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A Tensor Study of Physical 4-Vectors

SRQM 4-Vector Study:
The QM Aharonov-Bohm Effect
QM Potential $\Delta\Phi_{pot} = -(q/\hbar)\int_{path}\mathbf{A}\cdot d\mathbf{X}$

4-Vector SRQM Interpretation of QM

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Aharonov-Bohm Effect

The EM 4-VectorPotential gives the Aharonov-Bohm Effect.

$\Phi_{pot} = -(q/\hbar)\mathbf{A}\cdot\mathbf{X} = -\mathbf{K}_{pot}\cdot\mathbf{X}$

or taking the differential...

$d\Phi_{pot} = -(q/\hbar)\mathbf{A}\cdot d\mathbf{X}$

over a path...

$\Delta\Phi_{pot} = \int_{path} d\Phi_{pot}$

$\Delta\Phi_{pot} = -(q/\hbar)\int_{path}\mathbf{A}\cdot d\mathbf{X}$

$\Delta\Phi_{pot} = -(q/\hbar)\int_{path}[(\varphi/c)(cdt) - \mathbf{a}\cdot d\mathbf{x}]$

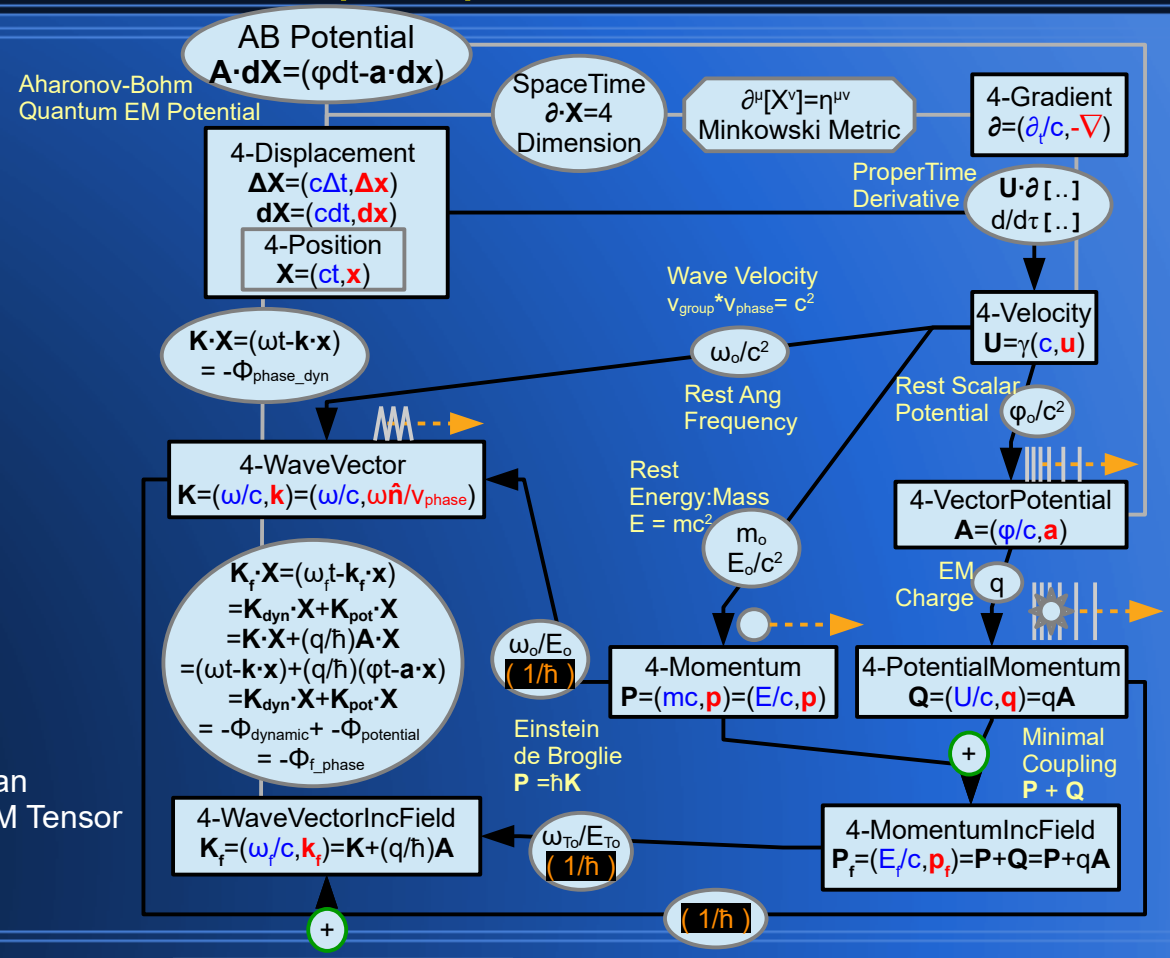
$\Delta\Phi_{pot} = -(q/\hbar)\int_{path}(\varphi dt - \mathbf{a}\cdot d\mathbf{x})$

Note that both the Electric and Magnetic effects come out by using the 4-Vector notation.

Electric AB effect: $\Delta\Phi_{pot\_Elec} = -(q/\hbar)\int_{path}(\varphi dt)$

Magnetic AB effect: $\Delta\Phi_{pot\_Mag} = +(q/\hbar)\int_{path}(\mathbf{a}\cdot d\mathbf{x})$

Proves that the 4-VectorPotential **A** is more fundamental than **e** and **b** fields, which are just components of the Faraday EM Tensor



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SRQM 4-Vector Study:
The QM Josephson Junction Effect = SuperCurrent
EM 4-VectorPotential A = -(ħ/q)∂[ΔΦ$_{pot}$]

4-Vector SRQM Interpretation of QM

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Josephson Effect

The EM 4-VectorPotential gives the Aharonov-Bohm Effect.
Phase $\Phi_{pot} = -(q/\hbar)\mathbf{A}\cdot\mathbf{X} = -\mathbf{K}_{pot}\cdot\mathbf{X}$

Rearrange the equation a bit:
$-(\hbar/q)\Delta\Phi_{pot} = \mathbf{A}\cdot\Delta\mathbf{X}$
$\mathbf{A}\cdot\Delta\mathbf{X} = -(\hbar/q)\Delta\Phi_{pot}$
$d/d\tau[\mathbf{A}\cdot\Delta\mathbf{X}] = d/d\tau[-(\hbar/q)\Delta\Phi_{pot}] = d/d\tau[\mathbf{A}]\cdot\Delta\mathbf{X} + \mathbf{A}\cdot d/d\tau[\Delta\mathbf{X}] = d/d\tau[\mathbf{A}]\cdot\Delta\mathbf{X} + \mathbf{A}\cdot\mathbf{U}$

Assume that ($d/d\tau[\mathbf{A}]\cdot\Delta\mathbf{X} \sim 0$)
$[\mathbf{A}\cdot\mathbf{U}] = d/d\tau[-(\hbar/q)\Delta\Phi_{pot}]$
$[\mathbf{U}\cdot\mathbf{A}] = (\mathbf{U}\cdot\partial)[-(\hbar/q)\Delta\Phi_{pot}]$
$[\mathbf{A}] = -(\hbar/q)(\partial)[\Delta\Phi_{pot}]$
$\mathbf{A} = -(\hbar/q)\partial[\Delta\Phi_{pot}]$
$(\varphi/c,\mathbf{a}) = -(\hbar/q)(\partial_t/c,-\nabla)[\Delta\Phi_{pot}]$

Which explains Josephson Effect criteria :
$\Delta\mathbf{X} \sim 0$: small gap
$d/d\tau[\mathbf{A}] \sim 0$: "critical current" & no voltage
$d/d\tau[\mathbf{A}]\cdot\Delta\mathbf{X}$ ~ orthogonal: ??

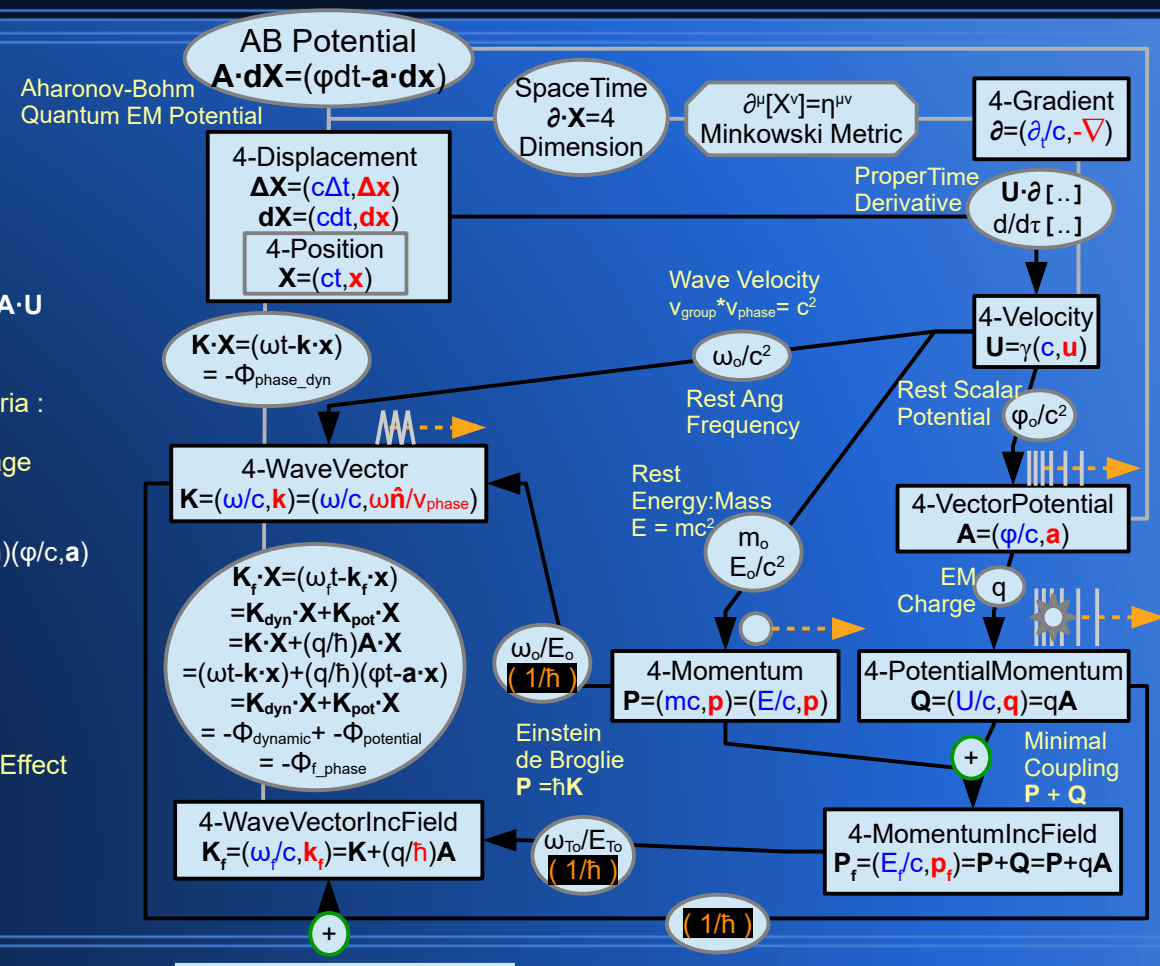
$\mathbf{A} = (\hbar/q)\mathbf{K}$; $\mathbf{K} = (\omega/c,\mathbf{k}) = (q/\hbar)\mathbf{A} = (q/\hbar)(\varphi/c,\mathbf{a})$

Take the temporal part:
EM ScalarPotential $\varphi = -(\hbar/q)(\partial_t)[\Delta\Phi_{pot}]$; $\omega = (q/\hbar)\varphi$

If the charge (q) is a Cooper-electron-pair: { q = -2e }

Voltage $V(t) = \varphi(t) = (\hbar/2e)(\partial/\partial t)[\Delta\Phi_{pot}]$; AngFreq $\omega = -2eV/\hbar$
This is the superconducting phase evolution equation of the Josephson Effect

$(\hbar/2e)$ is defined to be the Magnetic Flux Quantum Φ_o



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SRQM Symmetries:
Hamilton-Jacobi vs Relativistic Action
Josephson vs Aharonov-Bohm
Differential (4-Vector) vs Integral (4-Scalar)

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Differential Formats : 4-Vectors : HJ

Notice the Symmetry:

Integral Formats : 4-Scalars : Action

SR Hamilton-Jacobi Equation

$$\mathbf{P}_T = \mathbf{P}+q\mathbf{A} = \mathbf{P}+\mathbf{Q} = -\partial[\Delta S_{action}] = -\partial[\hbar\Delta\Phi_{phase}]$$
$$= -\partial[\hbar(\Delta\Phi_{phase,dyn} + \Delta\Phi_{phase,potential})]$$

Inverse

SR Action Equation

$$\Delta S_{action} = -\int_{path}\mathbf{P}_T\cdot d\mathbf{X} = -\int_{path}(\mathbf{P}+q\mathbf{A})\cdot d\mathbf{X} = -\int_{path}(\mathbf{P}+\mathbf{Q})\cdot d\mathbf{X}$$
$$= \hbar\Delta\Phi_{phase} = \hbar(\Delta\Phi_{phase,dyn} + \Delta\Phi_{phase,potential})$$

+

4-TotMomentum Conservation
$\mathbf{P}_T = (\mathbf{P}+\mathbf{Q}) = (\mathbf{P}+q\mathbf{A})$
Minimal Coupling
$\mathbf{P} = (\mathbf{P}_T-q\mathbf{A}) = (\mathbf{P}_T-\mathbf{Q})$

Dynamic Part

4-Momentum(free part)

$$\mathbf{P} = -\partial[\Delta S_{act,dynamic}]$$
$$-\partial[\hbar\Delta\Phi_{phase,dynamic}]$$

Inverse

Dynamic Part

Action(free part)

$$\Delta S_{act,dynamic} = \hbar\Delta\Phi_{phase,dynamic}$$
$$= -\int_{path}(\mathbf{P})\cdot d\mathbf{X}$$

+

4-TotMomentum Conservation
$\mathbf{P}_T = (\mathbf{P}+\mathbf{Q}) = (\mathbf{P}+q\mathbf{A})$
Minimal Coupling
$\mathbf{P} = (\mathbf{P}_T-q\mathbf{A}) = (\mathbf{P}_T-\mathbf{Q})$

(ħ)

4-WaveVector

$$\mathbf{K} = -\partial[\Delta S_{act,dync}]/\hbar$$
$$-\partial[\Delta\Phi_{phase,dynamic}]$$

Inverse

SR Phase(free part)

$$\Delta\Phi_{phase,dyn} = \Delta S_{act,dyn}/\hbar$$
$$= -\int_{path}(\mathbf{K})\cdot d\mathbf{X}$$

(ħ)

Potential Part

4-PotentialMomentum

$$\mathbf{Q} = q\mathbf{A} = -\partial[\Delta S_{act,potential}]$$
$$-\partial[\hbar\Delta\Phi_{phase,potential}]$$

Inverse

Potential Part

Action(potential part)

$$\Delta S_{act,pot} = \hbar\Delta\Phi_{phase,potential} =$$
$$-\int_{path}(q\mathbf{A})\cdot d\mathbf{X} = -\int_{path}(\mathbf{Q})\cdot d\mathbf{X}$$

Technically, the standard Josephson Junction uses just the temporal part { $\mathbf{A} = (\varphi/c,\mathbf{a})$ } & Cooper-pair-electrons { $q = -2e$ } giving $V(t) = \varphi = (\hbar/2e)\partial/\partial t[\Delta\Phi_{pot}]$.
There should be a spatial part as well.

q

Josephson Junction Relation

$$\mathbf{A} = -(\hbar/q)\partial[\Delta\Phi_{potential}]$$
$$= -(1/q)\partial[\Delta S_{act,pot}]$$
$$=\mathbf{Q}/q$$

Inverse

(ħ)

Aharonov-Bohm Relation

$$\Delta\Phi_{potential} = -(q/\hbar)\int_{path}\mathbf{A}\cdot d\mathbf{X}$$
$$= -(1/\hbar)\int_{path}\mathbf{Q}\cdot d\mathbf{X}$$
$$=\Delta S_{act,pot}/\hbar$$

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$\mathbf{P}=P^\mu=(mc,\mathbf{p})=(mc,m\mathbf{u})=m_o\mathbf{U}$
$=(E/c,\mathbf{p})=(E_o/c^2)\mathbf{U}$

[kg·m/s] = [J·s/m]

1/ħ

Einstein de Broglie
$\mathbf{P}=\hbar\mathbf{K}$

4-WaveVector
$\mathbf{K}=K^\mu=(\omega/c,\mathbf{k})=(\omega_o/c^2)\mathbf{U}$
$=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})=(1/c\mathcal{T},\hat{\mathbf{n}}/\lambda)$

[1/m]

-i = 1/i

Complex Plane-Waves
$\mathbf{K}=i\partial$

4-Gradient
$\partial=\partial_R=\partial/\partial R_\mu=\partial^\mu=(\partial_t/c,-\nabla)$
$\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$
$=(\partial/c\partial t,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

[1/m]

ħ

i

$S_{action}=-\int[\mathbf{P}_T\cdot d\mathbf{R}]$
$=-\int[\mathbf{P}_T\cdot\mathbf{U}]d\tau$
$=-\int[(H/c,\mathbf{p}_T)\cdot\gamma(c,\mathbf{u})]d\tau$
$=-\int[\gamma(H-\mathbf{p}_T\cdot\mathbf{u}]d\tau$
$=-\int[H_o]d\tau$

Einstein-de Broglie: $\mathbf{P}=\hbar\mathbf{K}\rightarrow\{E=\hbar\omega : \mathbf{p}=\hbar\mathbf{k}\}$
Complex Plane-Wave: $\mathbf{K}=i\partial\rightarrow\{\omega=i\partial_t : \mathbf{k}=-i\nabla\}$
==========
Schrödinger Relations: $\mathbf{P}=i\hbar\partial\rightarrow\{E=i\hbar\partial_t : \mathbf{p}=-\hbar\nabla\}$

Wick Rotation: $\mathbf{R}=-i\mathbf{R}_{im}\rightarrow\{t=-i\tau : \mathbf{r}=-i(i\mathbf{r})\}$
CyclicTemp: $\mathbf{R}_{im}=\hbar\boldsymbol{\Theta}\rightarrow\{\tau=\hbar/k_BT : i\mathbf{r}=\hbar\mathbf{u}/k_BT\}$
==========
TimeTemp: $\mathbf{R}=-i\hbar\boldsymbol{\Theta}\rightarrow\{t=-i\hbar/k_BT : \mathbf{r}=-i\hbar\mathbf{u}/k_BT\}$

Inverses

4-Position
$\mathbf{R}=R^\mu=(ct,\mathbf{r})\in$<Event>
$\rightarrow(ct,x,y,z)$
alt. notation $\mathbf{X}=X^\mu$

[m]

i

Covariant Wick Rotation
$\mathbf{R}=-i\mathbf{R}_{im}$

4-ImaginaryPosition
$\mathbf{R}_{im}=R_{im}{}^\mu=i(ct,\mathbf{r})$
$=(ict,i\mathbf{r})=(c\tau,i\mathbf{r})$

[m]

-i = 1/i

Covariant Euclidean Time ~ Inv Temp
$\mathbf{R}_{im}=\hbar\boldsymbol{\Theta}$

4-ThermalVector
4-InverseTemperatureMomentum
$\boldsymbol{\Theta}=\Theta^\mu=(\theta^0,\boldsymbol{\theta})=(c/k_BT,\mathbf{u}/k_BT)=(\theta_o/c)\mathbf{U}$
$=(1/k_BT)(c,\mathbf{u})=(1/k_B\gamma T)\mathbf{U}=(1/k_BT_o)\mathbf{U}$

[s/kg·m] = [m/J·s]

1/ħ

ħ

Boltzmann Distribution
$\mathbf{P}\cdot\boldsymbol{\Theta}=(E/c,\mathbf{p})\cdot(c/k_BT,\boldsymbol{\theta})$
$=(E/k_BT-\mathbf{p}\cdot\boldsymbol{\theta})=(E_o/k_BT_o)$

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.

$\mathrm{Trace}[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Symmetries: Wave-Particle

4-Vector SRQM Interpretation of QM

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SciRealm@aol.com
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$\mathbf{P}\cdot\mathbf{P} = (m_o c)^2 = (E_o/c)^2$

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$
$\mathbf{P}=-\partial[S_{action,free}]$

$\int \mathbf{P}\cdot d\mathbf{R} = -S_{action,free}$

4-Position
$\mathbf{R}=(ct,\mathbf{r})$

$\mathbf{U}\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$

Rest Mass:Energy Einstein
$E=\gamma E_o=\gamma m_o c^2=mc^2$

m_o
E_o/c^2

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$

$\mathbf{U}\cdot\mathbf{U} = (c)^2$

ω_o/c^2

Wave Velocity
$v_{group}*v_{phase}=c^2$
RestAngFrequency

$\mathbf{P}\cdot\mathbf{U} = E_o$

Einstein
de Broglie
$\mathbf{P} = \hbar\mathbf{K}$

$\mathbf{K}\cdot\mathbf{U} = \omega_o$

$\omega_o/E_o = (1/\hbar)$
or
$E_o/\omega_o = (\hbar)$

$\mathbf{P}\cdot\mathbf{K} = m_o\omega_o$

$\int \mathbf{K}\cdot d\mathbf{R} = -\Phi_{phase,plane}$

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$
$\mathbf{K}=-\partial[\Phi_{phase,plane}]$

$\mathbf{K}\cdot\mathbf{K} = (\omega_o/c)^2$

Treating motion like a particle
Moving particles have a 4-Velocity
4-Momentum is the negative 4-Gradient of the SR Action (S)

$..[-S_{action,free}]$

Hamilton-Jacobi
$\mathbf{P} = -\partial[S_{action,free}]$

4-Gradient
$\partial=(\partial_t/c,-\nabla)\rightarrow(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

d'Alembertian
$\partial\cdot\partial = (\partial_t/c)^2 - \nabla\cdot\nabla = (\partial_\tau/c)^2$

$..[-\Phi_{phase,plane}]$

$\mathbf{K} = -\partial[\Phi_{phase,plane}]$
WaveVector Gradient

SpaceTime
$\partial\cdot\mathbf{R}=4$
Dimension

$\partial[\mathbf{R}]=\eta^{\mu\nu}\rightarrow$Diag[1,-1,-1,-1]
Minkowski Metric

ProperTime
$\mathbf{U}\cdot\partial=d/d\tau=\gamma d/dt$
Derivative

Treating motion like a wave
Moving waves have a 4-Velocity
4-WaveVector is the negative 4-Gradient of the SR Phase (Φ)

See Hamilton-Jacobi Formulation of Mechanics
for info on the Lorentz Scalar Invariant SR Action.
{ $\mathbf{P} = (E/c,\mathbf{p}) = -\partial[S] = (-\partial/c\partial t[S],\nabla[S])$ }
{temporal component} $E = -\partial/\partial t[S] = -\partial_t[S]$
{spatial component} $\mathbf{p} = \nabla[S]$
Note This is the Action (S_{action}) for a free particle.
Generally Action is for the 4-TotalMomentum $\mathbf{P}_T$ of a system.

See SR Wave Definition
for info on the Lorentz Scalar Invariant SR WavePhase.
{ $\mathbf{K} = (\omega/c,\mathbf{k}) = -\partial[\Phi] = (-\partial/c\partial t[\Phi],\nabla[\Phi])$ }
{temporal component} $\omega = -\partial/\partial t[\Phi] = -\partial_t[\Phi]$
{spatial component} $\mathbf{k} = \nabla[\Phi]$
Note This is the Phase (Φ) for a single free plane-wave.
Generally WavePhase is for the 4-TotalWaveVector $\mathbf{K}_T$ of a system.

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Lorentz Scalar

Existing SR Rules
Quantum Principles

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}{}_{\mu} = T$
$\mathbf{V}\cdot\mathbf{V} = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Symmetries:
Relativistic Euler-Lagrange Equation
The Easy Derivation ($\mathbf{U}=(d/d\tau)\mathbf{R}$)→($\partial_R=(d/d\tau)\partial_U$)

4-Vector SRQM Interpretation of QM

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Note Similarity:

4-Velocity is ProperTime Derivative of 4-Position
$\mathbf{U} = (d/d\tau)\mathbf{R}$ [m/s] = [1/s]*[m]

Relativistic Euler-Lagrange Eqn
$\partial_R = (d/d\tau)\partial_U$ [1/m] = [1/s]*[s/m]

The differential form just inverses the dimensional units, so the placement of the **R** and **U** switch.

That is it: so simple! Much, much easier than how I was taught in Grad School.

To complete the process and create the Equations of Motion, one just applies the base form to a Lagrangian.

This can be:
a classical Lagrangian
a relativistic Lagrangian
a Lorentz scalar Lagrangian
a quantum Lagrangian

Relativistic Particle Dynamics Eqn (4-Vector)
$\mathbf{U} = (d/d\tau)\mathbf{R}$
Classical limit, spatial component (3-vector)
$\mathbf{u} = (d/dt)\mathbf{r}$

[m]

4-Position
$\mathbf{R}=(ct,\mathbf{r})=(\mathbf{R}\cdot\partial_U)\mathbf{U}$
$=R^\alpha=(\int d\tau)\mathbf{U}$

$\mathbf{U}\cdot\partial_R$ [..]
$\gamma d/dt$ [..]
$d/d\tau$ [..]

[1/s]

[m/s]

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})=(\mathbf{U}\cdot\partial_R)\mathbf{R}$
$=U^\alpha=(d/d\tau)\mathbf{R}$

Natural 4-Vector
= 4D (1,0)-Tensor

$\partial_R\cdot\mathbf{R}=4$
SpaceTime Dimension

[1]

$\partial_U[\mathbf{U}]=\eta^{\alpha\beta}\rightarrow$Diag[1,-1,-1,-1]
Minkowski Metric

[1]

[1/s]

[s]

Proper Time
$\mathbf{R}\cdot\partial_U=\int d\tau$
Integral

Proper Time
$\mathbf{U}\cdot\partial_R=d/d\tau=\gamma d/dt$
Derivative

$\mathbf{U}\cdot\partial_R = d/d\tau = \gamma d/dt$
Interestingly, this has it's own similar inverse relations.
$d/d\tau = \gamma d/dt$
$dt = \gamma d\tau$

$\partial_U\cdot\mathbf{U}=4$
SpaceTime Dimension

[1]

[1]

$\partial_R[\mathbf{R}]=\eta^{\alpha\beta}\rightarrow$Diag[1,-1,-1,-1]
Minkowski Metric

4

4-VelocityGradient
$\partial_U{}^\beta=\partial_U=\partial/\partial U_\beta=(\partial_{U_t}/c,-\nabla_U)$
$\rightarrow(\partial/\partial\gamma c,-\partial/\partial\gamma u_x,-\partial/\partial\gamma u_y,-\partial/\partial\gamma u_z)$

$\mathbf{U}\cdot\partial_R$ [..]
$\gamma d/dt$ [..]
$d/d\tau$ [..]

[1/s]

4-PositionGradient:4-Gradient
$\partial_R{}^\beta=\partial_R=\partial/\partial R_\beta=\partial=(\partial_t/c,-\nabla)$
$\rightarrow(\partial/\partial ct,-\partial/\partial x,-\partial/\partial y,-\partial/\partial z)$

4-Vector
{ Index-raised-from One-Form }
= 4D (1,0)-Tensor

[s/m]

[1/m]

$\eta^{\alpha\beta}$
Raise index

$\eta^{\alpha\beta}$
Raise index

Relativistic Euler-Lagrange Eqn
$\partial_R = (d/d\tau)\partial_U$
$\partial/\partial\underline{\mathbf{R}} = (d/d\tau)\partial/\partial\underline{\mathbf{U}}$
$\partial[L]/\partial\underline{\mathbf{R}} = (d/d\tau)\partial[L]/\partial\underline{\mathbf{U}}$
Classical limit, spatial component
$\partial[L]/\partial\mathbf{r} = (d/dt)\partial[L]/\partial\mathbf{u}$
$\partial[L]/\partial\mathbf{x} = (d/dt)\partial[L]/\partial\mathbf{u}$

VelocityGradient One-Form
$\partial_{U^\alpha}=\partial/\partial U^\alpha=(\partial_{U_t}/c,\nabla_U)$
$\rightarrow(\partial/\partial\gamma c,\partial/\partial\gamma u_x,\partial/\partial\gamma u_y,\partial/\partial\gamma u_z)$

PositionGradient One-Form
Gradient One-Form
$\partial_{R^\alpha}=\partial/\partial R^\alpha=(\partial_t/c,\nabla)$
$\rightarrow(\partial/\partial ct,\partial/\partial x,\partial/\partial y,\partial/\partial z)$

One-Form
= 4D (0,1)-Tensor

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

$\mathbf{P_T} = -\partial_U[L_o] = -\partial_U[\gamma L] = -\partial_U[-(\mathbf{P_T}\cdot\mathbf{U})] = \mathbf{P_T}$
$(E_T/c,\mathbf{p_T}) = -(\partial_{\gamma c},-\partial_{\gamma u})[\gamma L] = (-\partial_c,\partial_u)[L]$
$\mathbf{p_T} = \partial_u[L] = (\partial/\partial\mathbf{u})[L]$

$\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^\mu{}_\mu = T$
$\mathbf{V}\cdot\mathbf{V} = V^\mu\eta_{\mu\nu}V^\nu = [(v^0)^2 - \mathbf{v}\cdot\mathbf{v}] = (v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

4

A Tensor Study of Physical 4-Vectors

4-Vector SRQM Interpretation of QM

SRQM Symmetries:
Lorentz Transform Connection Map – Trace Identification
CPT, Big-Bang, (Matter↔AntiMatter), Arrow(s)-of-Time

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All Lorentz Transforms have Tensor Invariants: Determinant = ±1 and InnerProduct = 4.
However, one can use the Tensor Invariant Trace to Identify CPT Symmetry & AntiMatter

Tr[NM-Rotate] = {0...+4} Tr[NM-Identity] = +4 Tr[NM-Boost] = {+4...+∞}
Tr[AM-Rotate] = {0.....-4} Tr[AM-Identity] = -4 Tr[AM-Boost] = {-4.....-∞}

Discrete NormalMatter (NM) Lorentz Transform Type

Minkowski-Identity : AM-Flip-txyz=AM-ComboPT

Flip-t=TimeReversal, Flip-x, Flip-y, Flip-z
AM-Flip-xyz=AM-ParityInverse

Flip-xy=Rotate-xy(π), Flip-xz=Rotate-xz(π), Flip-yz=Rotate-yz(π)

AM-Flip-xy=AM-Rotate-xy(π), AM-Flip-xz=AM-Rotate-xz(π), AM-Flip-yz=AM-Rotate-yz(π)

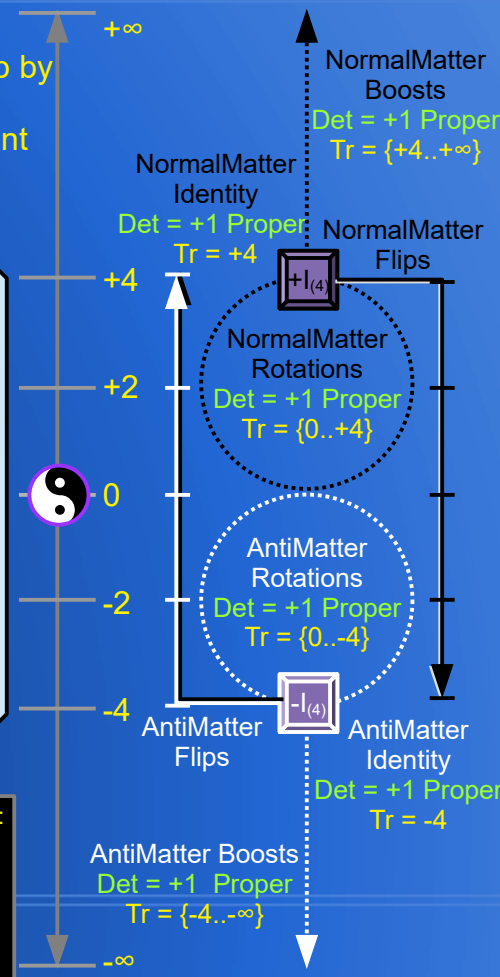
Flip-xyz=ParityInverse
AM-Flip-t=AM-TimeReversal, AM-Flip-x, AM-Flip-y, AM-Flip-z

AM-Minkowski-Identity : Flip-txyz=ComboPT

Discrete AntiMatter (AM) Lorentz TransformType

Trace : Determinant
Tr = +4 : Det = +1 Proper
Tr = +2 : Det = -1 Improper
Tr = 0 : Det = +1 Proper
Tr = 0 : Det = +1 Proper
Tr = -2 : Det = -1 Improper
Tr = -4 : Det = +1 Proper
Trace : Determinant

Line up by Trace Invariant values



SR:Lorentz Transform

$\partial_\nu[R^{\mu'}] = \partial R^{\mu'}/\partial R^\nu = \Lambda^{\mu'}{}_\nu$

$\Lambda^\mu{}_\nu = (\Lambda^{-1})_\nu{}^\mu : \Lambda^\mu{}_\alpha\Lambda^\alpha{}_\nu = \eta^\mu{}_\nu = \delta^\mu{}_\nu$

$\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta = \eta_{\alpha\beta}$

$\mathrm{Det}[\Lambda^\mu{}_\nu]=\pm1$ $\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4=\Lambda^\mu{}_\nu\Lambda_\mu{}^\nu$

$\mathrm{Tr}[\Lambda^\mu{}_\nu]=\{-\infty..+\infty\}$ =Lorentz Transform Type

Two interesting properties of (1,1)-Tensors, of which the Lorentz Transform is an example:
Trace = Sum (Σ) of EigenValues : Determinant = Product (Π) of EigenValues
As 4D Tensors, each Lorentz Transform has 4 EigenValues (EV's).
Create an Anti-Transform which has all EigenValue Tensor Invariants negated.
$\Sigma[-(EV's)] = -\Sigma[EV's]$: The Anti-Transform has negative Trace of the Transform.
$\Pi[-(EV's)] = (-1)^4\,\Pi[EV's] = \Pi[EV's]$: The Anti-Transform has equal Determinant.

The Trace Invariant identifies a "Dual" Negative-Side for all Lorentz Transforms.

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM 4-Vector Study: Einstein-de Broglie The (ħ) Connection

4-Vector SRQM Interpretation of QM

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The ħ Connection

$\mathbf{P} = \hbar\mathbf{K}$: Basic Einstein-de Broglie

$\mathbf{P}+\mathbf{Q} = \mathbf{P} + \mathbf{Q}$

$\mathbf{P}+\mathbf{Q} = \hbar\mathbf{K}_{dyn}+\hbar\mathbf{K}_{pot}$

$\mathbf{P}+\mathbf{Q} = \hbar(\mathbf{K}_{dyn}+\mathbf{K}_{pot})$

Sum over n particles: $\mathbf{P}_T = \Sigma_n(\mathbf{P}+\mathbf{Q}), \mathbf{K}_T = \Sigma_n(\mathbf{K}_{dyn}+\mathbf{K}_{pot})$

$\mathbf{P}_T = \hbar\mathbf{K}_T$

$\mathbf{P}_T\cdot\mathbf{X} = \hbar\mathbf{K}_T\cdot\mathbf{X}$

$(\mathbf{P}_T\cdot\mathbf{X}) = \hbar(\mathbf{K}_T\cdot\mathbf{X})$

$-S_{action} = -\hbar\Phi_{phase}$

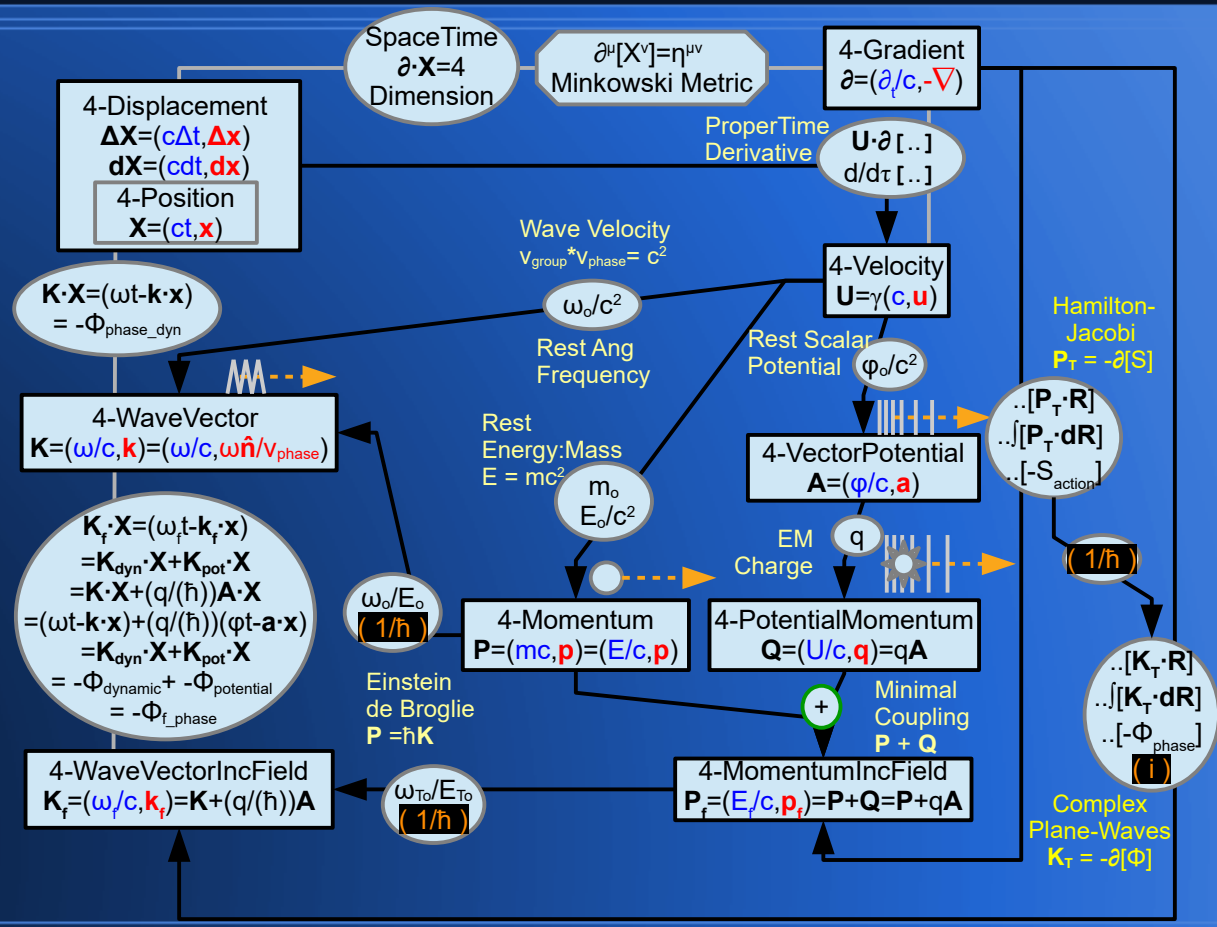
$S_{action} = \hbar\Phi_{phase}$

$-\partial[S_{action}] = -\hbar\partial[\Phi_{phase}]$

$\mathbf{P}_T = \hbar\mathbf{K}_T$

{SR Hamilton-Jacobi} = ħ{QM Complex Plane-Waves}

The SR Hamilton-Jacobi Equation, and the QM idea of Complex Plane-Waves, are related by a simple constant (ħ) relation.



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(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM 4-Vector Study: Dimensionless Physical Objects

4-Vector SRQM Interpretation of QM

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Dimensionless Physical Objects

There are a number of unit-dimensionless physical objects in SR that can be constructed from Physical 4-Vectors. There are examples with 4-Scalars, 4-Vectors, and 4-Tensors.

$\partial\cdot\mathbf{X}=4$: SpaceTime Dimension
$\partial^\mu[X^\nu]=\eta^{\mu\nu}$: The SR Minkowski Metric
$\partial^{\mu'}[X_\nu]=\Lambda^{\mu'}{}_\nu$: Lorentz Transformation Tensors
$d\mathbf{X}\cdot\partial$: Calculus Total Derivative Chain Rule
$[\partial^\mu,X^\nu] = \partial^\mu X^\nu - X^\nu\partial^\mu = \eta^{\mu\nu}$: SRQM Non-Zero Commutation
$V^{\mu\nu}$ & $H^{\mu\nu}$: Projection Tensors; (V)ertical $V^{\mu\nu} = \bar{T}^\mu\bar{T}^\nu$ & (H)orizontal $H^{\mu\nu} = \eta^{\mu\nu}-\bar{T}^\mu\bar{T}^\nu$

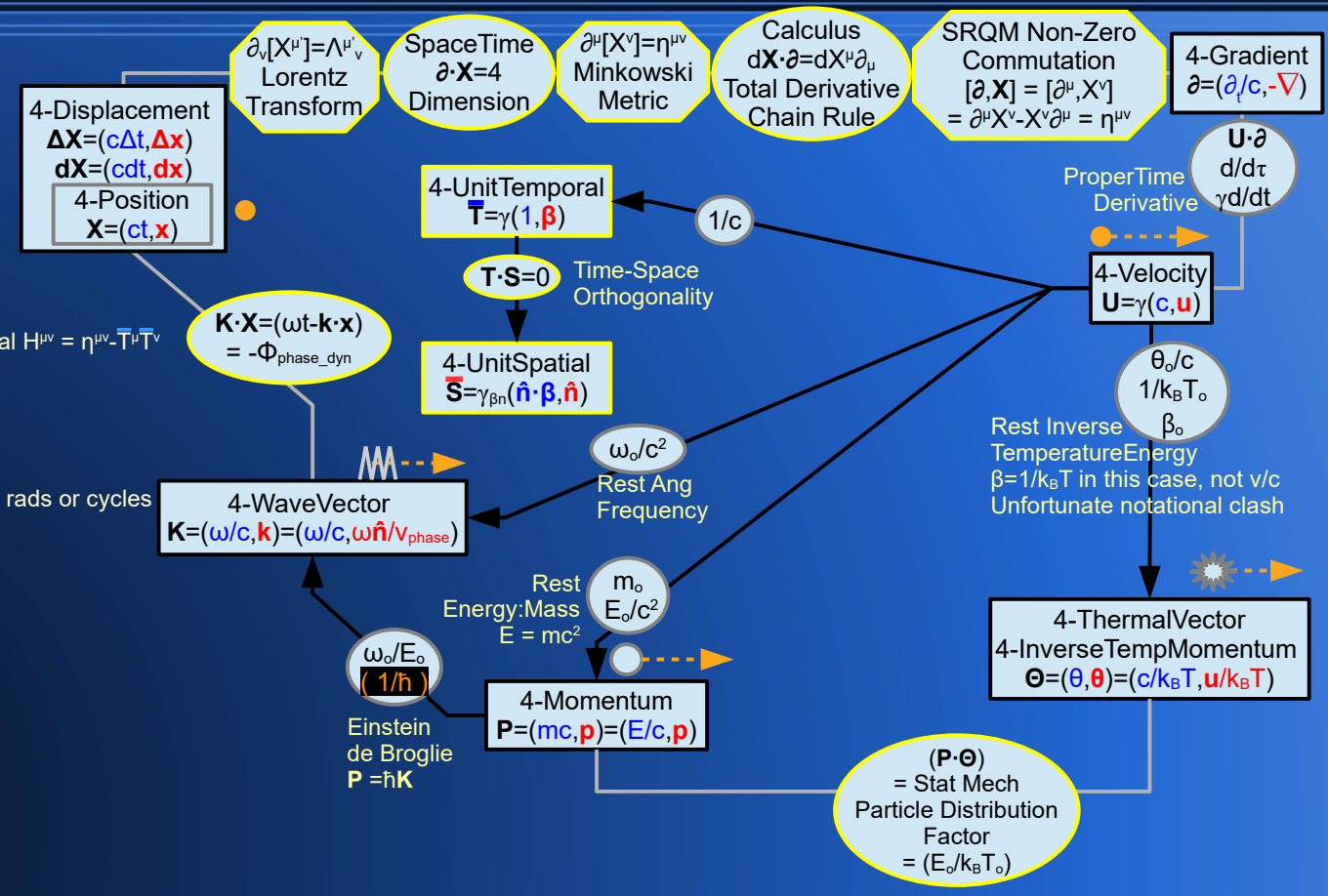
$\bar{\mathbf{T}}\cdot\bar{\mathbf{T}}$= +1: Lorentz Scalar Magnitude^2 of the 4-UnitTemporal
$\bar{\mathbf{T}}\cdot\bar{\mathbf{S}}$= 0: Lorentz Scalar of 4-UnitTemporal with 4-UnitSpatial
$\bar{\mathbf{S}}\cdot\bar{\mathbf{S}}$= -1: Lorentz Scalar Magnitude^2 of the 4-UnitSpatial

$\mathbf{K}\cdot\mathbf{X}=(\omega t-\mathbf{k}\cdot\mathbf{x}) = -\Phi_{phase\_dyn}$: Phase of an SR Wave, technically in rads or cycles used in SRQM wave functions $\psi=a^*e^{-(\mathbf{K}\cdot\mathbf{X})}$

$(\mathbf{P}\cdot\mathbf{\Theta}) = (E_o/k_BT_o)$: 4-Momentum with 4-InvThermalMomentum used in statistical mechanics particle distributions
$F(\text{state}) \sim e^{-(\mathbf{P}\cdot\mathbf{\Theta})} = e^{-(E_o/k_BT_o)}$

$\alpha = (1/4\pi\varepsilon_o)(e^2/\hbar c) = (\mu_o/4\pi)(ce^2/\hbar)$: Fine Structure Constant constructed from Lorentz 4-Scalars, which are themselves constructed from 4-Vectors via the Lorentz Scalar Product.
ex. $\hbar=(\mathbf{P}\cdot\mathbf{X})/(\mathbf{K}\cdot\mathbf{X})$; $q=(\mathbf{Q}\cdot\mathbf{X})/(\mathbf{A}\cdot\mathbf{X}) \rightarrow e$ for electron; $c=(\mathbf{T}\cdot\mathbf{U})$
$\mu_o=\{(\partial\cdot\partial)[\mathbf{A}]\cdot\mathbf{X}\}/(\mathbf{J}\cdot\mathbf{X})$ when $(\partial\cdot\mathbf{A})=0$

$\{\gamma^\mu\}$: Dirac Gamma Matrix ("4-Vector") {4 component}
$\{\sigma^\mu\}$: Pauli Spin Matrix ("4-Vector") {2 component}
Components are matrices of numbers, not just numbers



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= Lorentz Scalar Invariant

SRQM: QM Axioms Unnecessary

QM Principles emerge from SR

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SciRealm@aol.com
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QM is derivable from SR plus a few empirical facts – the "QM Axioms" aren't necessary
These properties are either empirically measured or are emergent from SR properties...

3 "QM Axioms" are really just empirical constant relations between purely SR 4-Vectors:
- Particle-Wave Duality [(**P**) = ħ(**K**)]
- Unitary Evolution [∂ = (-i)**K**]
- Operator Formalism [(∂) = -i**K**]

2 "QM Axioms" are just the result of the Klein-Gordon Equation being a linear wave PDE:
- Hilbert Space Representation (<bra|,|ket>, wavefunctions, etc.) & The Principle of Superposition

3 "QM Axioms" are a property of the Minkowski Metric and the empirical fact of Operator Formalism
- The Canonical Commutation Relation
- The Heisenberg Uncertainty Principle (time-like-separated measurement exchange)
- The Pauli Exclusion Principle (space-like-separated particle exchange)

1 "QM Axiom" only holds in the NRQM case
- The Born QM Probability Interpretation – Not applicable to RQM, use Conservation of Worldlines instead

1 "QM Axiom" is really just another level of limiting cases, just like SR → CM in limit of low velocity
- The QM Correspondence Principle (QM → CM in limit of $\{\nabla^2[\phi] << (\nabla[\phi])^2\}$)

SRQM Interpretation: Relational QM & EPR

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The SRQM interpretation fits fairly well with Carlo Rovelli's Relational QM interpretation:

Relational QM treats the state of a quantum system as being observer-dependent, that is, the QM State is the relation between the observer and the system. This is inspired by the key idea behind Special Relativity, that the details of an observation depend on the reference frame of the observer.

All systems are quantum systems: no artificial Copenhagen dichotomy between classical/macroscopic/conscious objects and quantum objects.

The QM States reflect the observers' information about a quantum system.
Wave function "collapse" is informational – not physical. A particle always "knows" its complete properties – it "is" its properties. An observer has at best only partial information about the particle's properties.

No Spooky Action at a Distance. When a measurement is done locally on an entangled system, it is only the partial information about the distant entangled state that "changes/becomes-available-instantaneously". There is no superluminal signal. Measuring/physically-changing the local particle does not physically change the distant particle.

ex. Place two identical-except-for-color marbles into a box, close lid, and shake. Without looking, pick one marble at random and place it into another box. Send that box very far away. After receiving signal of the far box arrival at a distant point, open the near box and look at the marble. You now instantaneously know the far marble's color as well. The information did not come by signal. You already had the possibilities (partial knowledge). Looking at the near marble color simply reduced the partial knowledge of both marble's color to complete knowledge of both marbles' color. No signal was required, superluminal or otherwise.

ex. The quantum version of the same experiment uses the spin of entangled particles. When measured on the same axis, one will always be spin-up, the other will be spin-down. It is conceptually analogous. Entanglement is only about correlations of system that interacted in the past and are determined by conservation laws.

SRQM Interpretation:
Interpretation of EPR-Bell Experiment

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Einstein and Bohr can both be "right" about EPR:
Per Einstein: The QM State measured is not a "complete" description, just one observer's point-of-view.
Per Bohr: The QM State measured is a "complete" description, it's all that a single observer can get.

The point is that many observers can all see the "same" system, but see different facets of it. But a single measurement is the maximal information that a single observer can get without re-interacting with the system, which of course changes the system in general. Remember, the Heisenberg Uncertainty comes from non-zero commutation properties which *require separate measurement arrangements*. The properties of a particle are always there. Properties define particles. We as observers simply have only partial information about them.

Relativistic QM, being derived from SR, should be local – The low-velocity limit to QM may give unexpected anomalous results if taken out of context, or out of the applicable validity range, such as with velocity addition $v_3 = v_1+v_2$, where the correct formula should be the relativistic velocity composition $v_3 = (v_1+v_2)/[1+v_1v_2/c^2]$

These ideas lead to the conclusion that the wavefunction is just one observer's state of information about a physical system, not the state of the physical system itself. The "collapse" of the wavefunction is simply the change in an observer's information about a system brought about by a measurement or, in the case of EPR, an inference about the physical state.

EPR doesn't break Heisenberg because measurements are made on different particles. The happy fact is that those particles interacted and became correlated in the causal past. The EPR-Bell experiments prove that it is possible to maintain those correlations over long distances. It does *not* prove superluminal (FTL) signaling

SRQM Interpretation: Range-of-Validity Facts & Fallacies

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

We should not be surprised by the "quantum" probabilities being correct instead of "classical" in the EPR and Bell Inequalities experiments.
Classical thinking (in both CM and QM) has a number of fallacies when it is mistakenly applied outside of its range-of-validity.

Examples

*Classical Physics as the limit of $\hbar \rightarrow 0$ {Fallacy}:
$\hbar$ is a Lorentz Scalar Invariant and Fundamental Physical Constant. It never becomes 0. {Fact}

*The classical commutator being zero $[p^k,x^j] = 0$ {Fallacy}:
$[P^\mu,X^\nu] = i\hbar\eta^{\mu\nu}$; $[p^k,x^l] = -i\hbar\delta^{kj}$; $[p^0,x^0] = [E/c,ct] = [E,t] = i\hbar(1)$; Again, it never becomes 0, it's just really small. {Fact}

*Using Maxwell-Boltzmann (distinguishable) statistics for counting probabilities of (indistinguishable) quantum states {Fallacy}:
One must use Fermi-Dirac (FD) statistics for Fermions:Spin=(n+1/2); Bose-Einstein (BE) statistics for Bosons:Spin=(n) {Fact}

*Using sums of classical probabilities on quantum states {Fallacy}:
Must use sums of quantum probability-amplitudes {Fact}

*Ignoring phase cross-terms and interference effects in calculations {Fallacy}:
Quantum systems and entanglement require phase cross-terms {Fact}

*Assuming that one can simultaneously "measure" non-commuting properties at a single spacetime event {Fallacy}:
Particle properties always exist. However, non-commuting ones require separate measurement arrangements to get information about the properties.
The required measurement arrangements on a single particle/worldline are at best sequential events, where the temporal order plays a role; {Fact}
However, EPR allows one to "infer (not measure)" the other property of a particle by the separate measurement of an entangled partner. {Fact}
This does not break Heisenberg Uncertainty, which is about the order of operations (measurement events) on a single worldline. {Fact}
In the entangled case, both/all of the entangled partners share common past-causal entanglement events, typically due to a conservation law. {Fact}
Information is not transmitted at FTL. The particles simply carried their normal respective "correlated" properties (no hidden variables) with them. {Fact}

*Assuming that QM is a generalization of CM, or that classical probabilities apply to QM {Fallacy}:
CM is a limiting-case of QM for when changes in a system by a few quanta have a negligible effect on the whole/overall system. {Fact}

SRQM Interpretation: Quantum Information

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{from Wikipedia}

No-Communication Theorem/No-Signaling:
A no-go theorem from quantum information theory which states that, during measurement of an entangled quantum state, it is not possible for one observer, by making a measurement of a subsystem of the total state, to communicate information to another observer. The theorem shows that quantum correlations do not lead to what could be referred to as "spooky communication at a distance". SRQM: There is no FTL signaling/communication.

No-Teleportation Theorem:
The no-teleportation theorem stems from the Heisenberg uncertainty principle and the EPR paradox: although a qubit |ψ> can be imagined to be a specific direction on the Bloch sphere, that direction cannot be measured precisely, for the general case |ψ>. The no-teleportation theorem is implied by the no-cloning theorem.
SRQM: Ket states are informational, not physical.

No-Cloning Theorem:
In physics, the no-cloning theorem states that it is impossible to create an identical copy of an arbitrary unknown quantum state. This no-go theorem of quantum mechanics proves the impossibility of a simple perfect non-disturbing measurement scheme. The no-cloning theorem is normally stated and proven for pure states; the no-broadcast theorem generalizes this result to mixed states. SRQM: Measurements are arrangements of particles that interact with a subject particle.

No-Broadcast Theorem:
Since quantum states cannot be copied in general, they cannot be broadcast. Here, the word "broadcast" is used in the sense of conveying the state to two or more recipients. For multiple recipients to each receive the state, there must be, in some sense, a way of duplicating the state. The no-broadcast theorem generalizes the no-cloning theorem for mixed states. The no-cloning theorem says that it is impossible to create two copies of an unknown state given a single copy of the state.
SRQM: Conservation of worldlines.

No-Deleting Theorem:
In physics, the no-deleting theorem of quantum information theory is a no-go theorem which states that, in general, given two copies of some arbitrary quantum state, it is impossible to delete one of the copies. It is a time-reversed dual to the no-cloning theorem, which states that arbitrary states cannot be copied.
SRQM: Conservation of worldlines.

No-Hiding Theorem:
the no-hiding theorem is the ultimate proof of the conservation of quantum information. The importance of the no-hiding theorem is that it proves the conservation of wave function in quantum theory.
SRQM: Conservation of worldlines. RQM wavefunctions are Lorentz 4-Scalars (spin=0), 4-Spinors (spin=1/2), 4-Vectors (spin=1), all of which are Lorentz Invariant.

SRQM Interpretation: Quantum Information

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Classical thinking (in both CM and QM) has a number of fallacies when it is mistakenly applied outside of its range-of-validity.
{from Wikipedia}
Quantum information (qubits) differs strongly from classical information, epitomized by the bit, in many striking and unfamiliar ways. Among these are the following:

A unit of quantum information is the qubit. Unlike classical digital states (which are discrete), a qubit is continuous-valued, describable by a direction on the Bloch sphere. Despite being continuously valued in this way, a qubit is the smallest possible unit of quantum information, as despite the qubit state being continuously-valued, it is impossible to measure the value precisely.

A qubit cannot be (wholly) converted into classical bits; that is, it cannot be "read". This is the no-teleportation theorem.

Despite the awkwardly-named no-teleportation theorem, qubits can be moved from one physical particle to another, by means of quantum teleportation. That is, qubits can be transported, independently of the underlying physical particle. SRQM: Ket states are informational, not physical.

An arbitrary qubit can neither be copied, nor destroyed. This is the content of the no-cloning theorem and the no-deleting theorem. SRQM: Conservation of worldlines.

Although a single qubit can be transported from place to place (e.g. via quantum teleportation), it cannot be delivered to multiple recipients; this is the no-broadcast theorem, and is essentially implied by the no-cloning theorem. SRQM: Conservation of worldlines.

Qubits can be changed, by applying linear transformations or quantum gates to them, to alter their state. While classical gates correspond to the familiar operations of Boolean logic, quantum gates are physical unitary operators that in the case of qubits correspond to rotations of the Bloch sphere.

Due to the volatility of quantum systems and the impossibility of copying states, the storing of quantum information is much more difficult than storing classical information. Nevertheless, with the use of quantum error correction quantum information can still be reliably stored in principle. The existence of quantum error correcting codes has also led to the possibility of fault tolerant quantum computation.

Classical bits can be encoded into and subsequently retrieved from configurations of qubits, through the use of quantum gates. By itself, a single qubit can convey no more than one bit of accessible classical information about its preparation. This is Holevo's theorem. However, in superdense coding a sender, by acting on one of two entangled qubits, can convey two bits of accessible information about their joint state to a receiver.

Quantum information can be moved about, in a quantum channel, analogous to the concept of a classical communications channel. Quantum messages have a finite size, measured in qubits; quantum channels have a finite channel capacity, measured in qubits per second.

Minkowski still applies in local GR

QM is a local phenomenon

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

The QM Schrodinger Equation is not fundamental. It is just the low-energy limiting-case of the RQM Klein-Gordon Equation. All of the standard QM Axioms are shown to be empirically measured constants or emergent properties of SR. It is a bad approach to start with NRQM as an axiomatic starting point and try to generalize it to RQM, in the same way that one cannot start with CM and derive SR. Since QM *can* be derived from SR, this partially explains the difficulty of uniting QM with GR:
QM is not a "separate formalism" outside of SR that can be used to "quantize" just anything...

Strictly speaking, the use of the Minkowski space to describe physical systems over finite distances applies only in the SR limit of systems without significant gravitation. In the case of significant gravitation, SpaceTime becomes curved and one must abandon SR in favor of the full theory of GR.

Nevertheless, even in such cases, based on the GR Equivalence Principle, Minkowski space is still a good description in a local region surrounding any point (barring gravitational singularities). More abstractly, we say that in the presence of gravity, SpaceTime is described by a curved 4-dimensional manifold for which the tangent space to any point is a 4-dimensional Minkowski Space. Thus, the structure of Minkowski Space is still essential in the description of GR.

So, even in GR, at the local level things are considered to be Minkowskian:
i.e. SR → QM "lives inside the surface" of this local SpaceTime, GR curves the surface.

SRQM Interpretation: Main Result

QM is derivable from SR!

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

- Hopefully, this interpretation will shed light on why Quantum Gravity has been so elusive. Basically, QM rules of "quantization" don't apply to GR. They are a manifestation-of/derivation-from SR. Relativity *is* the "Theory of Measurement" that QM has been looking for.

  This would explain why no one has been able to produce a successful theory of "Quantum Gravity", and why there have been no violations of Lorentz Invariance, CPT, or the Equivalence Principle.

  If quantum effects "live" in Minkowski SpaceTime with SR, then GR curvature effects are at a level above the RQM description, and two levels above standard QM. SR+QM are "in" SpaceTime, GR is the "shape" of SpaceTime…

  Thus, this SRQM Treatise explains the following:

- Why GR works so well in it's realm of applicability {large scale systems}.
- Why QM works so well in it's realm of applicability {micro scale systems and certain macroscopic systems}.
  i.e. The tangent space to any point in GR curvature is locally Minkowskian, and thus QM is typically found in small local volumes...
- Why RQM explains more stuff than QM without SR {because QM is just an approximation: the low-velocity limiting-case of RQM}.
- Why all attempts to "quantize gravity" have failed {essentially, everyone has been trying to put the cart (QM) before the horse (GR)}.
- Why all attempts to modify GR keep conflicting with experimental data {because GR is apparently fundamental – passed all tests to-date}.
- Why QM works perfectly well with SR as RQM but not with GR {because QM is derivable from SR, hence a manifestation of SR rules}.
- How Minkowski Space, 4-Vectors, and Lorentz Invariants play vital roles in RQM, and give the SRQM Interpretation of Quantum Mechanics.

SRQM: Special Relativistic Quantum Measurement, Special Relativistic Quantum Mechanics

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^{\mu}{}_{\nu}$ or $T_{\mu}{}^{\nu}$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^{\mu} = \mathbf{V} = (v^0, \mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_{\mu} = (v_0, -\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
Special Relativity → Quantum Mechanics
RoadMap of SR→QM

4-Vector SRQM Interpretation of QM

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4-Gradient=***Alteration*** of SR <Events>
SR SpaceTime "Flat" Minkowski 4D Metric
SR SpaceTime Dimension=4
SR Lorentz Transforms
SR Action → 4-Momentum
SR Phase → 4-WaveVector
SR ProperTime Derivative
SR & QM Invariant Waves

START HERE: 4-Position=***Location*** of SR <Events> in SpaceTime
follow the arrows

$\partial^\mu[R^\nu]=\eta^{\mu\nu}$ Minkowski Metric

$\partial_\nu[R^{\mu'}]=\Lambda^{\mu'}{}_\nu$ Lorentz Transform

$\Lambda_{\mu\nu}\Lambda^{\mu\nu}=4$

$Tr[\eta^{\mu\nu}]=4$

SpaceTime $\partial\cdot R=\partial_\mu R^\mu=4$ Dimension

4-Position R^μ
$R=(ct,r)\in$<Event>

$U\cdot\partial[..]$
$\gamma d/dt[..]$
$d/d\tau[..]$
ProperTime Derivative

4-Velocity=***Motion*** of SR <Events> in SpaceTime as both particles & waves

$R\cdot R=(ct)^2-r\cdot r$
$=(c\tau)^2=(i|r_o|)^2$

$R\cdot U=c^2\tau$

$\partial\cdot\partial=(\partial_t/c)^2-\nabla\cdot\nabla$
$=-(m_oc/\hbar)^2=-(\omega_o/c)^2$
$=(\partial_\tau/c)^2$

4-Gradient ∂^μ
$\partial=\partial/\partial R_\mu$
$=(\partial_t/c,-\nabla)=-iK$

ProperTime
$U\cdot\partial=d/d\tau=\gamma d/dt$
Derivative

4-Velocity U^μ
$U=\gamma(c,u)=dR/d\tau$

Matter Wave
$v_{group}*v_{phase}=c^2$
Rest Angular Frequency ω_o

$-K\cdot R=\Phi_{phase,free}$
SR Phase

$-P\cdot R=S_{action,free}$
SR Action

Phase & Action Lorentz Scalars

$U\cdot U=\gamma^2(c^2-u\cdot u)$
$=(c)^2$

$K\cdot U=\omega_o$

ω_o/c^2

SR d'Alembertian & Klein-Gordon Relativistic Quantum Wave Relation
Schrödinger QWE is {|v|<<c} limit of KG QWE
[SR → QM]

$-\partial[\Phi_{phase,free}]=K$

$-\partial[S_{action,free}]=P$

4-WaveVector Complex Plane-Waves
$K_T=-\partial[\Phi]$
$K=i\partial$

Hamilton-Jacobi
$P_T=-\partial[S]$

(-i)

Einstein
$E=mc^2=\gamma m_oc^2=\gamma E_o$
Rest Mass m_o : Rest Energy E_o

$P\cdot U=E_o$

m_o
E_o/c^2

4-WaveVector=***Substantiation*** of SR Wave <Events>
oscillations proportional to mass:energy & 3-momentum

4-WaveVector K^μ
$K=(\omega/c,k)=(\omega/c,\omega\hat{n}/v_{phase})$
$=(1/c\mathcal{T},\hat{n}/\lambda)=(\omega_o/c^2)U=P/\hbar$

4-Momentum P^μ
$P=(mc,p)=(E/c,p)=m_oU$

4-Momentum=***Substantiation*** of SR Particle <Events>
mass:energy & 3-momentum

Einstein, de Broglie Relation
Dirac:Planck Constant $\hbar=h/2\pi$
$P=\hbar K$

$P\cdot K=m_o\omega_o$

$K\cdot K=(\omega/c)^2-k\cdot k$
$=(m_oc/\hbar)^2=(\omega_o/c)^2=(1/c\mathcal{T}_o)^2$

$P\cdot P=(E/c)^2-p\cdot p$
$=(m_oc)^2=(E_o/c)^2$

ω_o/E_o
($1/\hbar$)

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SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
(QM Principles)

$Trace[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Chart:
Special Relativity → Quantum Mechanics
SR→QM Interpretation Simplified

4-Vector SRQM Interpretation of QM

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SRQM: The [SR→QM] Interpretation of Quantum Mechanics

Special Relativity (SR) Axioms: Invariant Interval + LightSpeed (c) as Universal Physical Constant lead to SR, although technically SR is itself the Minkowski-SpaceTime low-curvature:"flat" limiting-case of GR.

{ $c,\tau,m_o,\hbar,i$ } = {c: SpeedOfLight, τ: ProperTime, m_o: RestMass, $\hbar$: Dirac/PlanckReducedConstant($\hbar=h/2\pi$), i: ImaginaryNumber}: are all Empirically-Measured SR Lorentz-Invariant Physical and/or Mathematical Constants $i = +\sqrt{[-1]} = (0,1)_{complex\#}$

Standard SR 4-Vectors:		*Related by these SR Lorentz Invariants:*	
4-Position	$\mathbf{R} = (ct,\mathbf{r})$	$\in$ <Event> $\in$ <Time·Space>	$(\mathbf{R}\cdot\mathbf{R}) = (c\tau)^2 = (i\lvert\mathbf{r_o}\rvert)^2$
4-Velocity	$\mathbf{U} = \gamma(c,\mathbf{u})$	$= (\mathbf{U}\cdot\partial)\mathbf{R} = (d/d\tau)\mathbf{R} = d\mathbf{R}/d\tau$	$(\mathbf{U}\cdot\mathbf{U}) = (c)^2$
4-Momentum	$\mathbf{P} = (E/c,\mathbf{p})$	$= m_o\mathbf{U}$	$(\mathbf{P}\cdot\mathbf{P}) = (m_oc)^2$
4-WaveVector	$\mathbf{K} = (\omega/c,\mathbf{k})$	$= \mathbf{P}/\hbar$	$(\mathbf{K}\cdot\mathbf{K}) = (m_oc/\hbar)^2$
4-Gradient	$\partial = (\partial_t/c,-\nabla)$	$= -i\mathbf{K}$	$(\partial\cdot\partial) = (-im_oc/\hbar)^2 = -(m_oc/\hbar)^2$ = QM Relation → RQM → QM

KG Equation: $|v|<<c$

SR + Empirically Measured Physical Constants lead to RQM via the (KG) Klein-Gordon Quantum Eqn, and thence to QM via the low-velocity limit { $|\mathbf{v}| << c$ }, giving the Schrödinger Eqn. This fundamental KG Relation also leads to the other

Quantum Wave Equations:	RQM (massless, no rest-frame, Lorentzian) { $\lvert\mathbf{v}\rvert = c : m_o = 0$ }	RQM (with non-zero mass, Lorentzian) { $0 <= \lvert\mathbf{v}\rvert < c : m_o > 0$ }	QM (limit-case from RQM, Galilean) { $0 <= \lvert\mathbf{v}\rvert << c : m_o > 0$ }
spin=0 boson field = 4-Scalar:	Free Scalar Wave (Higgs)	Klein-Gordon (KG)	Schrödinger (regular QM)
spin=1/2 fermion field = 4-Spinor:	Weyl	Dirac (RQM w/ EM charge)	Pauli (QM w/ EM charge)
spin=1 boson field = 4-Vector:	Maxwell (EM photonic)	Proca	

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SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)

(+,-,-,-) SR → QM Physics

A Tensor Study of Physical 4-Vectors

SRQM Diagram: SRQM 4-Vectors and Lorentz Scalars / Physical Constants

4-Vector SRQM Interpretation of QM

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4-Displacement
$\Delta\mathbf{R}=(c\Delta t,\Delta\mathbf{r})$
$d\mathbf{R}=(cdt,d\mathbf{r})$

4-Position
$\mathbf{R}=(ct,\mathbf{r})\in$<Event>

Invariant Interval
$\mathbf{R}\cdot\mathbf{R}=(ct)^2-\mathbf{r}\cdot\mathbf{r}=(c\tau)^2$

$\partial\cdot\mathbf{R}=4$ SpaceTime Dimension

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

ProperTime Derivative

4-Acceleration
$\mathbf{A}=\gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$
$=d\mathbf{U}/d\tau$

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

4-UnitTemporal
$\bar{\mathbf{T}}=\gamma(1,\boldsymbol{\beta})$

$\bar{\mathbf{T}}\cdot\bar{\mathbf{T}}=+1$

Time:Space Orthogonal

$\bar{\mathbf{T}}\cdot\bar{\mathbf{S}}=0$

$\bar{\mathbf{S}}\cdot\bar{\mathbf{S}}=-1$

4-UnitSpatial
$\bar{\mathbf{S}}=\gamma_{\beta n}(\hat{\mathbf{n}}\cdot\boldsymbol{\beta},\hat{\mathbf{n}})_{\perp}$

Speed of Light c

$\mathbf{U}\cdot\mathbf{A}=\mathbf{U}\cdot\mathbf{U}'=0$

4-Velocity
$\mathbf{U}=\gamma(c,\mathbf{u})$
$=d\mathbf{R}/d\tau$

$\mathbf{U}\cdot\mathbf{U}=c^2$

Rest Number Density n_o

$\rho_{prob^o}=\chi^*\psi$
$=|\langle\chi|\psi\rangle|^2$

Born Probability Rule
Rest Prob Density

Rest Charge Density ρ_o

EM φ_o/c^2

Rest Scalar Potential

4-NumberFlux
$\mathbf{N}=(nc,\mathbf{n})=n(c,\mathbf{u})$

4-ProbCurrDensity
4-ProbabilityFlux
$\mathbf{J}_{prob}=(\rho_{prob}c,\mathbf{j}_{prob})$

EM Charge q

4-ChargeFlux
4-CurrentDensity
$\mathbf{J}=(\rho c,\mathbf{j})=\rho(c,\mathbf{u})$

4-Polarization
$\mathbf{E}=(\varepsilon^0,\boldsymbol{\varepsilon})=(\boldsymbol{\varepsilon}\cdot\boldsymbol{\beta},\boldsymbol{\varepsilon})$

4-Spin
$\mathbf{S}=(s^0,\mathbf{s})=(\mathbf{s}\cdot\boldsymbol{\beta},\mathbf{s})$

Spin is actually an outcome of Poincaré Invariance, not QM

$\mathbf{U}\cdot\mathbf{E}=0$
$\mathbf{U}\cdot\mathbf{S}=0$

Polarization:Spin is Rest Spatial

ω_o/c^2

4-WaveVector
$\mathbf{K}=(\omega/c,\mathbf{k})=(\omega/c,\omega\hat{\mathbf{n}}/v_{phase})$

$\{\omega_o=0\}\leftrightarrow\{\mathbf{K}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{K}$ is null$\}$

Wave Velocity
$v_{group}*v_{phase}=c^2$
Rest AngFrequency

E_o/ω_o $(\hbar)$

Einstein de Broglie
$\mathbf{P}=\hbar\mathbf{K}$

m_o E_o/c^2

$E=mc^2$
Rest Energy:Mass

4-Momentum
$\mathbf{P}=(mc,\mathbf{p})=(E/c,\mathbf{p})$

$\{m_o=0\}\leftrightarrow\{\mathbf{P}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{P}$ is null$\}$

4-EMVectorPotential
$\mathbf{A}=(\varphi/c,\mathbf{a})$

$\{\varphi_o=0\}\leftrightarrow\{\mathbf{A}\cdot\mathbf{U}=0\}\leftrightarrow\{\mathbf{A}$ is null$\}$

EM Charge q

4-EMPotentialMomentum
$\mathbf{Q}=(U/c,\mathbf{q})=q\mathbf{A}$

$\mathbf{U}\cdot\partial[..]$ $\gamma d/dt[..]$ $d/d\tau[..]$

ProperTime Derivative

4-Force
$\mathbf{F}=\gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}})$
$=d\mathbf{P}/d\tau$

Minkowski
$\partial[\mathbf{R}]=\partial^\mu[R^\nu]=\eta^{\mu\nu}$
Metric

Lorentz
$\partial_\nu[R^{\mu'}]=\Lambda^{\mu'}{}_\nu$
Transform

SpaceTime Dim
$Tr[\eta^{\mu\nu}]=4=\Lambda_{\mu\nu}\Lambda^{\mu\nu}$

Conservation of 4-TotalWaveVector Sum of Plane-Waves

$\sum_n[..]$

4-TotalWaveVector
$\mathbf{K}_T=(\omega_T/c,\mathbf{k}_T)$
$=-\partial[\Phi_{phase}]$

$..[\mathbf{K}_T\cdot\mathbf{R}]$
$..\int[\mathbf{K}_T\cdot d\mathbf{R}]$
$..[-\Phi_{phase}]$
(i)

Complex Plane-Waves
$\mathbf{K}_T=-\partial[\Phi]$, $\mathbf{K}=i\partial$

4-Gradient
$\partial=(\partial_t/c,-\nabla)$
$=(\partial_t/c,-\partial_x,-\partial_y,-\partial_z)$

Hamilton-Jacobi
$\mathbf{P}_T=-\partial[S]$

$..[\mathbf{P}_T\cdot\mathbf{R}]$
$..\int[\mathbf{P}_T\cdot d\mathbf{R}]$
$..[-S_{action}]$

E_{To}/ω_{To} $(\hbar)$

Einstein de Broglie
$\mathbf{P}_T=\hbar\mathbf{K}_T$

4-TotalMomentum
$\mathbf{P}_T=(E_T/c,\mathbf{p}_T)=(H/c,\mathbf{p}_T)$
$=-\partial[S_{action}]$

Conservation of 4-TotalMomentum Sum of Momenta

$\sum_n[..]$

Minimal Coupling
$\mathbf{P}+\mathbf{Q}$

$+$

4-MomentumIncField
$\mathbf{P}_f=(E_f/c,\mathbf{p}_f)=\mathbf{P}+\mathbf{Q}=\mathbf{P}+q\mathbf{A}$

SRQM Diagram

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $T^\mu{}_\nu$ or $T_\mu{}^\nu$
(0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu=\mathbf{V}=(v^0,\mathbf{v})$
SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu=(v_0,-\mathbf{v})$

SR 4-Scalar
(0,0)-Tensor S or S_o
Lorentz Scalar

Existing SR Rules
Quantum Principles

$Trace[T^{\mu\nu}]=\eta_{\mu\nu}T^{\mu\nu}=T^\mu{}_\mu=T$
$\mathbf{V}\cdot\mathbf{V}=V^\mu\eta_{\mu\nu}V^\nu=[(v^0)^2-\mathbf{v}\cdot\mathbf{v}]=(v^0{}_o)^2$
= Lorentz Scalar Invariant

Special Relativity → Quantum Mechanics

The SRQM Interpretation: Links

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

See also:

http://scirealm.org/SRQM.html (alt discussion)
http://scirealm.org/SRQM-RoadMap.html (main SRQM website)
http://scirealm.org/4Vectors.html (4-Vector study)
http://scirealm.org/SRQM-Tensors.html (Tensor & 4-Vector Calculator)
http://scirealm.org/SciCalculator.html (Complex-capable RPN Calculator)

or Google "SRQM"

http://scirealm.org/SRQM.pdf (this document: most current ver. at SciRealm.org)

The 4-Vector SRQM Interpretation

QM is derivable from SR!

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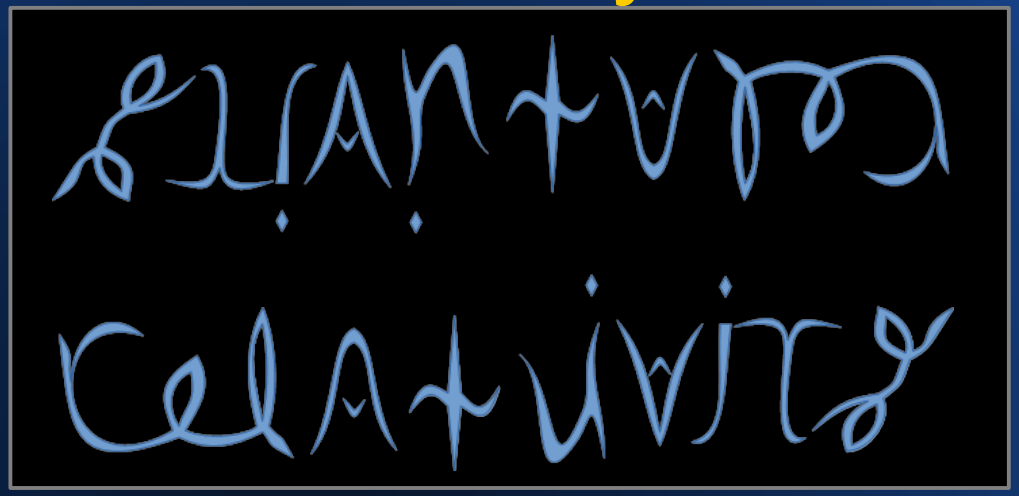
The SRQM or [SR→QM] Interpretation of Quantum Mechanics
A Tensor Study of Physical 4-Vectors

quantum
relativity



SRQM = SciRealm QM? A happy coincidence… :) Ambigrams

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
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SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)